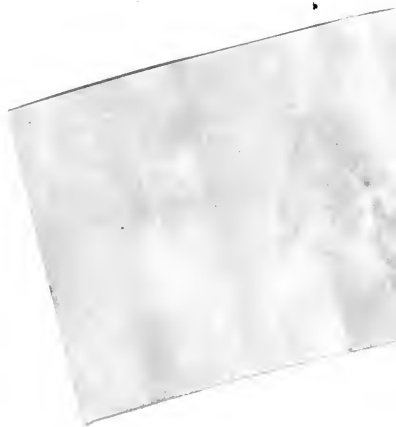


The Bancroft Library

University of California • Berkeley

The Theodore P. Hill Collection
of
Early American Mathematics Books





32612
A NEW AND

COMPLETE SYSTEM

OF

ARITHMETIC.

COMPOSED FOR THE

USE OF THE CITIZENS OF THE UNITED STATES.

BY NICOLAS PIKE, A. M.

QUID MUNUS REIPUBLICÆ MAJUS MELIUSVE AFFERRE POSSUMUS, QUAM
SI JUVENTUTEM DOCEMUS, ET BENE ERUDIMUS.

— E VARIIS SUMENDUM EST OPTIMUM. — *CICERO.*

SECOND EDITION, ENLARGED.

REVISED AND CORRECTED,
BY EBENEZER ADAMS, A. M.
PRECEPTOR OF LEICESTER ACADEMY.

PRINTED AT WORCESTER, MASSACHUSETTS,
AT THE PRESS OF
ISAIAH THOMAS,

By LEONARD WORCESTER, for said THOMAS.

Sold by THOMAS, SON & THOMAS, in WORCESTER; by THOMAS and ANDREWS,
D. WEST, E. LARKIN, S. HALL, J. WHITE, and J. WEST, in BOSTON, and
by all the BOOKSELLERS in the UNITED STATES.

RECOMMENDATIONS.



Dartmouth University, A. D. 1786.

AT the request of Nicolas Pike, Esq. we have inspected his System of Arithmetic, which we cheerfully recommend to the public, as easy, accurate and complete. And we apprehend there is no treatise of the kind extant, from which so great utility may arise to Schools.

B. WOODWARD, Math. and Phil. Prof.

JOHN SMITH, Prof. of the Learned Languages.

I do most sincerely concur in the preceding recommendation.

J. WHELOCK, President of the University.

Providence, State of Rhodeisland, 1785.

WHOEVER may have the perusal of this treatise on Arithmetic may naturally conclude I might have spared myself the trouble of giving it this recommendation, as the work will speak more for itself than the most elaborate recommendation from my pen can speak for it : But as I have always been much delighted with the contemplation of mathematical subjects, and at the same time fully sensible of the utility of a work of this nature, was willing to render every assistance in my power to bring it to the public view : And should the student read it with the same pleasure with which I perused the sheets before they went to the Press, am persuaded he will not fail of reaping that benefit from it which he may expect, or wish for, to satisfy his curiosity in a subject of this nature. The Author, in treating on numbers, has done it with so much perspicuity and singular address, that I am convinced the study thereof will become more a pleasure than a task.

The arrangement of the work, and the method by which he leads the *tyro* into the first principles of numbers, are novelties I have not met with in any book I have seen. Wingate, Hatton, Ward, Hill, and many other Authors, whose names might be adduced, if necessary, have claimed a considerable share of merit ; but when brought into a comparative point of view with this treatise, they are inadequate and defective. This volume contains, besides what is useful and necessary in the common affairs of life, a great fund for amusement and entertainment. The Mechanic will find in it much more than he may have occasion for ; the Lawyer, Merchant and Mathematician will find an ample field for the exercise of their genius ; and I am well assured it may be read to great advantage by students of every class, from the lowest School to the University. More than this need not be said by me, and to have said less, would be keeping back a tribute justly due to the merit of this Work.

BENJAMIN WEST.

University in Cambridge, A. D. 1786.

HAVING, by the desire of Nicolas Pike, Esq. inspected the following volume in manuscript, we beg leave to acquaint the public, that in our opinion it is a work well executed, and contains a complete system of Arithmetic. The rules are plain, and the demonstrations perspicuous and satisfactory ; and we esteem it the best calculated, of any single piece we have met with, to lead youth, by natural and easy gradations, into a methodical and thorough acquaintance with the science of figures. Persons of all descriptions may find in it every thing, respecting numbers, necessary to their business ; and not only so, but if they have a speculative turn and mathematical taste, may meet with much for their entertainment at a leisure hour.

We are happy to see so useful an American production, which, if it should meet with the encouragement it deserves, among the inhabitants of the United States, will save much money in the country, which would otherwise be sent to Europe, for publications of this kind.

We heartily recommend it to schools, and to the community at large, and wish that the industry and skill of the Author may be rewarded, for so beneficial a work, by meeting with the general approbation and encouragement of the public.

JOSEPH WILLARD, D. D. President of the University.

E. WIGGLESWORTH, S. T. P. Hollis.

S. WILLIAMS, L. L. D. Math. et Phil. Nat. Prof. Hollis.

Yale College, 1786.

UPON examining Mr. Pike's System of Arithmetic and Geometry in manuscript, I find it to be a work of such mathematical ingenuity, that I esteem myself honored in joining with the Rev. President Willard, and other learned gentlemen, in recommending it to the public as a production of genius, interspersed with originality in this part of learning, and as a book suitable to be taught in schools—of utility to the merchant, and well adapted even for the university instruction. I consider it of such merit, as that it will probably gain a very general reception and use throughout the republic of letters.

EZRA STILES, President.

Boston, 1786.

FROM the known character of the Gentlemen who have recommended Mr. Pike's System of Arithmetic, there can be no room to doubt, that it is a valuable performance ; and will be, if published, a very useful one. I therefore wish him success in its publication.

JAMES BOWDOIN.

P R E F A C E

TO THE FIRST EDITION.

IT may, perhaps, by some, be thought needless, when Authors are so multiplied, to attempt publishing any thing further on Arithmetic, as it may be imagined there can be nothing more than the repetition of a subject already exhausted. It is however the opinion of not a few, who are conspicuous for their knowledge, in the Mathematics, that the books, now in use among us, are generally deficient in the illustration and application of the rules ; of the truth of which, the general complaint among Schoolmasters is a strong confirmation. And not only so, but as the United States are now an independent nation, it was judged that a System might be calculated more suitable to our meridian, than those heretofore published.

Although I had sufficient reason to distrust my abilities for so arduous a task, yet not knowing any one who would take upon himself the trouble, and apprehending I could not render the public more essential service, than by an attempt to remove the difficulties complained of, with diffidence I devoted myself to the work.

I have availed myself of the best Authors which could be obtained, but have followed none particularly, except Bonnycastle's Method of Demonstration.

Although I have arranged the Work in such order as appeared to me the most regular and natural, the student is not obliged to pay a strict adherence to it ; but
may

may pass from one Rule to another, as his inclination, or opportunity for study, may require.

The Federal Coin, being purely decimal, most naturally falls in after Decimal Fractions.

I have given several methods of extracting the Cube Root, and am indebted to a learned friend, who declines having his name made public, for the investigation of two very concise Algebraic Theorems for the extraction of all Roots, and of a particular Theorem for the Surfsolid.

Among the Miscellaneous Questions, I have given some of a philosophical nature, as well with a view to inspire the pupil with a relish for philosophical studies, as to the usefulness of them in the common businesses of life.

The short introduction to Algebra, which is subjoined, was abstracted principally from Bonnycastle, and that of Conic Sections, from Emerson's Works.

Being sensible the following Treatise will stand or fall, according to its real merit or demerit, I submit it to the judgment of the *candid*.

With pleasure I embrace this opportunity, to express my gratitude to those learned Gentlemen, who have honored this Treatise with their approbation, as well as to such Gentlemen as have encouraged it by their subscriptions ; and to request the reader to excuse any errors he may meet with ; for although great pains have been taken in correcting, yet it is difficult to prevent errors from creeping into the press, and some may have escaped my own observation ; in either case, a hint from the candid will much oblige their

Most obedient,

And humble Servant,

THE AUTHOR.

P R E F A C E

TO THIS EDITION.

THE demand for the former Edition of Mr. PIKE'S New and Complete System of Arithmetic, has induced the Proprietor of that work to re-publish it.

The numerous Recommendations, by characters celebrated for their knowledge of the Mathematics, prefixed to this and the former Edition, and its introduction as a Classic into several of our Universities, speak its eulogy ; and the estimation in which it is now held is superior to any publication of the kind extant.

Explanatory of Decimal Calculation in the Federal Currency, the Rules which are adduced under this head, stamp an additional value on this Edition. Dollars and Cents are coming gradually into use ; but pounds, shillings and pence will continue to be in practice, and be the basis of many Arithmetical Questions, both in the United States and elsewhere.

Much attention has been given to the Revision and Correction of this valuable work, at the request of the Proprietor, and recommendation of the Author, by the ingenious Mr. Ebenezer Adams, Preceptor of Leicester Academy ; and, as far as his other engagements

ments would permit, the Author himself has aided in making the work as perfect as possible, and in the detection of many errors which escaped notice in the former, and which are corrected in this, edition, now presented to the public.

With respect, the Public's most obedient and humble servant,

ISALAH THOMAS.

Worcester, February, 1797.

C O N T E N T S.

	Page
NUMERATION	17
Simple Addition	20
—— Subtraction	22
—— Multiplication	23
—— Division	34
Supplement to Contraction in Multiplication	42
Tables in Compound Addition	42
Compound Addition	49
Problems resulting from the preceding Rules	56
Reduction	61
Vulgar Fractions	70
Decimal Fractions	85
Federal Money	96
Decimal Tables	99
Compound Multiplication	101
—— Division	106
Rules for reducing all the Coins, from Canada to Georgia ; also English, Irish and French Coins, and Spanish Dollars, each to the par of all the others	111
Duodecimals, or Cross Multiplication	124
Single Rule of Three Direct	126
The Method of making Taxes, in a Note	133
Single Rule of Three Direct in Vulgar Fractions	137
To find the Value of Gold in the currency of Newengland and Virginia	139
Single Rule of Three Direct in Decimals	143
Rule of Three Inverse	145
Abbreviation of Vulgar Fractions	148
Double Rule of Three	148
Conjoined Proportion	154
Arbitration of Exchanges	156
Single Fellowship	156
Double Fellowship, or Fellowship with Time	159
Fellowship by Decimals	161
Practice	162
—— by Decimals	190
Form of a Bill of Parcels	192
Tare and Tret	193
Involution	195
Evolution	196
Table of Powers	197

	Page
<i>Extraction of the Square Root</i>	198
<i>Application and Use of the Square Root</i>	201
<i>Extraction of the Cube Root</i>	204
<i>Application and Use of the Cube Root</i>	210
<i>Extraction of the Biquadrate Root</i>	211
----- of the Sur-solid, by Approximation	212
----- of the Roots of all Powers	213
<i>Proportion in General</i>	217
<i>Arithmetical Proportion</i>	218
----- Progression	220
<i>Geometrical Proportion</i>	235
----- Progression	237
<i>Simple Interest</i>	252
----- by Decimals	256
<i>Annuities, or Pensions in Arrears, at Simple Interest</i>	267
<i>Present Worth of Annuities, at Simple Interest</i>	268
<i>Discount</i>	269
----- by Decimals	273
<i>Barter</i>	274
<i>Loss and Gain</i>	277
<i>Equation of Payments</i>	285
----- by Decimals	287
<i>Commission, or Factorage</i>	288
<i>Brokerage</i>	289
<i>Policies of Insurance</i>	290
<i>Compound Interest</i>	296
----- by Decimals	298
<i>Discount at Compound Interest</i>	303
<i>Annuities, or Pensions in Arrears, at Compound Interest</i>	304
<i>Present Worth of Annuities at Compound Interest</i>	309
<i>Annuities in Reversion at Compound Interest</i>	313
<i>Purchasing Annuities forever, or Freehold Estates</i>	318
<i>Table, shewing the Amount of £1 from 1 year to 50</i>	322
----- shewing the the present worth of £1 from 1 year to 40	323
----- shewing the amount of £1 Annuity, &c.	324
----- shewing the Present Worth of £1 Annuity, &c.	325
----- shewing the Annuity, which £1 will purchase, &c.	326
<i>Circulating Decimals</i>	327
<i>Alligation Medial</i>	333
<i>Alligation Alternate</i>	333
<i>Single Position</i>	338
<i>Double Position</i>	340
<i>Permutations and Combinations</i>	343
<i>A short Method of reducing a Vulgar Fraction to a Decimal</i>	349
----- of finding the Duplicate, Triplicate, &c. Ratio	
of two Numbers, whose difference is small	349
<i>To estimate the Distance of Objects</i>	351

C O N T E N T S.

	Page
<i>To estimate the Height of Objects</i>	352
<i>Miscellaneous Questions</i>	352
— <i>Of Gravity</i>	361
— <i>Of the Fall of Bodies</i>	363
— <i>Of Pendulums</i>	366
— <i>Of the Lever, or Steelyard</i>	368
— <i>Of the Wheel and Axle</i>	369
— <i>Of the Screw</i>	369
— <i>Of the Specific Gravities of Bodies</i>	372
<i>Table of Specific Gravities</i>	373
<i>Use of the Barometer in measuring Heights</i>	377
<i>Table of the Weight of Money</i>	379
— <i>of Exchange</i>	380
<i>Ditto</i>	381
<i>Table of the Value of sundry Pieces in the several States</i>	382
— <i>of Commission</i>	383
— <i>of the net Proceeds, after the Commissions at 2½ and 5 per cent. are deducted</i>	384
<i>Table shewing the number of Days from any day in one month to any day in any other month</i>	385
<i>Table of the Measure of Length of the principal places in Europe compared with the American Yard</i>	386
<i>Table directing how to buy and sell by the Hundred weight</i>	387
<i>Comparison of the American Foot with the Feet of other Countries</i>	388
<i>Table to cast up Wages or Expenses for a year, at so much per day, week, or month</i>	389
<i>Table to find Wages, or Expenses for a month, week, or day, at so much per year</i>	389
<i>Table of Simple Interest, at 6 per cent. from 1s. to £1000, and from 1 day to a year, the intermediate months consisting of 30 days each</i>	390
<i>A Perpetual Almanac</i>	394
<i>Tables reducing Troy Weight to Avoirdupois, and the contrary</i>	395
<i>An Account of the Gregorian Calendar, or New Style</i>	396
<i>Chronological Problems</i>	396
<i>Problem 1. To find in which Century the last year is to be Leap year, and the contrary</i>	396
<i>Prob. 2. To find, with regard to any other Years, whether any Year be Leap year, or not</i>	397
<i>Prob. 3. To find the Dominical Letter according to the Julian Method</i>	397
<i>Prob. 4. To find the same according to the Gregorian Method</i>	398
<i>Prob. 5. To find the Prime, or Golden Number</i>	399
<i>Prob. 6. To find the Julian Epact</i>	399
<i>Prob. 7. To find the Gregorian Epact</i>	400
<i>To find the same forever</i>	401
<i>Prob.</i>	401

	Page
Prob. 8. To calculate the Moon's Age on any given Day	401
Prob. 9. To find the Times of the New and Full Moon, and first and last Quarters	402
Prob. 10. Having the time of the Moon's Southing given, to find her Age	403
Prob. 11. To find the Time of the Moon's Southing	403
Prob. 12. To find on what Day of the Week any given day in any month will fall	404
Prob. 13. To find the Cycle of the Sun	405
Table of the Dominical Letters according to the Cycle of the Sun	406
Prob. 14. To find the Year of the Dionysian Period	406
Prob. 15. To find the Year of Indiction	406
Prob. 16. To find the Julian Period	407
Prob. 17. To find the Cycle of the Sun, Golden Number and Indiction, for any current year	407
Prob. 18. Having the Cycle of the Sun, Golden Number, and Indiction, to find the Year of the Christian Æra	408
Prob. 19. To find the Time of High Water	408
Prob. 20. To find on what day Easter will happen	408
Table to find Easter from the year 1753 to 1999	410
The Use of Logarithms	411
Plain Geometry	412
Plain Rectangular Trigonometry	416
Plain Oblique Angular Trigonometry	421
Trigonometry applied to the Mensuration of Heights and Distances	425
Mensuration of Superficies and Solids	427
Section 1. Of Superficies	427
Article 1. To measure a Square	427
Art. 2. To measure a Parallelogram, or long Square	428
Art. 3. When the Breadth of a Superficies is given, to find how much in Length will make a Square Foot, Yard, &c.	428
Art. 4. To measure a Rhombus	428
Art. 5. To measure a Rhomboides	429
Art. 6. To measure a Triangle	429
To find the Sides of a Triangle by the Square Root	430
Art. 7. Another Method of finding the Area of a Triangle	431
Art. 8. To measure a Trapezium	432
Art. 9. To measure any irregular Figure	433
Art. 10. To measure a Trapezoid	433
Art. 11. To measure any regular Polygon	434
Art. 12. Having the Diameter of a Circle, to find the Circumference	435
Art. 13. Having the Circumference, to find the Diameter	437
Art. 14. To find the Area of a Circle	437
Art. 15. Having the Diameter, to find the Area, without finding the Circumference	438
	Art.

	Page
Art. 16. Having the Circumference, to find the Area without finding the Diameter	438
Art. 17. Having the Dimensions of any of the parts of a Circle, to find the Side of a Square, equal to the Circle	438
Art. 18. Having the Area of a Circle, to find the Diameter	439
Art. 19. Having the Area, to find the Circumference	439
Art. 20. Having the Side of a Square, to find the Diameter of a Circle, which shall be equal to the Square whose Side is given	440
Art. 21. Having the Side of a Square, to find the Circumference of a Circle equal to the given Square	440
Art. 22. Having the Diameter of a Circle, to find the Area of a Semicircle	440
Art. 23. Having the Segment of a Circle, to find the Length of the Arch Line	440
Art. 24. Having the Chord and Versed Sine of a Segment, to find the Diameter of a Circle	441
Art. 25. To measure a Sector	442
Art. 26. To measure the Segment of a Circle	443
Art. 27. To measure an Ellipsis	444
Directions for applying Superficies to Surveying	445
Section 2. Of Solids	445
Art. 28. To measure a Cube	445
Art. 29. To measure a Parallelopipedon	447
Having the Side of a square Solid, to find what Length will make a Foot solid	449
Art. 30. To measure a Cylinder	450
Having the diameter of a Cylinder given, to find what length will make a solid Foot	151
To find how much a round tree, which is equally thick from end to end, will hew to when made square	451
Art. 31. To measure a Prism	451
Art. 32. To measure a Pyramid, or Cone	452
Art. 33. To measure the Frustrum of a Pyramid or Cone	454
Art. 34. To measure a Sphere or Globe	456
Art. 35. To measure a Frustrum or Segment of a Globe	457
Art. 36. To measure the middle Zone of a Globe	458
Art. 37. To measure a Spheroid	458
Art. 38. To measure the middle Frustrum of a Spheroid	458
Art. 39. To measure a Segment, or Frustrum of a Spheroid	458
Art. 40. To measure a Parabolic Conoid	459
Art. 41. To measure the lower Frustrum of a Parabolic Conoid	459
Art. 42. To measure a Parabolic Spindle	459
Art. 43. To measure the middle Zone, or middle Frustrum of a Parabolic Spindle	459
Art. 44. To measure a Cylinderoid or Prismoid	460
Art. 45. To measure a solid Ring	460
Art.	

	Page
Art. 46. To measure the Solidity of any irregular Body, whose Dimensions cannot be taken	461
Of the five Regular Bodies	461
Art. 47. To measure a Tetraedron	462
Art. 48. To measure an Octaedron	462
Art. 49. To measure a Dodecaedron	462
Art. 50. To measure an Eicofiedron	462
Art. 51. To gauge a Cask	464
Art. 52. To gauge a Mash Tub	464
Art. 53. Having the Difference of Diameters, Height and con- tent of a Mash Tub, to find the Diameters, at Top and Bottom	465
Art. 54. To ullage a Cask, lying on one side, by the Gauging Rod	465
Art. 55. To find a Ship's Tonnage	465
The Proportions and Tonnage of Noah's Ark	467
Questions in Mensuration	467
Algebra	473
Conic Sections	507

EXPLANATION of the CHARACTERS made Use of in this TREATISE.

$=$ { THE sign of equality : As, $12 \text{ pence} = 1$ shilling, signifies that 12 pence are equal to 1 shilling ; and, in general, that whatever precedes it is equal to what follows.

$+$ { The sign of addition : As $5+5 = 10$, that is, 5 added to 5 is equal to 10. Read 5 plus 5, or 5 more 5 equal to 10.

$-$ { The sign of subtraction : As, $12-4 = 8$, that is, 12 lessened by 4 is equal to 8, or 4 from 12 and 8 remains. Read 12 minus 4, or 12 less 4 equal to 8.

\times { The sign of multiplication : As $6 \times 5 = 30$, that is, 6 multiplied by 5 is equal to 30. Read 6 into 5 equal to 30.

\div or $6)30($ { The sign of division : As, $30 \div 5 = 6$, that is, 30 divided by 5 is equal to 6. Read 30 by 5 equal to 6.

$\frac{875}{25}$ { Numbers placed fractionwise, do likewise denote division, the numerator or upper number being the dividend, and the denominator or lower number, the divisor, thus, $\frac{875}{25}$ is the same as $875 \div 25 = 35$.

$:$ $::$ $:$ { The sign of proportion, thus, $2 : 4 :: 8 : 16$; that is, As 2 is to 4 so is 8 to 16.

$\overline{9-2+6}=13$ { Shews that the difference between 2 and 9 added to 6 is equal to 13. Read 9 minus 2 plus 6 equal to 13. And that the line atop (called a *Vinculum*) connects all the numbers over which it is drawn.

$12-\overline{3+5}=4$ { Signifies that the sum of 3 and 5 taken from 12 leaves or is equal to 4.

$\overline{\quad}^2$ Signifies the second power, or square.

$\overline{\quad}^3$ Signifies the third power, or cube.

$\overline{\quad}^m$ { Signifies any power in general, as $\overline{6}^2 =$ square of 6 ; and $\overline{50}^3 =$ cube of 50, &c. thus m signifies either the square or cube, or any other power.

$\sqrt{\quad}$

$$\sqrt{}, \text{ or } \sqrt[n]{}$$

Prefixes to any number or quantity, signifies that the square root of that number is required. It likewise (as also the character for any other root) stands for the expression of the root of that number or quantity to which it is prefixed. As $\sqrt{36} = 6$, and $\sqrt{108+36} = 12$, or $36^{\frac{1}{2}} = 6$, &c.

$$\sqrt[3]{}, \text{ or } \sqrt[n]{}$$

Prefixes to any number, signifies that the cube root of that number is required, or expressed. As $\sqrt[3]{216} = 6$, and $\sqrt[3]{513+216} = 9$, &c.—or $216^{\frac{1}{3}} = 6$, &c.

$$\sqrt[n]{}, \text{ or } \sqrt[m]{}$$

Signifies any root in general. As $36^{\frac{1}{2}} =$ square root, $216^{\frac{1}{3}} =$ cube root, &c. Thus, $\frac{n}{m}$ signifies either the square root, cube root, or any other root whatever,

$$a b c d$$

When several letters are set together, they are supposed to be multiplied into each other; as those in the margin are the same as $a \times b \times c \times d$, and represent the continual product of quantities or numbers.

$$\frac{1}{a}$$

Is the reciprocal of a , and $\frac{a}{b}$ is the reciprocal of $\frac{b}{a}$.

If a be the root, then $a \times a = aa$ or a^2 is the square of a , and $a \times a \times a = aaa$ or a^3 is the cube of a , &c.

Note, The figure atop is called the index of the power.

It is usual to write shillings at the left hand of a stroke, and pence at the right; thus, $13\frac{4}{4}$ is thirteen shillings and four pence.

Note, The use of these characters must be perfectly understood by the pupil, as he may have occasion for them.



A

NEW AND COMPLETE SYSTEM OF ARITHMETICK.



RITHMETICK is the Art or Science of computing by numbers, and consists both in Theory and Practice.—The Theory considers the nature and quality of numbers, and demonstrates the reason of practical operations.—The Practice is, that which shews the method of working by numbers, so as to be most useful and expeditious for business, and is comprised under five principal or

fundamental Rules, viz. NOTATION or NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION ; the knowledge of which is so necessary, that, scarcely any thing in life, and nothing in trade, can be done without it.

NUMERATION

Teaches the different value of figures by their different places, and to read or write any sum or number by these ten characters. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.—0 is called a cypher, and all the rest are called figures or digits. The names and significations of these characters, and the origin or generation of the numbers they stand for, are as follow ; 0 nothing ; 1 one, or a single thing called an unit ; $1+1=2$, two ; $2+1=3$, three ; $3+1=4$, four ; $4+1=5$, five ; $5+1=6$, six ; $6+1=7$, seven ; $7+1=8$, eight ; $8+1=9$, nine ; $9+1=10$, ten, which has no single character ; and thus, by the continual addition of one, all numbers are generated.

2. Beside the simple value of figures, as above noted, they have, each, a local value, according to the following law ; viz. In a combination of figures, reckoning from right to left, the figure, in the first place, represents its primitive simple value ; that in

C

the

the second place, ten times its simple value, and so on; the value of the figure, in each succeeding place, being ten times the value of it, in that immediately preceding it.

3. The values of the places are estimated according to their order: The first is denominated the place of units; the second, tens; the third, hundreds, and so on, as in the table. Thus in the number—5293467: 7, in the first place, signifies only seven; 6, in the second place, signifies 6 tens, or sixty; 4, in the third place, four hundred; 3, in the fourth place, three thousand; 9, in the fifth place, ninety thousand; 2, in the sixth place, two hundred thousand; 5, in the seventh place, is five millions; and the whole, taken together, is read thus; five millions, two hundred and ninety three thousand, four hundred and sixty seven.

4. A cypher, though it is of no signification, itself, yet, it possesses a place, and, when set on the right hand of figures, in whole numbers, increases their value in the same tenfold proportion; thus, 9 signifies only nine; but, if a cypher is placed on its right hand, thus, 90, it then becomes ninety; and, if two cyphers be placed on its right, thus, 900, it is nine hundred; &c.

To enumerate any parcel of figures, observe the following Rule.

First, commit the words at the head of the Table, viz. units, tens, hundreds, &c. to memory; then, to the simple value of each figure, join the name of its place, beginning at the left hand, and reading towards the right.—*More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over such dot will, universally, have the name of thousands.—2. Place the figures 1, 2, 3, 4, &c. as indices, over the 2d, 3d, 4th, &c. period: These indices will then shew the number of times the millions are involved—the figure under 1, bearing the name of millions, that under 2, the name of billions (or millions of millions) that under 3, trillions (or millions of millions of millions.)

E X A M P L E.

Sextillions.		Quintilli.		Quatrill.		Trillions.		Billions.		Millions.		Units.
th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	c.x.t.c.x.u.
6		5		4		3		2		1		
9	13,208,000,341	6	20,057,219,356	8	09,379,120,406	1	29,763					
	Thousands		Thousands		Thousands		Thousands		Thousands		Thousands	

NOTE 1. Billions is substituted for millions of millions: Trillions, for millions of millions of millions: Quatrillions, for millions of millions of millions of millions.

Quintillions,

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. answer to millions so often involved as their indices respectively denote.

NOTE 2. The right hand figure of each half period has the place of units of that half period ; the middle one, that of tens, and the left hand one, that of hundreds.

The A P P L I C A T I O N.

Write down, in proper figures, the following numbers.

Fifteen.

Two hundred and seventy nine.

Three thousand, four hundred and three.

Thirty seven thousand, five hundred and sixty seven.

Four hundred, one thousand and twenty eight.

Nine millions, seventy two thousand and two hundred.

Fifty five millions, three hundred, nine thousand and nine.

Eight hundred millions, forty four thousand, and fifty five.

Two thousand, five hundred and forty three millions, four hundred and thirty one thousand, seven hundred and two.

Write down, in words at length, the following numbers.

8	437	709040	3476194	7584397647
17	3010	879066	84094007	49163189186
129	76506	4091875	690748591	500098400700

Notation by Roman Letters.

I. One.	XV. Fifteen.	CC. Two hundred.
II. Two.	XVI. Sixteen.	CCC. Three hundred.
III. Three.	XVII. Seventeen.	CCCC. Four hundred.
IV. Four.	XVIII. Eighteen.	D or I \overline{D} . Five hundred.
V. Five.	XIX. Nineteen.	DC. Six hundred.
VI. Six.	XX. Twenty.	DCC. Seven hundred,
VII. Seven.	XXX. Thirty.	DCCC. Eight hundred.
VIII. Eight.	XL. Forty.	DCCCC. Nine hundred.
IX. Nine.	L. Fifty.	M or C \overline{I} . One Thousand.
X. Ten.	LX. Sixty.	I \overline{D} . Five thousand.
XI. Eleven.	LXX. Seventy,	I \overline{D} 00. Fifty thousand.
XII. Twelve.	LXXX. Eighty.	I \overline{D} 000. Five hundred tho.
XIII. Thirteen.	XC. Ninety.	MDCCXCVI. One thous.
XIV. Fourteen.	C. Hundred.	seven hund. and ninety six.

A less literal number, placed after a greater, always augments the value of the greater ; if put before, it diminishes it. Thus, VI is 6 ; IV is 4 ; XI is 11 ; IX is 9, &c.

ADDITION

SIMPLE ADDITION.

ADDITION

Is the putting together of two or more numbers, or sums, to make them one total, or whole sum.

SIMPLE ADDITION

Is the adding of several integers or whole numbers together, which are all of one kind, or sort ; as 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or sum total, is 39 pounds.

R U L E.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units : After adding up every figure in that column, consider how many tens are contained in their sum, and, placing the excess under the units, carry so many, as you have tens, to the next column, of tens : Proceed in the same manner through every column, or row, and set down the whole amount of the last row.*

PROOF. Begin at the top of the sum, and reckon the figures downwards, in the same manner as they were added upwards, and, if it be right, this aggregate will be equal to the first. Or, cut off the upper line of figures, and find the amount of the rest ; then, if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

ADDITION

* This Rule, as well as the method of Proof, is founded on the known Axiom, "The whole is equal to the sum of all its parts." The method of placing the numbers, and carrying for the tens, is evident from the nature of notation ; for any other disposition of the numbers would alter their value ; and carrying one, for every ten, from an inferior to a superior column, is, evidently, right, because one unit in the latter case is equal to the value of ten units in the former.

Beside the method of proof, here given, there is another, by casting out the nines ; thus :

1. Add the figures in the upper row together, and find how many nines are contained in their sum.

2. Reject the nines, and set down the remainder, directly even with the figures, in the row.

3. Do the same with each of the given numbers, and set all the excesses of nines in a column, and find their sum ; then, if the excess of nines in this sum, found as before, is equal to the excess of nines in the sum total ; the question is supposed to be right.

EXAMPLE.

5738	5
9156	3
8471	2
5324	5
28689	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever ; viz. that any number, divided by 9, will leave the same remainder, as the sum of its figures, or digits, divided by 9 : Which may be thus demonstrated.

Demonstration. Let there be any number, as 5432 ; this, separated into its several parts, becomes $5000 + 400 + 30 + 2$; but $5000 = 5 \times 1000 = 5 \times 999 + 1 = 5 \times 999 + 5$. In like manner $400 = 4 \times 99 + 4$, and $30 = 3 \times 9 + 3$. Therefore, $5432 = 5 \times 999 + 5, + 4 \times 99 + 4, + 3 \times 9 + 3 + 2 = 5 \times 999 + 4 \times 99 + 3 \times 9 + 5 + 4 + 3 + 2$. And $\frac{5432}{9} = \frac{5 \times 999 + 4 \times 99 + 3 \times 9 + 5 + 4 + 3 + 2}{9}$; but $\frac{5 \times 999 + 4 \times 99}{9} + 3$

SIMPLE ADDITION.

21

ADDITION and SUBTRACTION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column, and the other atop, and in the common angle of meeting, or, at the right hand of the first, and under the second, you will find the sum—as, 5 and 8 is 13.

When you would subtract : Find the number to be subtracted in the left hand column, run your eye along to the right hand till you find the number from which it is taken, and right over it, atop, you will find the difference—as, 8, taken from 13, leaves 5.

1.	2.	3.	4.	5.	6.
£.	℥.	Cwt.	Miles.	Yards.	£.
1	12	123	1234	12345	987654321
2	34	456	5678	67890	123456789
3	56	789	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	69	67890	567879287
7	23	901	4713	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4567	2102865
4567890	4566	89093	4321234
5678209	333	654321	5682098
6789098	90	1234567	6543218

SUBTRACTION

+3X9 is divisible by 9; therefore, 5432, divided by 9, will leave the same remainder as 5+4+3+2, divided by 9; and the same will hold good of any other number whatever.

The same property belongs to the number 3 : However, this inconveniency attends this method, that, although the work will always prove right, when it is so ; it will not, always, be right, when it proves so ; I have, therefore, given this demonstration more for the sake of the curious, than for any real advantage.

M U L T I P L I C A T I O N

May be accounted the most serviceable Rule in Arithmetick. It teaches how to increase the greater of two numbers given, as often as there are units in the less; performs the work of many additions in the most compendious manner; brings numbers of great denominations into small, as pounds into shillings, pence or farthings, &c. and, by knowing the value of one thing, we find the value of many.

It consists of three parts.

1. The Multiplicand, or number given to be multiplied, and, commonly, the largest number.
2. The multiplier, or number to multiply by, commonly, the least number.
3. The Product is the result of the work, or the answer to the question.

SIMPLE MULTIPLICATION

Is the multiplying any two numbers together, without having regard to their signification; as 7 times 8 is 56, &c.

MULTIPLICATION and DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table, for Multiplication : Find your multiplier in the left hand column, and your multiplicand atop, and in the common angle of meeting, or against your multiplier, along at the

the greater, accordingly; which is only taking from one place, and adding as much to another, whereby the total is never changed: And, by this mean, the greater is resolved into such parts, as are, each, greater than, or equal to, the similar parts of the less; and the difference of the correspondent figures, taken together, will, evidently, make up the difference of the whole.

The truth of the method of proof is evident; for the difference of two numbers, added to the less, is, manifestly, equal to the greater.

24 SIMPLE MULTIPLICATION.

the right hand, and under your multiplicand, you will find the product, or answer.

To learn it, for Division : Find the divisor in the left hand column, and run your eye along the row to the right hand until you find the dividend; then, directly over the dividend, atop, you will find the quotient, shewing how often the divisor is contained in the dividend.

C A S E I.

When the multiplier is not more than 12, always placing the greatest number uppermost, set the multiplier underneath, units under units, &c. and begin as the Table directs, setting down the unit figure under units, and carrying the tens to the next place, in all respects as in simple addition.*

P R O O F.

Multiply the multiplier by the multiplicand.

E X A M P L E S.

1. 37934 2 ----- Prod. ----- 5. 67589 6 ----- -----	2. 769308 3 ----- ----- 6. 503764 7 ----- -----	3. 4980076 4 ----- ----- 7. 3918295 8 ----- -----	4. 763896 5 ----- ----- 8. 9164785 9 ----- -----
9. 4879567 10 ----- -----	10. 5864734 11 ----- -----	11. 8583478649 12 ----- -----	

C A S E II.

When the multiplier consists of more places than one, multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the first figure of each product exactly under its multiplier: Lastly, add these several products together,

* *Dem.* When the multiplier is a single digit, it is plain that we find the Product; for, by multiplying every figure, that is, every part of the multiplicand, we multiply the whole; and, the writing down the products, which are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens, to the product of the next place, is only gathering together the similar parts of the respective products, and is, therefore, the same, in effect, as though we wrote down the multiplicand as often as the multiplier expresses, and added them together; for the sum of every column is the product of the figures in the place of that column; and the products, collected together, are evidently equal to the whole required product.

SIMPLE MULTIPLICATION. 25

ther, in the same order as they stand, and their sum will be their total product.*

EXAMPLES.

<p>1.</p> $\begin{array}{r} 6357534 \\ 47 \\ \hline 44502738 \\ 25430136 \\ \hline \text{Prod. } 298804098 \end{array}$	<p>2.</p> $\begin{array}{r} 8324629 \\ 59 \\ \hline \end{array}$	<p>3.</p> $\begin{array}{r} 46293845 \\ 76 \\ \hline \end{array}$
<p>4.</p> $\begin{array}{r} 647906 \\ 4873 \\ \hline 3157245938 \end{array}$	<p>5.</p> $\begin{array}{r} 760483 \\ 9152 \\ \hline 6959940416 \end{array}$	<p>6.</p> $\begin{array}{r} 91867584 \\ 6875 \\ \hline 631589640000 \end{array}$

CASE

* If the multiplier be a number, made up of more than one figure ; after we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and, after the same manner, find the product of the multiplicand by the second figure of the multiplier ; but, as the figure, by which we are multiplying, stands in the place of tens, the product must be ten times its simple value ; and, therefore, the first figure in this product must be noted in the place of tens, or, which is the same, directly under the figure we are multiplying by. And, proceeding in the same manner with all the figures of the multiplier, separately, it is evident we shall multiply all the parts of the multiplicand by all the parts of the multiplier ; therefore, these several products, being added together, will be equal to the whole required product.

The reason of the method of proof, depends upon this proposition, that if two numbers are to be multiplied together, either of them may be made the multiplier or multiplicand; and the product will be the same.

A small attention to the nature of numbers will make this truth evident ; for $5 \times 9 = 45 = 9 \times 5$; and, in general, $2 \times 3 \times 4 \times 5 \times 6$, &c. $= 3 \times 2 \times 6 \times 5 \times 4$, &c. without any regard to the order of the terms ; and this is true of any number of factors whatever.

N. B. By factors are meant the multiplier and multiplicand.

The following examples are subjoined, to make the reason of the rule appear as clearly as possible.

$\begin{array}{r} 64753 \\ 5 \\ \hline 15 = 3 \times 5 \\ 25 = 50 \times 5 \\ 35 = 700 \times 5 \\ 20 = 4000 \times 5 \\ 30 = 60000 \times 5 \\ \hline 323765 = 64753 \times 5 \end{array}$	$\begin{array}{r} 237956 \\ 3728 \\ \hline 1903648 = 8 \text{ times the multiplicand.} \\ 475912 = 20 \text{ times ditto.} \\ 1665692 = 700 \text{ times ditto.} \\ 713868 = 3000 \text{ times ditto.} \\ \hline 887099968 = 3728 \text{ times ditto.} \end{array}$
---	---

Multiplication may also be proved, by casting out the nines ; but is liable to the inconvenience before mentioned.

C A S E III.

When the multiplier is a composite number, that is, when it is produced by the multiplication of any two numbers in the Table, multiply the multiplicand by one of those figures, first, and that product by the other : And the last product will be the total required.*

E X A M P L E S.

<p>1. Mult. 59375 by 35.</p> $ \begin{array}{r} 7 \times 5 = 35 \quad \underline{\hspace{1cm}} \\ 415625 \\ 5 \\ \hline 2078125 \end{array} $	<p>2. 39187 by 48.</p>	<p>3. 91632 by 56.</p>
<p>4. Mult. 91738 by 81.</p>	<p>5. 35963 by 72.</p>	<p>6. 847396 by 99.</p>
<p>7. Mult. 38462 by 108.</p>	<p>8. 749357 by 121.</p>	<p>9. 9043278 by 144.</p>

C A S E IV.

When there are cyphers on the right hand of, either the multiplicand, or multiplier, or both, neglect those cyphers ; then place the significant figures under one another, and multiply by them only ; add them together, as before directed, and place to the right hand as many cyphers as there are in both the factors.

E X A M P L E S.

It may likewise be, very naturally, proved by division ; for the product, being divided by either of the factors, will, evidently, give the other ; and it might not be amiss for the pupil to be taught division, at the same time with multiplication ; as it not only serves for proof ; but also gives him a readier knowledge of them both. But it would have been contrary to good method to have given this rule in the text, because the pupil is supposed, as yet, to be unacquainted with division.

* The reason of this method is obvious : For any number, multiplied by the component parts of another number, must give the same product, as though it were multiplied by that number at once : Thus, in example first, 5 times the product of 7, multiplied into the given number, makes 35 times that given number, as plainly, as 5 times 7 makes 35.

SIMPLE MULTIPLICATION.

27

EXAMPLES.

1. 67910 5600 <hr/> Prod. 380296000	2. 956700 320 <hr/> 306144000	3. 930137000 9500 <hr/> 8836301500000
4. 359260 7364 <hr/> Prod. 2645590640	5. 8196000 59180 <hr/> 485039280000	6. 623000 589000 <hr/> 366947000000

CASE V.

When there are cyphers between the significant figures of the multiplier, omit multiplying by them, and place the first figure of each product of the significant figures, exactly under that figure by which you multiply; lastly, add them together, and their sum will be the total product.

EXAMPLES.

1. 5397 8009 <hr/> 48573 43176 <hr/> Prod. 43224573	2. 85630 7005 <hr/> 599838150	3. 48976850 400030 <hr/> 19592209305500
--	--	--

CASE VI.

To multiply by 10, 100, 1000, &c. set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.

EXAMPLES.

1. 57935 10 <hr/> Prod. 579350	2. 84935 100 <hr/>	3. 613975 1000 <hr/>	4. 8473965 10000 <hr/>
---	-----------------------------	-------------------------------	---------------------------------

CASE

C A S E VII.

To multiply by 99, 999, &c. in one line ; place as many dots at the right hand of the multiplicand, as there are figures of 9 in your multiplier, which dots suppose to be cyphers ; then, beginning with the right hand dot, subtract the *given* multiplicand from the *new* one, and the remainder will be the total product.*

E X A M P L E S.

1.	2.	3.
6473..	857389...	5384976....
99	999	9999
<hr/>	<hr/>	<hr/>
640827	856531611	53844375024

That these examples may appear as clear as possible, I will illustrate them by giving another.

Mult. 371967 ...	{ According to the rule, }	371967 ... Minuend.
by 999	{ it will stand thus, }	371967 Subtrah.
<hr/>		<hr/>
371595033		371595033 Rem. or to-
		tal Prod.

C A S E VIII.

To multiply by 13, 14, 15, &c. to 19 *inclusively*, at one multiplication.

R U L E I.

Multiply the multiplicand by the unit figure of the multiplier, and add to the product of each multiplication that figure which stands next on the right hand of that which you multiplied, and, to the last figure in the multiplicand, add what you carry.

E X A M P L E S.

* Here it may easily be seen that, if you multiply any sum by 9, the product will be but 9 tenths of the product of the same sum, multiplied by 10 ; and as the annexing of a dot or cypher, to the right hand of the multiplicand, supposes it to be increased tenfold ; therefore, subtracting the given multiplicand from the tenfold multiplicand, it is evident that the remainder will be ninefold the said given multiplicand, equal to the product of the same by 9 ; and the same will hold true of any number of nines.

Note, When the multiplicand has a fraction added to it, as one fourth, one half, &c. add such a part of the multiplier as the fraction makes, to the last product : But when such fraction belongs to the multiplier, add to the last product such a part of the multiplicand as the fraction denotes.

E X A M P L E S.

1. 65497 13	2. 84916 14	3. 19345 15	4. 7398 16	5. 9108 17	6. 6173 18	7. 54937 19
Prod. 851461						

In example first, I say, 3 times 7 is 21 ; I put down 1 and carry 2 ; saying, 3 times 9 is 27, and 2 that I carried, makes 29, and 7, the right hand figure to 9, makes 36 ; I put down 6, and carry 3 ; then 3 times 4 is 12, and 3, which I carried, makes 15, and 9, its right hand figure, makes 24 ; therefore, I put down 4, and carry 2 ; saying, 3 times 5 is fifteen, and 2 which I carried, makes 17, and 4, its right hand figure, makes 21 ; I, therefore, put down 1 and carry 2 ; saying, 3 times 6 is 18, and 2 which I carried, makes 20, and 5, its right hand figure, makes 25 ; I, therefore, set down 5 and carry 2 ; lastly, the 2 which I carry, and 6, the last figure in the multiplicand, make 8, which gives the total product.

R U L E II.

To multiply by 13, 14, 15, &c. to 19 : Place your multiplier at the right of the multiplicand, with the sign of multiplication between them, and multiply with the unit figure, only, of the multiplier, removing the product one figure to the right hand of the multiplicand ; then add all together, and their sum will be the total product.

E X A M P L E S.

1. 75964 × 13 227892	2. 7598 × 14	3. 76013 × 15	4. 8196 × 16	5. 3179 × 17
Prod. 987532				

C A S E IX.

To multiply by 111, 112, 113, &c. to 119, so as to have the product in one line : Multiply the multiplicand by the unit figure, only, of the multiplier, and add to each multiplication the two figures which stand next on the right hand to that which you multiplied, and to the two last figures, separately, add what you carry.

E X A M P L E S.

EXAMPLES.

1. 52976 111	2. 58975 112	3. 89193 113	4. 76435 114	5. 781572 115
Prod.				
6. 43958 116	7. 647358 117	8. 499789 118	9. 9417 119	
Prod.			1120623	

In the last example I say, 9 times 7 is 63 ; I set down 3 and carry 6 ; then, 9 times 1 is 9, and 6 I carried makes 15, and 7, its right hand figure, makes 22 ; I set down 2, and carry 2 ; then, 9 times 4 is 36, and 2 I carry is 38, and its right hand figures, 1 and 7, make 46 ; I set down 6 and carry 4 ; then, 9 times 9 is 81, and 4 I carried, is 85, and 4 and 1, its right hand figures, make 90 ; I put down 0, and carry 9, which I add to 9 and 4, the 2 last figures, and they make 22 ; I then put down 2 and carry 2 ; lastly, this 2 and 9 make 11, which I set down, and the product is complete.

CASE X.

To multiply by 101, 102, 103, &c. to 109, so as to have the product in one line :

RULE I.

Multiply the multiplicand by the unit figure of the multiplier, and add to it the next right hand figure, but one, to that which you multiplied, remembering to add to the two last figures in your multiplicand, separately, what you carry.

EXAMPLES.

1. 57691 101	2. 89726 102	3. 75964 103	4. 84975 104	5. 635918 105
Prod.				
6. 64791 106	7. 58493 107	8. 8493294 108	9. 6479 109	
Prod.			706211	

SIMPLE MULTIPLICATION. 31

In the last example I say, 9 times 9 is 81 ; I set down 1 and carry 8 ; then, 9 times 7 is 63, and 8 I carry is 71 ; I set down 1 and carry 7 ; then, 9 times 4 is 36, and 7 I carry is 43, and 9, its right hand figure but one, makes 52 ; I set down 2 and carry 5 ; saying, 9 times 6 is 54, and 5 I carry is 59, and 7, its next right hand figure but one, makes 66 ; I set down 6 and carry 6, which I add to 4, the last figure but one in the multiplicand, and it makes 10 ; I set down 0, and carry 1, which I add to 6, the last figure, and it makes 7, which I set down, and I have the whole product.

R U L E II.

To multiply by 101, 102, 103, &c. to 109 : Multiply by the right hand figure, only, of the multiplier, removing the product two figures to the right hand of the multiplicand : Add all together, and the sum will be the total product.

E X A M P L E S.

<p>1. $\begin{array}{r} 64795 \times 101 \\ 64795 \\ \hline \end{array}$ </p> <p>Product, $\underline{6544295}$</p>	<p>2. $\begin{array}{r} 79164 \times 102 \\ \hline \end{array}$ </p>	<p>3. $\begin{array}{r} 37598 \times 103 \\ \hline \end{array}$ </p>
<p>4. $\begin{array}{r} 73967 \times 104 \\ \hline \end{array}$ </p> <p>Prod. $\underline{\hspace{2cm}}$</p>	<p>5. $\begin{array}{r} 84973 \times 105 \\ \hline \end{array}$ </p>	<p>6. $\begin{array}{r} 748794 \times 106 \\ \hline \end{array}$ </p>
<p>7. $\begin{array}{r} 87958 \times 107 \\ \hline \end{array}$ </p>	<p>8. $\begin{array}{r} 395867 \times 108 \\ \hline \end{array}$ </p>	<p>9. $\begin{array}{r} 5916379 \times 109 \\ \hline \end{array}$ </p>

C A S E XI.

To multiply by 21, 31, 41, &c. to 91, in one line :

R U L E.

First, bring down the unit figure of the multiplicand, which will, always, be the unit figure of the product, then multiply every figure of the multiplicand by the ten's figure of the multiplier, and to each product add the figure, which stands next on the left hand to that which you multiplied.

E X A M P L E S.

SIMPLE MULTIPLICATION.

E X A M P L E S.

1. 6493587 21	2. 935846 31	3. 3584956 41	4. 716298543 51
Prod. _____	_____	_____	_____
5. 537984 61	6. 326478 71	7. 938467 81	8. 4793 91
Prod. _____	_____	_____	436163

In the last example, I first bring down the unit figure 3, of the multiplicand, for the unit figure of the product; and, then, I say, 9 times 3 is 27, and 9, its left hand figure, makes 36; I set down 6 and carry 3; then, 9 times 9 is 81, and 3 I carry is 84, and 7, its left hand figure, is 91; I set down 1 and carry 9; saying, 9 times 7 is 63, and 9 I carry is 72, and 4, its left hand figure, is 76; I set down 6 and carry 7; lastly, 9 times 4 is 36, and 7 I carry makes 43, which I set down, and have the product complete.

R U L E II.

To multiply by 21, 31, 41, &c. to 91: Multiply by the ten's figure, only, of the multiplier, and set the unit figure of the product under the place of tens; add them all together, and their sum will be the total product.

E X A M P L E S.

1. 73918 × 21 147836	2. 56934 × 31	3. 86789 × 41	4. 759846 × 51
_____	_____	_____	_____
1552278	_____	_____	_____
5. 37954 × 61	6. 73958 × 71	7. 84937 × 81	8. 54937 × 91
_____	_____	_____	_____
_____	_____	_____	_____

C A S E XII.

To multiply by 22, 23, 24, &c. to 29; so as to have the product in one line: Multiply every figure of the multiplicand by the unit figure

figure of the multiplier, and add to each product *twice* that figure which stands next on the right hand of that figure you multiplied; and to *twice* the last figure of the multiplicand, add what you carry.

E X A M P L E S.

$$\begin{array}{r} 1. \\ 649378 \\ 22 \end{array}$$

Prod.

$$\begin{array}{r} 2. \\ 46795 \\ 23 \end{array}$$

$$\begin{array}{r} 3. \\ 64839 \\ 24 \end{array}$$

$$\begin{array}{r} 4. \\ 83964723 \\ 25 \end{array}$$

$$\begin{array}{r} 5. \\ 73758 \\ 26 \end{array}$$

$$\begin{array}{r} 6. \\ 91357 \\ 27 \end{array}$$

$$\begin{array}{r} 7. \\ 849358 \\ 28 \end{array}$$

$$\begin{array}{r} 8. \\ 7657 \\ 29 \end{array}$$

Prod.

222053

In the last example, I say, 9 times 7 is 63; I set down 3 and carry 6; then, 9 times 5 is 45, and 6 I carry is 51, and 7, its right hand figure, added *twice*, makes 65; I, therefore, set down 5 and carry 6, saying 9 times 6 is 54, and 6 I carry is 60, and 5, its right hand figure, added *twice*, makes 70; I set down 0 and carry 7; then, 9 times 7 is 63, and 7 I carry is 70, and 6, its right hand figure, added *twice*, makes 82; I set down 2 and carry 8; lastly, I add this 8 to *twice* the last multiplicand figure, and they make 22, and the whole product stands as above.

To multiply any number, viz. whole or decimal, by any number, giving only the Product.

Put down the Product figure of the first figure of the multiplicand by the *first* of the multiplier. To the product of the *second* of the multiplicand by the *first* of the multiplier, add the number to be carried, and the product of the *first* of the multiplicand by the *second* of the multiplier; then, carrying for the tens in the sum, put down the rest. To the product of the *third* of the multiplicand by the *first* of the multiplier add the number to be carried, and the product of the *second* of the multiplicand by the *second* of the multiplier, also the product of the *first* of the multiplicand by the *third* of the multiplier, carry the tens, and put down the rest, and so proceed till you have multiplied the *highest* of the multiplicand by the *lowest* of the multiplier. Then multiply the *highest* of the multiplicand by the *second* of the multiplier: Add the number to be carried, and the product of the *last but one* of the multiplicand by the *third* of the multiplier, and the product of the *last but two* of the multiplicand by the *fourth* of the multiplier, &c. Then to the product of the *last but one* of the multiplicand by the *fourth* of the multiplier; and so proceed till you have multiplied the *last* of the multiplicand by the *last* of the multiplier, which finishes the work.

E

Example.

SIMPLE DIVISION.

*Example.**Explanation.*

Mult. 5321415
 By 2354
 Prod. 12526610910

$$\begin{array}{r}
 5 \times 4 = 20 \\
 1 \times 4 + 2 + 5 \times 5 = 31 \\
 4 \times 4 + 3 + 1 \times 5 + 5 \times 3 = 39 \\
 1 \times 4 + 3 + 4 \times 5 + 1 \times 3 + 5 \times 2 = 40 \\
 2 \times 4 + 4 + 1 \times 5 + 4 \times 3 + 1 \times 2 = 31 \\
 3 \times 4 + 3 + 2 \times 5 + 1 \times 3 + 4 \times 2 = 36 \\
 5 \times 4 + 3 + 3 \times 5 + 2 \times 3 + 1 \times 2 = 46 \\
 5 \times 5 + 4 + 3 \times 3 + 2 \times 2 = 42 \\
 5 \times 3 + 4 + 3 \times 2 = 25 \\
 5 \times 2 + 2 = 12
 \end{array}$$

D I V I S I O N

Teaches to separate any number, or quantity given, into any number of parts assigned ; or to find how often one number is contained in another ; or from any two numbers given, to find a third, which shall consist of so many units, as the one of those given numbers is comprehended in the other ; and is a concise way of performing several Subtractions.

There are four principal parts to be noticed in Division, viz.

1. The Dividend, or number given to be divided.
2. The Divisor, or number given to divide by.
3. The Quotient, or answer to the question, which shews how often the divisor is contained in the dividend.
4. The Remainder (which is always less than the Divisor, and of the same name with the Dividend) is very uncertain, as there is sometimes a Remainder, and sometimes none.

Division is both simple and compound.

P R O O F.

Multiply the divisor and quotient together, and add the remainder, if there be any, to the product ; if the work be right, that sum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values : As, 56, divided by 8, produces 7 in the quotient : That is, 8 is contained 7 times in 56.*

CASE

* According to the rule, we resolve the dividend into parts, and find, by trial, the number of times the divisor is contained in each of those parts ; and the only thing which remains to be proved, is, that the several figures of the quotient, taken as one number, according to the order, in which they are placed, is the true quotient of the whole dividend by the divisor ; which may be thus demonstrated.

Dem.

CASE I.

RULE. First, seek how many times the divisor is contained in a competent number of the first figures of the dividend; when found, place the figure in the quotient; multiply the divisor by this quotient figure, place the product under the left hand figures of the dividend; then subtract it therefrom, and bring down the next

Dem. The complete value of the first part of the dividend, is, by the nature of notation, 10, 100, 1000, &c. times the simple value of what is taken in the operation; accordingly, as there are 1, 2, or 3, &c. figures standing before it; and, consequently, the true value of the quotient figure, belonging to that part of the dividend, is also 10, 100, 1000, &c. times its simple value; but the true value of the quotient figure, belonging to that part of the dividend, found by the rule, is also 10, 100, 1000, &c. times its simple value; for there are as many figures set before it, as the number of remaining figures in the dividend; therefore, this first quotient figure, taken in its complete value from the place it stands in, is the true quotient of the divisor in the complete value of the first part of the dividend. For the same reason, all the rest of the figures of the quotient, taken according to their places, are, each, the true quotient of the divisor, in the complete value of the several parts of the dividend belonging to each; because, as the first figure, on the right hand of each succeeding part of the dividend, has a less number of figures standing before it, so ought their quotients to have; and so they are actually ordered; consequently, taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the sum of the true quotients of all the several parts of the dividend; and is, therefore, the true quotient of the whole dividend by the divisor.

That no obscurity may remain, in this demonstration, it is illustrated by the following example.

	Divisor 25	74503	Dividend	
1st. part of the dividend is	=	74000		
	25 × 2000	= 50000	— —	2000 the 1st. quotient.
		—————		
1st. remainder	=	24000		
	add	500		
		—————		
2d. part of the dividend	=	24500		
	25 × 900	= 22500	— —	900 the 2d quotient.
		—————		
2d. remainder	=	2000		
	add	00		
		—————		
3d. part of the dividend	=	2000		
	25 × 80	= 2000	— —	80 the 3d quotient.
		—————		
		00		
	add	3		
		—————		
4th. part of the dividend	=	3		
	25 × 0	= 0	— —	0 the 4th quotient.
		—————		
Last remainder	=	3	— —	2980 = Sum of all the quo-

tients, or, the Answer.

Explan. It is evident the dividend is resolved into these parts, 74000 + 500 + 00 + 3; for the first part of the dividend is considered only as 74; but yet it is, truly, 74000; and therefore its quotient, instead of 2, is 2000, and the remainder 24000; and so of the rest; as may be seen in the operation.

When there is no remainder to a division, the quotient is the absolute and perfect answer to the question; but where there is a remainder, it may be observed, that

next figure of the dividend to the right hand of the remainder : If when you have brought down a figure to the remainder, it is still less than the divisor, a cypher must be placed in the quotient, and

that it goes so much towards another time as it approaches the divisor ; thus, if the remainder be half the divisor, it will go half of a time more, and so on ; in order, therefore, to complete the quotient, put the last remainder to the end of it, above a line, and the divisor below it.

It is sometimes difficult to find how often the divisor may be had in the numbers of the several steps of the operation : The best way will be to find how often the first figure of the divisor may be had in the first, or two first figures of the dividend, and the answer, made less by one or two, is, generally, the figure wanted ; but if, after subtracting the product of the divisor and quotient from the dividend, the remainder be equal to, or exceed the divisor, the quotient figure must be increased accordingly.

The reason of the method of proof is plain ; for, since the quotient is the number of times the dividend contains the divisor, the product of the quotient and divisor, must, evidently, be equal to the dividend.

There are several other methods made use of to prove division ; as follow, viz.

R U L E I.

Subtract the remainder from the dividend ; divide this number by the quotient, and the quotient, found by this division, will be equal to the former divisor, when the work is right.

R U L E II.

Add the remainder and all the products of the several quotient figures multiplied by the divisor together, according to the order in which they stand in the work, and the sum, when the work is right, will be equal to the dividend.

Here, the numbers to be added are the products of the divisor by every figure of the quotient, separately ; and each, by its place, possesses its complete value ; therefore, the sum of the parts, together with the remainder, must be equal to the whole. I will illustrate the whole by an example proved according to the several different methods.

$$\begin{array}{r} 79 \overline{) 987654321} (12501953 \\ 79^* \qquad \qquad \qquad 79 + 34 \text{ remainder.} \end{array}$$

$$\begin{array}{r} 197 \\ 158^* \\ \hline \end{array}$$

$$\begin{array}{r} 112517577 \\ 87513671 \\ + 34 \\ \hline \end{array}$$

$$\begin{array}{r} .396 \\ .395^* \\ \hline \end{array}$$

$$987654321 \text{ Proof by multiplication.}$$

$$\begin{array}{r} . . . 154 \\ . . . 79^* \\ \hline \end{array}$$

$$\begin{array}{r} 987654321 \\ - 34 \\ \hline \end{array}$$

$$\begin{array}{r} 753 \\ 711^* \\ \hline \end{array}$$

$$12501953 \overline{) 987654321} 79 \text{ Proof by division.}$$

$$87513671$$

$$\begin{array}{r} 422 \\ 395^* \\ \hline \end{array}$$

$$\begin{array}{r} 112517577 \\ 112517577 \\ \hline \end{array}$$

$$\begin{array}{r} 271 \\ 237^* \\ \hline 34^* \\ \hline \end{array}$$

$$987654321 \text{ Proof by addition.}$$

We need only to refer to the example ; except for the proof of addition ; where, it may be remarked, that the Asterisks shew the numbers to be added, and the dotted lines their order.

and another figure be brought down; after which, you must seek, multiply and subtract, till you have brought down every figure of the dividend.

EXAMPLES.

1.
Divisor. Dividend. Quotient.

3)175817(58605

15

25

24

18

18

17

15

2 Rem.

Proof

58605 Quotient

× 3 Divisor + 2

175817

In this example, I find that 3, the divisor, cannot be contained in the first figure of the dividend; therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend, &c. I then subtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz. 5; then, I inquire how often the divisor 3, is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when, finding I cannot have the divisor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the first.

Observe, that, in multiplying by 3, I add in the 2.

2.
29)153598(5296
145

85

58

279

261

188

174

14

3.
6493)91876375(14150
6493

26946

25972

9743

6493

32507

32465

425

4.
28)503775(

5.
35)197184(

6.
85)994466(

7.
236)8798567(

8.
3479)483956795(

9.
5679)19647394(

EXAMPLES,

SIMPLE DIVISION.

EXAMPLES.

$$\begin{array}{r} 10. \\ 38473 \overline{)119184693} \end{array} \quad \begin{array}{r} 11. \\ 641976 \overline{)9187642959} \end{array}$$

$$\begin{array}{r} 12. \\ 5823789 \overline{)791822376496} \end{array}$$

$$\begin{array}{r} 13. \\ 123456789 \overline{)121932631112635269} \end{array}$$

CASE II.

When there is one cypher, or more, at the right hand of the divisor, it or they must be cut off; also, cut off the same number of figures from the dividend, and then proceed as in Case first: But the figures which were cut off from the dividend must be placed at the right hand of the remainder.*

EXAMPLES.

$$\begin{array}{r} 1. \\ 65 \overline{)003794326} \overline{)7558374} \\ 325 \end{array}$$

$$\begin{array}{r} 2. \\ 5193 \overline{)0008937643} \overline{)893} \end{array}$$

$$\begin{array}{r} 544 \\ 520 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \\ 917 \overline{)047658} \overline{)3} \end{array}$$

$$\begin{array}{r} 243 \\ 195 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \\ 875 \overline{)00091764789430} \overline{)000} \end{array}$$

$$\begin{array}{r} 482 \\ 455 \\ \hline \end{array}$$

$$\begin{array}{r} 276 \\ 260 \\ \hline \end{array}$$

1675 Rem.

$$\begin{array}{r} 5. \\ \text{Quot. Rem.} \\ 1 \overline{)09584} \overline{)6} \end{array}$$

$$\begin{array}{r} 6. \\ \text{Quot. Rem.} \\ 1 \overline{)0076495} \overline{)80} \end{array}$$

$$\begin{array}{r} 7. \\ \text{Quot. Rem.} \\ 1 \overline{)00093751839} \overline{)462} \end{array}$$

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divisor, your work is done; those figures, cut off at the right hand, are the remainder, and those on the left, the quotient, as above.

CASE

* The reason of this contraction is easy to conceive; for the cutting off the same figures from each, is the same as dividing each of them by 10, 100, 1000, &c. and it is evident, that as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in the like part of the dividend; this method is only to avoid a needless repetition of cyphers, which would happen in the common way.

C A S E III.

To perform division without setting down the multiplication : First, seek how often the divisor may be contained, as before directed ; place the figure in the quotient, and multiply it by the divisor ; subtract the unit figure of the multiplication from the dividend, and, if you are obliged to borrow in subtracting, you must add one extraordinary to the next multiplication, and proceed as before.*

E X A M P L E S.

$$\begin{array}{r}
 1. \\
 756 \overline{) 647395} (856 \\
 \underline{4259} \\
 4795 \\
 \underline{259}
 \end{array}$$

$$\begin{array}{r}
 2. \\
 8197 \overline{) 9167846} (
 \end{array}$$

$$\begin{array}{r}
 3. \\
 293 \overline{) 5017846} (
 \end{array}$$

$$\begin{array}{r}
 4. \\
 5840 \overline{) 8976439} (
 \end{array}$$

$$\begin{array}{r}
 5. \\
 75000 \overline{) 9840000} (
 \end{array}$$

In the first example I find the first quotient figure to be 8 ; then, I say, 8 times 6 is 48 ; subtract the unit figure 8 from the 3 in the dividend, and 5 remains ; then, as I was obliged to borrow 1 in subtracting, I carry 5, that is, 4 tens in 48, and one ten I borrowed, make 5 tens ; saying, 8 times 5 is 40, and 5 I carried, makes 45 ; then, 5 from 7 and 2 remains ; I now carry only 4, as I borrowed none in subtracting ; next, I say 8 times 7 is 56, and 4 I carry, is 60, which, subtracted from 64, leaves 4 ; I now bring down the next figure, and proceed, in the same manner, through the whole.

C A S E IV.

Short division is, when the divisor does not exceed 12.

R U L E.

First, seek how often the divisor can be had in the first figure, or figures, of the dividend ; which, when found, place in the quotient ;

* The reason of this rule is the same as that of the general rule, page 34.

SIMPLE DIVISION.

quotient ; then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left hand figures of your dividend, and the units, which remain, must be accounted so many tens, which you must suppose to stand at the left hand of the next figure in the dividend, and to be reckoned with it ; then, seek how often you can have your divisor in those two figures ; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Divisor. Dividend.	2.	3.	4.	5.
2)71935	3)51903	5)633795	6)8471937	7)193847
Quot. 35967—1				
6.	7.	8.	9.	
8)5437846	9)45963784	11)91843756	12)1196437847536	

CASE V.

When the divisor is such a number, that any two, or more, figures in the Table, being multiplied together, will produce it, divide the given dividend by one of those figures ; the quotient, thence arising, by the other, and so on ; and the last quotient will be the answer.*

EXAMPLES.

* This follows from the contraction in case 3d. of simple multiplication, of which it is only the reverse ; for the fourth part of the half of any thing is evidently the same as the eighth part of the whole ; and so of any other number.

As the learner, at present, is supposed to be unacquainted with the nature of fractions, and as the quotient is incomplete without the remainder ; I shall here give a rule for finding the true remainder, without having recourse to fractions.

RULE.

Multiply the quotient by the divisor : Subtract the product from the dividend, and the result will be the true remainder.

The Rule, which is most commonly made use of, when the divisor is a composite number, is

RULE II.

Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder ; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder ; and so on, till you have gone through all the divisors and remainders, to the first.

EXAMPLES.

EXAMPLES.

<i>1st. method.</i> 9)196473 <hr/> 8)21830 <hr/> Quot. 2728—57	<i>2d. method.</i> 8)196473 <hr/> 9)24559 <hr/> Quot. 2728—57	<i>3d. method..</i> 72)196473(2728 Quot. 144 <hr/> 524 504 <hr/> 207 144 <hr/> 633 576 <hr/>
---	--	---

57 Remainder.

I have wrought the above question three ways, that the learner may understand the method of finding the true remainder, according to this case. In the *first*, in dividing by 9, 3 remains, and, by 8, 6 remains; which, being the last remainder, I multiply it by the first divisor 9, and add in the first remainder 3, and they make 57, the true remainder. In the *second* method, dividing by 8, 1 remains, and by 9, 7 remains; I, therefore, multiply 7, the last remainder, by 8, adding in the 1, and they make 57, as before. The *third* method is selfevident, and shews that the other remainders are true.

2. 36)79638	3. 25)197835	4. 84)93975	5. 54)93738764
6. 221)75328939	7. 132)38473692	8. 144)891376429732	Supplement

EXAMPLE.

6)85397 divided by 150

*5)14232—5

5)2846—2

569—1

Ans. 569 $\frac{47}{150}$

1 the last remainder.
multiply by 5 the last divisor but one.

5
add 2 the second remainder.

7
multiply by 6 the first divisor.

42
add 5 the first remainder.

47 the true remainder.

To explain this rule from the example, we may observe, that every unit in the first quotient may be looked upon as containing 6 of the units in the given dividend; consequently, every unit, which remains, will contain the same; therefore, this remainder must be multiplied by 6, to find the units it contains of the given dividend. Again, each unit in the next quotient will contain 5 of the preceding ones, or 30 of the first, that is, 6 times 5; therefore, what remains must be multiplied by 30, or, which

Supplement to Contractions in Multiplication.

1. The shortest method of multiplication, when the multiplier is in any even part of 100, 1000, &c. is by division: For if the multiplicand be increased by a number of cyphers equal to the places in the multiplier, and a part of that product taken for the same proportion, which the multiplier bears to 1, and the same number of cyphers annexed to it, the quotient will be the true product.

1. Multiply 39756 into 125.

$125 = \frac{1}{8}$ of 1000, wherefore,

$\begin{array}{r} 8)39756000 \\ \hline \end{array}$

4969500 Product.

2. Multiply 57638 by $33\frac{1}{3}$.

$33\frac{1}{3} = \frac{1}{3}$ of 100, therefore,

$\begin{array}{r} 3)5763800 \\ \hline \end{array}$

1921266 $\frac{2}{3}$ Product.

3. Multiply 91378 by $333\frac{1}{3}$.

$333\frac{1}{3} = \frac{1}{3}$ of 1000, therefore,

$\begin{array}{r} 3)91378000 \\ \hline \end{array}$

30459333 $\frac{1}{3}$ Product.

2. From the aforementioned property peculiar to the digit 9, it follows, that whatever other digit, with any number of cyphers annexed, is divided by it, the quotient will consist, wholly, of such digits, and so many 9ths of an unit over; hence the following method of multiplying by repetends of any of the digits.

1.
645 by 8888.

$\begin{array}{r} 80000 \\ \hline \end{array}$

$\begin{array}{r} 9)51600000 \\ \hline \end{array}$

5733333

Subtract 573

Product. 5732760

2.
5394 by 66666.

$\begin{array}{r} 600000 \\ \hline \end{array}$

$\begin{array}{r} 9)3236400000 \\ \hline \end{array}$

359600000

Subt. 3596

Prod. 359596404

3.
3798 by 444

$\begin{array}{r} 4000 \\ \hline \end{array}$

$\begin{array}{r} 9)15192000 \\ \hline \end{array}$

1688000

Subt. 1688

Prod. 1686312

TABLES in COMPOUND ADDITION.

1. M O N E Y.*

Note, 4 Farthings

12 Pence

20 Shillings

} make one

Penny,

Shilling,

Pound,

marked,

qrs. d.

s.

£.

Farthings

which is the same thing, by 6 and 5 continually: Now, this is the same as the Rule; for, instead of finding the remainders, separately, they are reduced from the bottom, upwards, step by step, to one another, and the remaining units, of the same class, taken as they occur.

* As the Federal Money depends on the principles of decimal fractions, no tables will be inserted here. The pupil is referred to that head for a knowledge of it.

Farthings,

4 = 1 Penny.

48 = 12 = 1 Shilling.

960 = 240 = 20 = 1 Pound.

P E N C E T A B L E S.

d.	s.	d.	d.	s.	d.	s.	d.	s.	d.
20 =	1	8	120 =	10	0	2 =	24	12 =	144
30 =	2	6	130 =	10	10	3 =	36	13 =	156
40 =	3	4	140 =	11	8	4 =	48	14 =	168
50 =	4	2	150 =	12	6	5 =	60	15 =	180
60 =	5	0	160 =	13	4	6 =	72	16 =	192
70 =	5	10	170 =	14	2	7 =	84	17 =	204
80 =	6	8	180 =	15	0	8 =	96	18 =	216
90 =	7	6	190 =	15	10	9 =	108	19 =	228
100 =	8	4	200 =	16	8	10 =	120	20 =	240
110 =	9	2				11 =	132		

2. T R O Y W E I G H T.*

24 Grains }
 20 Pennyweights } make 1 { Pennyweight, *marked* grs. pwt.
 12 Ounces } { Ounce, oz.
 { Pound, lb

Grains,

24 = 1 Pennyweight.

480 = 20 = 1 Ounce.

5760 = 240 = 12 = 1 Pound.

3. A V O I R D U P O I S W E I G H T.†

16 Drams }
 16 Ounces } make 1 { Ounce, *marked* dr. oz.
 28 Pounds } { Pound, lb
 4 Quarters } { Quarter of a hundred wt. qr.
 20 Hundred wt. } { Hundred wt. or 112 pounds, Cwt.
 { Ton, T.
 Drams.

* By this weight are weighed Gold, Silver, Jewels, Eleſtuaries, and all Liquors.

An ounce of gold is divided into 24 parts, called carats, and an ounce of ſilver, into 20 parts, called pennyweights; therefore, to diſtinguiſh fineneſs of metals, ſuch gold as will abide the fire without loſs, is accounted 24 carats fine: If it loſe 2 carats in trial, it is called 22 carats fine, &c.

A pound of ſilver, which loſes nothing in trial, is 12 ounces fine; but, if it loſe 3 pennyweights, it is 11oz. 17pwt. fine, &c.

Alloy is ſome baſe metal with which gold or ſilver is mixed to abate its fineneſs. 22 carats of gold, and 2 carats of copper, are eſteemed the true ſtandard for gold coin in England, the alloy being one eleventh of the fine gold: And 11oz. 2pwt. of fine ſilver, melted with 18pwt. of copper, make the true ſtandard for ſilver coin.

NOTE. 175 Troy ounces are preciſely equal to 192 Avoirdupois ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1 lb Troy = 5760 grains, and 1 lb Avoirdupois = 7000 grains.

† By Avoirdupois are weighed all coarſe and droſſy goods, grocery and chandlery wares; bread, and all metals, except gold and ſilver.

A barrel of pork weighs 220lb. A barrel of beef, 220lb. A quintal of fiſh, 1 Cwt. Avoirdupois. 12 particular things make 1 dozen; 12 dozen 1 groſs, and 144 dozen 1 great groſs. 20 particular things make 1 ſcore.

A Firkin

Drams,			
16 =	1	Ounce.	
256 =	16 =	1 Pound.	
7168 =	448 =	28 =	1 Quarter.
28672 =	1792 =	112 =	4 = 1 Hund. wt.
573440 =	35840 =	2240 =	80 = 20 = 1 Ton.

4. APOTHECARIES' WEIGHT.*

20 Grains	} make 1 {	Scruple,	marked gr. Θ
3 Scruples		Dram,	3
8 Drams		Ounce,	16
12 Ounces		Pound,	lb

Grains,

20 =	1	Scruple.
60 =	3 =	1 Dram.
480 =	24 =	8 = 1 Ounce.
5760 =	288 =	96 = 12 = 1 Pound.

5. CLOTH MEASURE.†

2 Inches, and one fourth	} make 1 {	Nail, marked in. na.
4 Nails, or 9 Inches		Quarter of a yd. qr.
4 Quarters of a yard, or 36 Inches		Yard, yd.
3 Quarters of a yard, or 27 Inches		Ell Flemish, E. Fl.
5 Quarters of a yard, or 45 Inches		Ell English, E. E.
6 Quarters of a yard, or 54 Inches		Ell French, E. Fr.
4 Quarters, 1 Inch & one 5th, or }		Ell Scotch, E. Sc.
37 Inches and one fifth	}	Spanish Var.
3 Quarters and two thirds		

Nails, 4 = 1 Quarter.

16 =	4 =	1 Yard.
12 =	3 =	1 Flemish Ell.
20 =	5 =	1 English Ell.
24 =	6 =	1 French Ell.

6. LONG

A Firkin of Foreign Butter	lb	A Stone of Iron, Shot, } lb
----- Soap	56	or horseman's weight, } 14
A Barrel of ----- Anchovies	94	----- Butcher's Meat, 8
----- Soap	30	A gallon of Train Oil 7 1/2
----- Raisins	256	A Tod is - - - 28
A Punch. of ----- Prunes	112	A Weigh - - - 182
A Fother of ----- Lead	1120	A Sack - - - 364
	19 1/2 Cwt.	A Last - - - 4368

* All the weights now used by Apothecaries, above grains, are Avoirdupois.

The Apothecaries' pound and ounce, and the pound and ounce Troy are the same, only differently divided and subdivided.

† All Scotch and Irish linens are bought by the English or American yard, which is the same, and all Dutch linens by the Ell Flemish: But are all sold in America by the American yard; though the Dutch linens are sold in England by the Ell English, and the Scotch and Irish linens, as in America.

The Scotch allow one English yard in every score yards.

6. LONG MEASURE.*

3 Barley corns	}	make 1	Inch,	marked bar. in.	
12 Inches			Foot,	feet.	
3 Feet			Yard,	yd.	
5½ Yards, or 16½ feet			Rod, Perch, or Pole,	pol.	
40 Poles			Furlong,	fur.	
8 Furlongs			Mile,	mile.	
69½ Statute miles, <i>nearly</i> ,	}		{ Degree of a great Circle,		deg.
360 Degrees			{ A great Circle of the Earth.		

Or in Measuring Distances.

7 ⁹² / ₁₀₀ Inches	}	make 1	Link.
25 Links			Pole.
100 Links			Chain.
10 Chains			Furlong.
8 Furlongs			Mile.

Bar. corns, 3 = 1 inch.

36 =	12 =	1 Foot.
108 =	36 =	3 = 1 Yard.
594 =	198 =	16½ = 5½ = 1 Pole.
23760 =	7960 =	660 = 220 = 40 = 1 Furlong.
190080 =	63360 =	5280 = 1760 = 320 = 8 = 1 M.

Inches, 7⁹²/₁₀₀ = 1 Link,

198 =	25 =	1 Pole or Perch.
792 =	100 =	4 = 1 Chain.
7920 =	1000 =	40 = 10 = 1 Furlong.
63360 =	8000 =	320 = 80 = 8 = 1 Mile.

7. TIME†

60 Seconds	}	make 1	Minute,	marked s. m.
60 Minutes			Hour,	h.
24 Hours			Day,	d.
7 Days			Week,	w.
4 Weeks			Month,	mo.
13 Months, 1 day & 6 hours			Julian year,	yr.
			Seconds	

* The use of Long Measure is to measure the distance of places, or any other thing, where length is considered without regard to breadth.

NOTE. 60 geometrical miles make a degree. 4 inches a hand. 5 feet a geometrical pace. 6 points make 1 line, 12 lines an inch, 12 inches a foot, and 6 feet one French toise, or fathom, equal to 6 feet 4 inches, 8,812,875 lines, English measure. 1 English foot equal to 11 inches, 31154 lines French. 66 feet, or 4 poles, make a Gunter's chain. 3 miles make a league.

† By the Calendar, the year is divided in the following manner.

Thirty days hath September, April, June and November;

February twenty eight alone, and all the rest have thirty one.

When you can divide the year of our Lord by 4, without any remainder, it is then Bissextile, or Leap Year, in which February has 29 days.

Seconds, 60 = 1 Minute.

3600 = 60 = 1 Hour.

86400 = 1440 = 24 = 1 Day.

604800 = 10080 = 168 = 7 = 1 Week.

2419200 = 40320 = 672 = 28 = 4 = 1 Month.

31557600 = 525960 = 8766 = 365 $\begin{smallmatrix} d. & h. & w. & d. & h. \\ 6 & 52 & 1 & 6 & \end{smallmatrix}$ = 1 Julian year.*

31558154 = 525969 = 8766 = 365 $\begin{smallmatrix} d. & h. & m. & s. \\ 6 & 9 & 14 & \end{smallmatrix}$ = 1 Periodical year.†

31556987 = 525948 = 8765 = 365 $\begin{smallmatrix} d. & h. & m. & s. \\ 5 & 48 & 57 & \end{smallmatrix}$ = 1 Tropical year.‡

8. M O T I O N.

60 Seconds	} make 1 {	Prime minute, marked " ' 0
60 Minutes		Degree,
30 Degrees		Sign, s
12 Signs, or 360 degrees		The whole great circle of the Zodiac.

Seconds, 60 = 1 Minute.

3600 = 60 = 1 Degree.

108000 = 1800 = 30 = 1 Sign.

1296000 = 21600 = 360 = 12 = Zodiac.

9. LAND or SQUARE MEASURE.

144 Inches	} make 1 {	Square foot.
9 Feet		— Yard.
30 $\frac{1}{4}$ Yards, or }		— Pole.
272 $\frac{1}{4}$ Feet		— Rood.
40 Poles		— Acre.
4 Roods, or 160 Rods, } or 4840 yards }		— Mile.

640 Acres

Inches, 144 = 1 Foot.

1296 = 9 = 1 Yard.

39204 = 272 $\frac{1}{4}$ = 30 $\frac{1}{4}$ = 1 Pole.

1568160 = 10890 = 1210 = 40 = 1 Rood.

6272640 = 43560 = 4840 = 160 = 4 = 1 Acre.

4014489600 = 27878400 = 3097600 = 102400 = 2560 = 640 = 1 Mile.

10. SOLID

* The civil solar year of 365 days being short of the true by 5h. 48m. 57f. occasioned the beginning of the year to run forwards through the seasons nearly one day in four years. On this account, Julius Cæsar ordained that one day should be added to February, every fourth year, by causing the 24th day to be reckoned twice; and because this 24th day was the sixth (sextilis) before the kalends of March, there were, in this year, two of these sextiles, which gave the name of Bissextile to this year, which, being thus corrected, was from thence called the Julian year.

† A just and equal measure of the year is called the periodical year, as being the time of the earth's period about the sun; in departing from any fixed point in the heavens, and returning to the same again.

‡ The several points of the Ecliptic having a retrograde, or backward motion, the Equinox will, as it were, meet the sun; by which mean the sun will arrive at the Equinox, or first point of Aries, before his revolution is completed, and this space of time is called the tropical year.

|| The Zodiac is a great circle of the sphere, containing the 12 signs, through which the sun passes.

10. SOLID MEASURE.*

1728 Inches	}	make 1	{	Foot,
27 Feet				Yard.
40 Feet of round Timber, or 50 feet of hewn Timber			{	Ton or Load.
128 Solid Feet, i. e. 8 in length, 4 in breadth, and 4 in height,				Cord of Wood.

11. WINE MEASURE.†

2 Pints	}	make one	{	Quart,	marked	pts.	qts.
4 Quarts				Gallon,		gal.	
10 Gallons				Anchor of Brandy,		anc.	
18 Gallons				Runlet,		run.	
31½ Gallons				Half an Hoghead,		½ hhd.	
42 Gallons				Tierce,		tier.	
63 Gallons				Hoghead,		hhd.	
2 Hogheads				Pipe or Butt,		P. or B.	
2 Pipes				Tun,		Tun.	

Cubic Inches.

287	=	1 Pint.							
57½	=	2	=	1 Quart.					
231	=	8	=	4	=	1 Gallon.			
9702	=	336	=	168	=	42	=	1 Tierce.	
14553	=	504	=	252	=	63	=	1½	= 1 Hoghead.
19404	=	672	=	336	=	84	=	2	= 1½ = 1 Puncheon.
29106	=	1008	=	504	=	126	=	3	= 2 = 1½ = 1 Pipe.
58212	=	2016	=	1008	=	252	=	6	= 4 = 3 = 2 = 1 Tun.

12. ALE or BEER MEASURE.‡

2 Pints	}	make 1	{	Quart,	marked	pts.	qts.
4 Quarts				Gallon,		gal.	
8 Gallons				Firkin of Ale in Lond. A. fir.			
8½ Gallons				Firkin of Ale or Beer,			
9 Gallons				Firkin of Beer in Lond. B. fir.			
2 Firkins				Kilderkin,		kil.	
2 Kilderkins				Barrel,		bar.	
1½ Barrel, or 54 Gallons				Hoghead of Beer,		hhd.	
2 Barrels	}		{	Puncheon,		pun.	
3 Barrels, or 2 Hogheads				Butt,		butt.	

BEER.

* By Solid Measure are measured all things that have length, breadth and depth.

† All Brandies, Spirits, Perry, Cyder, Mead, Vinegar, Honey, and Oil, are measured by Wine Measure: Honey is, commonly, sold by the pound Avoirdupois.

‡ Milk is sold by the Beer quart.

A barrel of Mackerel, and other barrelled fish, by an act of this Commonwealth, is to contain not less than 30 gallons.

In England, a barrel of Salmon or Eels is 42 gallons, and a barrel of Herring, 32 gallons. The gallon appointed to be used for measuring all kinds of Liquids in Ireland, is two hundred and seventeen Cubic inches, and six tenths.

BEER.

Cubic Inches.

$$35\frac{1}{4} = 1 \text{ Pint.}$$
$$70\frac{1}{2} = 2 = 1 \text{ Quart.}$$

282 = 8 = 4 = 1 Gallon.

2538 = 72 = 36 = 9 = 1 Firkin.

5076 = 144 = 72 = 18 = 2 = 1 Kilderkin.

10152 = 288 = 144 = 36 = 4 = 2 = 1 Barrel.

$1522^8 = 432 = 216 = 54 = 6 = 3 = 1\frac{1}{2} = 1$ Hogthead.

$$20304 = 576 = 288 = 72 = 8 = 4 = 2 = 1 \text{ Puncheon.}$$
$$30456 = 864 = 432 = 108 = 12 = 6 = 3 = 2 = 1 \text{ Butt.}$$

ALB.

Cubic Inches.

$$35\frac{1}{4} = 1 \text{ Pint.}$$
$$70\frac{1}{2} = 2 = 1 \text{ Quart.}$$

282 = 8 = 4 = 1 Gallon.

$2256 = 64 = 32 = 8 = 1$ Firkin.

$$45^{12} = 12^8 = 64 = 16 = 2 = 1 \text{ Kilderkin.}$$

$90^{24} = 256 = 128 = 32 = 4 = 2 = 1$ Barrel.

$13536 = 3^8 4 = 192 = 4^8 = 6 = 3 = 1\frac{1}{2} = 1$ Hoghead.

13. D R Y M E A S U R E.*

2 Pints	} make one {	Quart,	marked pts.	qts.
2 Quarts		Pottle,		pot.
2 Pottles		Gallon,		gal.
2 Gallons		Peck,		pk.
4 Pecks		Bushel,		bu.
2 Bushels		Strike,		str.
2 Strikes		Coom,		co.
2 Cooms		Quarter,		qr.
4 Quarters		Chaldron,		ch.
4½ Quarters		Chaldron in London.		
5 Quarters		Wey,		wey.
2 Weys		Last,		last.

Cubic Inches.

$$268\frac{4}{5} = 1 \text{ Gallon.}$$

537 $\frac{3}{5}$ = 2 = 1 Peck.

$$3150 \frac{2}{5} = \cdot 8 = 4 = 1 \text{ Bushel.}$$
$$43^{00\frac{4}{5}} = 16 = 8 = 2 = 1 \text{ Strike.}$$
$$86_{01\frac{3}{5}} = 3^2 = 16 = 4 = 2 = 1 \text{ Coom.}$$
$$17203\frac{1}{4} = 64 = 32 = 8 = 4 = 2 = 1 \text{ Quarter.}$$
$$860_{16}^9 = 3^{20} = 160 = 40 = 20 = 10 = 5 = 1 \text{ Wey.}$$
$$172032 = 640 = 320 = 80 = 40 = 20 = 10 = 2 = 1 \text{ Last.}$$

COMPOUND

* This measure is applied to all dry goods, as Corn, Seed, Fruit, Roots, Salt, Sand, Oysters and Coals.

A Winchester bushel is $18\frac{1}{2}$ inches diameter, and 8 inches deep.

COMPOUND ADDITION

Is the adding of several numbers together, having different denominations, as Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

R U L E.*

1. Place the numbers so that those of the same denomination may stand directly under each other.

2. Add the first Column or denomination together as in whole numbers ; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the column added, and carry the quotient to the next superior denomination, continuing the same to the last, which add as in simple addition.

1. M O N E Y.

E X A M P L E S.

1.	2.	3.	4.
£. s. d.	£. s. d. qr.	£. s. d. qr.	£. s. d. qr.
9 16 10	47 17 6 2	847 11 11 3	915 10 10 2
7 10 9	3 9 10 3	491 19 6 1	64 8 9 1
0 18 6	75 13 9 1	59 6 10 0	5 16 11 3
5 11 11	4 11 11 0	747 16 1 2	419 2 10 2
6 0 8	0 16 8 2	849 12 11 3	491 19 11 3
5 9 10	17 6 2 1	741 17 8 2	762 17 6 1
<hr/>	<hr/>	<hr/>	<hr/>
5.	6.	7.	8.
£. s. d. qr.	£. s. d. qr.	£. s. d. qr.	£. s. d. qr.
479 11 11 2	764 13 10 2	7 17 10 3	584 19 10 3
64 17 8 3	43 9 8 1	60 6 8 0	765 14 8 1
912 16 10 0	59 17 11 2	7 15 11 2	91 17 10 2
497 5 4 2	817 16 9 3	18 19 9 3	18 19 6 3
69 10 11 3	762 19 10 1	91 16 8 2	847 13 8 2
917 6 9 2	419 17 6 2	918 17 10 3	918 17 11 0
<hr/>	<hr/>	<hr/>	<hr/>

F R E N C H M O N E Y.

Note, 12 Deniers, or Pence }
 20 Sols, or Shillings } make 1 { Sol or Shilling.
 3 Livres, or Pounds }
 6 Livres, or Pounds } { Livre, or Pound.
 { Crown of exchange.
 { Real Crown, or ecu d'argent.
 Cr.

* The reason of this Rule is evident from what has been said in Simple Addition : For, in addition of money, as 1, in the pence, is equal to 4 in the farthings ; 1, in the shillings, to 12 in the pence ; and 1, in the pounds, to 20 in the shillings ; therefore, carrying as directed, is the arranging the money, arising from each column, properly, in the scale of denominations ; and this reasoning will hold good in the addition of compound numbers, of any denomination whatever.

COMPOUND ADDITION.

Cr. of exc. liv. sols. den.

976	2	17	10
379	1	12	6
491	1	0	11
592	2	15	8

Ecu d'ar. liv. sols. den.

567	5	13	11
389	1	19	6
548	4	17	10
632	3	11	9

DUTCH MONEY.

Note. 8 Phennings
2 Groats, or 16 Phennings
20 Stivers } make 1 { Groat.
Stiver.
Guilder or Florin.

A l s o,

12 Groats, or 6 Stivers
20 Schillings, or 6 Guilders } make 1 { Schilling.
Pound.

Guild.	stiv.	gr.	ph.
197	17	1	7
348	12	0	6
491	13	1	3
749	19	1	4

Guild.	stiv.	ph.
549	19	14
317	16	12
859	13	8
467	10	15

£.	sch.	gro.
357	18	11
508	12	6
497	13	10
618	17	8

2. TROY WEIGHT.

℥	oz.	pwt.	gr.
767	10	17	22
39	6	9	17
417	11	16	18
935	9	17	19
478	10	17	22
387	9	16	15

℥	oz.	pwt.	gr.
649	11	19	20
32	9	6	5
841	10	11	19
473	9	17	23
764	11	8	9
165	6	10	19

℥	oz.	pwt.	gr.
859	9	15	20
437	10	17	22
640	11	6	0
738	9	12	18
49	0	16	17
584	10	0	9

3. AVOIRDUPOIS WEIGHT.

℥	oz.	dr.
19	13	12
21	9	6
4	15	15
22	10	5
18	13	12
6	11	10

Cwt.	qrs.	℥
17	3	19
18	1	27
9	2	9
14	3	16
12	0	6
15	2	0

T. Cwt.	qrs.	℥
59	13	2 17
6	17	1 21
45	11	3 25
57	16	2 19
75	17	3 17
6	19	0 26

T. Cwt.	qrs.	℥	oz.	dr.
91	17	2 25	13	15
19	9	0 17	10	12
14	13	2 0	9	11
47	11	3 19	14	0
69	0	1 0	0	12
77	19	3 27	15	11

4. APOTHECARIES'

COMPOUND ADDITION.

51

4. APOTHECARIES' WEIGHT.

1.	2.	3.	4.
3 3 3 gr.	3 3 3 gr.	℥ 3 3 3 gr.	℥ 3 3 3 gr.
9 1 17	10 7 2 19	12 11 6 1 15	5 9 3 2 13
3 2 19	6 3 0 12	4 9 1 0 12	4 8 6 0 19
6 1 17	7 6 1 17	9 10 7 2 16	9 10 5 2 12
4 0 6	9 5 2 12	4 8 1 2 19	6 5 6 1 17
5 2 12	6 1 0 16	6 0 0 1 10	8 9 4 0 0
8 1 10	9 3 2 19	4 9 2 1 6	7 1 0 1 17

5. CLOTH MEASURE.

1.	2.	3.	4.	5.
Yd. qr. n.	℥. E. qr. n.	℥. Fl. qr. n.	℥. Fr. qr. n.	Yds. qr. n.
76 2 3	9 1 3 2	75 2 1	49 3 3	9 14 2 3
3 3 1	49 4 3	7 1 3	19 5 2	49 2 1
42 3 3	6 2 3	84 0 2	24 2 1	56 1 3 0
57 2 2	84 4 1	76 2 3	67 4 3	84 0 2
16 3 3	7 0 0	48 2 2	48 2 2	549 3 1
49 2 2	6 1 2 1	9 2 3	6 3 3	6 17 1 3

6. LONG MEASURE.

1.	2.	3.	4.	5.
Ft. in. bar.	Yd. ft. in.	Pol. ft. in.	Mil. fur. pol.	Deg. mi. fur. pol. ft. in. br.
9 11 2	7 2 11	12 11 10	9 7 36	759 56 6 20 15 10 2
6 9 1	4 1 6	9 16 9	7 3 19	3 17 39 1 36 11 6 1
7 0 2	6 0 10	8 12 11	4 1 24	497 63 7 24 9 8 1
8 10 0	7 2 9	7 15 6	6 5 12	562 17 0 11 13 11 0
9 6 2	8 1 10	4 14 9	4 6 9	64 48 5 17 9 4 2
7 10 2	9 2 11	5 11 11	5 1 10	764 52 4 19 15 11 1

7. TIME.

1.	2.	3.	4.
W. d. h. m. s.	Mo. d. h. m.	Y. mo. d.	Y. mo. w. d. h. m. s.
3 6 22 57 42	5 24 19 45	19 10 19	57 11 3 6 23 29 55
1 3 19 31 28	4 27 21 35	7 9 27	4 8 1 1 19 45 38
2 3 17 9 15	9 18 0 12	4 8 16	29 9 2 3 17 18 19
3 0 9 17 58	4 19 23 19	1 11 14	46 10 2 5 11 50 13
1 1 16 19 10	8 11 12 13	17 6 9	19 9 2 1 16 18 17
2 2 20 53 48	9 19 8 29	12 5 20	45 9 3 5 18 17 59

8. MOTION.

8. MOTION.

1.		
17°	55'	48"
1	37	51
28	19	45
19	19	37

2.		
25°	49'	51"
4	21	36
19	47	18
25	25	39

3.			
9s	29°	35'	53"
10	0	18	31
4	17	13	42
6	19	50	0

9. LAND or SQUARE MEASURE.

1.		
Pol.	feet.	in.
36	179	137
19	248	119
12	96	75
18	110	122
9	269	24
25	221	143

2.		
Yds.	ft.	in.
28	7	119
9	3	75
29	6	120
4	8	12
9	1	119
8	3	43

3.			
Acres.	rood.	pol.	feet. in.
756	3	37	245 128
29	1	28	93 25
416	2	31	128 119
37	1	19	218 20
61	0	0	92 103
191	1	25	129 136

10. SOLID MEASURE.

1.		
Ton.	feet.	in.
29	36	1229
12	19	64
8	11	917
19	8	1001
5	0	523
17	39	1119

2.		
Yds.	feet.	in.
75	22	1412
9	26	195
3	19	1091
28	15	1110
49	24	218
18	17	1225

3.		
Cord.	feet.	in.
37	119	1015
9	110	159
48	127	1071
8	111	956
21	9	27
9	28	1091

11. WINE MEASURE.

1.			
Tier.	gal.	qts.	pts.
37	39	3	1
9	17	2	1
4	28	0	0
32	19	1	1
9	0	3	1
12	40	1	1

2.			
Hhd.	gal.	qts.	pts.
51	53	1	1
27	39	3	0
9	18	0	1
0	9	2	1
16	24	1	1
5	0	3	0

3.			
Ton.	hhd.	gal.	qts.
37	2	37	2
19	1	59	1
28	2	0	0
19	0	47	1
37	1	17	3
14	2	48	2

COMPOUND SUBTRACTION. 53

12. ALE and BEER MEASURE.

1.	2.	3.
<i>A. B. fir. gal.</i>	<i>B. B. fir. gal.</i>	<i>Hhd. gal. qts.</i>
49 3 7	29 1 8	379 53 3
26 2 3	19 3 5	19 0 1
9 0 4	16 0 3	121 37 2
17 3 0	9 1 8	467 19 1
27 1 6	14 2 0	591 16 0
19 3 7	17 1 5	75 0 2

13. DRY MEASURE.

1.	2.	3.
<i>Qrs. bu. p. qt.</i>	<i>Bus. p. qt. pt.</i>	<i>Ch. bu. p. qts.</i>
64 7 3 7	37 2 5 1	37 27 3 5
9 4 1 5	19 3 7 0	6 29 1 7
19 6 2 1	16 2 0 1	15 30 0 0
4 0 2 0	5 1 6 1	4 11 3 0
17 3 0 6	9 0 3 0	5 0 1 0
9 5 3 4	19 3 0 1	2 0 2 1

COMPOUND SUBTRACTION

Teaches to find the difference, inequality, or excess, between any two sums of divers denominations.

R U L E.*

Place those numbers under each other, which are of the same denomination, the less being below the greater ; begin with the least denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater ; subtract it therefrom ; and to the difference add the upper figure, remembering, always, to add one to the next superior denomination, for that which you borrowed.

I. M O N E Y.

1.	2.
<i>£. s. d. gr.</i>	<i>£. s. d. gr.</i>
Borrowed 349 15 6 1	Lent 791 9 8 1
Paid 195 11 8 1	Received 197 16 4 2
Remains to pay 154 3 10 0	Due to me
Proof	

3.

* The reason of this Rule will readily appear, from what was said in Simple Subtraction ; for the borrowing depends upon the same principle, and is only different, as the numbers to be subtracted are of different denominations.

COMPOUND SUBTRACTION.

	3.					4.					5.			
	£.	s.	d.	qr.		£.	s.	d.	qr.		£.	s.	d.	qr.
From	439	9	10	1		843	12	1	3		569	7	5	2
Take	190	0	10	3		746	15	0	2		508	16	4	0

Rem.

	6.					7.			
	£.	s.	d.			£.	s.	d.	qr.
Borrowed	19372	12	6			Lent	27109	5	8 3
Paid at	293	16	8	0		Received	5196	15	10 0
	74	9	7	2			384	17	6 2
fundry	9413	11	0	1		at	4187	18	11 1
	1994	0	10	3		feveral	1649	16	8 0
times.	3914	19	0	0			917	9	10 3
	1064	17	9	1		times.	3196	0	2 1

Paid in all

Rec. in all

Remains to
be paid }

Remains due

2. TROY WEIGHT.

	1.					2.					3.			
	lb.	oz.	pwt.	gr.		lb.	oz.	pwt.	gr.		lb.	oz.	pwt.	gr.
Bought	749	5	13	16		379	8	12	10		543	3	9	13
Sold	96	9	19	13		148	4	16	19		179	1	15	18

Rem.

3. AVOIRDUPOIS WEIGHT.

	1.				2.				3.				4.		
	lb.	oz.	dr.		C. gr.	lb.			T. cwt.	qr.	lb.		T. cwt.	qr.	lb.
Bought	7	9	12		8	2	13		5	13	1	12	9	11	3
Sold	3	12	9		4	1	15		1	12	2	17	3	12	1

Rem.

4. APOTHECARIES' WEIGHT.

	1.					2.					3.			
	lb.	℥	ʒ	gr.		lb.	℥	ʒ	gr.		lb.	℥	ʒ	gr.
71	9	3	1	13		65	10	6	2		84	1	1	1
37	8	4	1	16		31	8	4	2		65	9	3	1

5. CLOTH

5. CLOTH MEASURE.

1.	2.	3.	4.
<i>Yds. qr. n.</i>	<i>E. E. qr. n.</i>	<i>E. Fl. qr. n.</i>	<i>E. Fr. qr. n.</i>
35 1 2	467 3 1	765 1 3	549 4 2
19 1 3	291 3 2	149 2 1	197 4 3
<hr/>	<hr/>	<hr/>	<hr/>

6. LONG MEASURE.

1.	2.	3.	4.
<i>Yds. ft. in.</i>	<i>Pol. ft. in.</i>	<i>Mil. fur. pol.</i>	<i>Deg. m. fur. p. yds. ft. in. bar.</i>
28 2 10	21 11 9	76 3 11	38 41 3 29 2 1 7 2
17 2 11	9 13 8	27 3 21	19 35 5 31 3 1 9 1
<hr/>	<hr/>	<hr/>	<hr/>

7. TIME.

1.	2.	3.	4.
<i>Mo. d. h. m. s.</i>	<i>Mo. w. d. h.</i>	<i>Y. mo. d.</i>	<i>Y. mo. w. d. h. m. s.</i>
6 17 18 27 19	9 2 5 15	7 3 13	48 9 2 5 19 27 31
1 21 16 41 35	4 3 5 15	4 2 19	19 9 3 4 20 19 49
<hr/>	<hr/>	<hr/>	<hr/>

8 MOTION.

1.	2.	3.
<i>79° 21' 31"</i>	<i>63 11° 12' 48"</i>	<i>45 19° 41' 22"</i>
41 41 52	3 8 39 29	1 22 19 45
<hr/>	<hr/>	<hr/>

9. LAND or SQUARE MEASURE.

1.	2.	3.
<i>A. R. Pol.</i>	<i>A. R. Pol.</i>	<i>A. R. Pol. ft. in.</i>
29 1 10	29 2 17	56 3 19 27 110
24 1 25	17 1 36	29 0 21 210 129
<hr/>	<hr/>	<hr/>

10. SOLID MEASURE.

1.	2.	3.
<i>Tons. ft. in.</i>	<i>Yds. ft. in.</i>	<i>Cords. ft. in.</i>
49 19 1100	79 11 917	349 97 1250
38 36 1295	17 25 1095	192 127 1349
<hr/>	<hr/>	<hr/>

11. WINE MEASURE.

1.				2.			3.			4.		
Tun.	gal.	qt.	pt.	Tier.	gal.	qt.	Hhd.	gal.	qt.	Tun.	hhd.	gal.
79	1	2	1	19	17	1	375	41	2	532	1	19
38	1	3	1	12	29	2	197	36	3	197	1	47

12. ALE and BEER MEASURE.

1.				2.					3.		
A. B.	fir.	gal.	qt.	B. B.	fir.	gal.	qts.	pts.	Hhds.	gal.	qts.
39	1	2	1	21	3	5	2	0	769	17	1
24	3	6	2	19	1	7	2	1	391	42	3

13. DRY MEASURE.

1.				2.				3.			
Qu.	bu.	pk.	qt.	Bu.	pk.	qts.	pts.	Chal.	bu.	pk.	qts.
56	2	2	1	91	1	3	2	39	12	2	1
39	3	1	2	29	2	1	1	24	25	3	2

P R O B L E M S

Resulting from a Comparison of the preceding Rules.

PROB. 1. Having the sum of two numbers, and one of them given, to find the other.

Rule. Subtract the given number from the given sum, and the remainder will be the number required.

Let 288 be the sum of two numbers; one of which is 115, the other is required? From 288 the Sum,
Take 115 the given number.
Remains 173 the other.

PROB. 2. Having the greater of two numbers, and the difference between that and the less given, to find the less.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other, 198: What is the other? From 325 the greater,
Take 198 the difference.
Rem. 127 the less.

PROB. 3. Having the least of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the less number.
198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule.

Rule. To half the sum add half the difference, and the sum is the greater, and from half the sum take half the difference, and the remainder is the less. Or, from the sum take the difference, and half the remainder is the least: To the least add the given difference, and the sum is the greatest.

What are those two numbers, whose sum is 48, and difference 14?

$$2)48 \quad 2)14 \quad 24+7=31 \text{ the greater, \& } 24-7=17 \text{ the less.}$$

$$\frac{1}{2} \text{ sum} = 24 \quad \frac{1}{2} \text{ diff.} = 7 \quad \text{Or } 48-14 \div 2 = 17, \& \ 17+14=31.$$

PROB. 5. Having the sum of two numbers and the difference of their squares given, to find those numbers.

Rule. Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference: You will then have their sum and difference to find the numbers by Prob. 4.

What two numbers are those, whose sum is 32, and the difference of whose squares is 256?

$\begin{array}{r} 32)256(8 \text{ difference.} \\ \underline{256} \end{array}$	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;">Half sum</td> <td style="width: 50%; text-align: right;">16</td> </tr> <tr> <td>Half diff.</td> <td style="text-align: right;">4</td> </tr> <tr> <td>Greater</td> <td style="text-align: right;"><u>20</u></td> </tr> <tr> <td>Less</td> <td style="text-align: right;">12</td> </tr> </table>	Half sum	16	Half diff.	4	Greater	<u>20</u>	Less	12
Half sum	16								
Half diff.	4								
Greater	<u>20</u>								
Less	12								

PROB. 6. Having the difference of two numbers and the difference of their squares given, to find those numbers.

Rule. Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum; then proceed by Prob. 4.

What are those two numbers, whose difference is 20, and the difference of whose squares is 2000?

$$20)2000(100 \text{ sum. } 50+10=60, \text{ the greater, \& } 50-10=40, \text{ the less.}$$

For more Questions of this nature, see Miscell. Quæst. Problems 46, 47, 48 and 49; but, as the extraction of the square root is there concerned, they could not be admitted here.

PROB. 7. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288, and one of them 8; I demand the other?

$$\begin{array}{r} 8)288 \\ \underline{288} \end{array} \quad \text{Answ. } 36$$

PROB. 8. Having the dividend and quotient, to find the divisor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving *Division*.

<p>Given { 288 the Dividend.</p> <p style="margin-left: 20px;">36 the Quotient.</p>	$\begin{array}{r} 36)288(8 \text{ Divisor.} \\ \underline{288} \end{array}$
---	---

Required the Divisor.

PROB. 9. Having the Divisor and Quotient given, to find the Dividend.

Rule. Multiply them together.

H

Given

Given $\left\{ \begin{array}{l} 8 \text{ the Divisor.} \\ 36 \text{ the Quotient.} \end{array} \right.$

$\begin{array}{r} 36 \\ 8 \end{array}$

Required the Dividend.

$\overline{288}$ the Dividend.

By a due consideration and application of these Problems only, many questions (of which kind are some of the following) may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

APPLICATION of the preceding Rules.

1. The least of two numbers is 19418, and the difference between them is 2384 : What is the greater, and sum of both ?

$19418 + 2384 = 21802$ greater, and $19418 + 21802 = 41220$ sum.

2. Suppose a man born in the year 1743 ; when will he be 57 years of age ?

$1743 + 57 = 1800$ *Anf.*

3. What number is that, which, being added to 19418, will make 21802 ?

$21802 - 19418 = 2384$ *Anf.*

4. Gen. Washington was born in 1732 ; what is his age in 1787 ?

$1787 - 1732 = 55$ *Anf.*

5. America was discovered by Columbus in 1492 and its Independence declared in 1776 : How many years have elapsed between those two Æras ?

$1776 - 1492 = 284$ *Anf.*

6. The Massacre at Boston, by the British Troops, happened, March 5th, 1770, and the Battle at Lexington, April 19th, 1775 : How long between ?

April 19th, 1775—March 5th, 1770=5y. 1m. 14d. *Anf.*

7. Gen. Burgoyne and his army were captured October 17th, 1777, and Earl Cornwallis and his army, October 19th, 1781 : What space of time between ?

Oct. 19th, 1781—Oct. 17th, 1777=4 years and 2 days, *Anf.*

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783 ; How long did the war continue ?

January 20th, 1783—April 19th, 1775=7y. 9m. 1d. *Anf.*

9. A, B, C and D purchased a quantity of goods in partnership ; A paid £.12 10s. a dollar and a crown piece ; B, 35s. C 29s. 10d. and D, 79d. : What did the goods cost ?

Anf. £.16 14 1.

10. A man borrowed, at different times, these several sums, viz. £.29 5s. £.18 17s. 6d. £.45 12s. £.98, 3 dollars, one crown piece and an half ; pray how much was he in debt ?

Anf. £.193 2 6.

11. There are 4 numbers ; the first 317, the second 912, the third 1229, and the fourth as much as the other three, abating 97 : What is the sum of them all ?

Anf. 4819.

12. Bought a quantity of goods for £.125 10s. paid for truckage 45s. for freight 79s. 6d. for duties 35s. 10d. and my expenses were 53s. 9d. : What did the goods stand me in ?

Anf. £.136 4 1.

13. A Gentleman left his son £.1725 more than his daughter, whose fortune was 15 thousand, 15 hundred and 15 pounds :

What

What was the son's portion, and what did the whole estate amount to ?

Ans. The son's fortune, £.18240, and the whole estate £.34755.

14. A merchant had 6 debtors, who together owed him £.2917 10s. 6d. A, B, C, D and E, owed him £.1675 13s. 9d. of it : What was F's debt ?

Ans. 1241 16 9.

15. What is the difference between £.1309 7s. 1d. and the amount of £.345 13s. 4d. and £.571 4s. 8d. ?

Ans. £.392 9 1.

16. A Merchant, at his first engaging in trade, owed £.937 15s. he had in cash £.1755 3s. 6d. in goods £.459 12s. 3d. in good debts £.197 16s. and he cleared the first year £.249 19 10. What was the neat balance at the year's end ?

Ans. £.1724 16 7.

17. What sum of money must be divided between 12 men, so as that each may receive £.155 ?

$£.155 \times 12 = 1860$ *Ans.*

18. What number must I multiply by 9, that the product may be 675 ?

$675 \div 9 = 75$ *Ans.*

19. A Privateer of 175 men took a prize, which amounted to £.59 per man, beside the owner's half : What was the value of the prize ?

$175 \times 59 \times 2 = £.20650$ *Ans.*

20. What is the difference between thrice five and thirty : and thrice thirty five ?

$35 \times 3 - 5 \times 3 + 30 = 60$ *Ans.*

21. The sum of two numbers is 750 ; the less 248 : What is their difference, product and the square of their difference ? *

$750 - 248 = 502$ the greater number, $502 - 248 = 254$ difference, $502 \times 248 = 124496$ product, and $254 \times 254 = 64516$ square of the difference.

22. What is the difference between six dozen dozen, and half a dozen dozen ; and what is their product, and the quotient of the greater by the less ?

Ans. $6 \times 12 \times 12 - 6 \times 12 = 792$ diff.

$6 \times 12 \times 12 \times 6 \times 12 = 62208$ product, and $6 \times 12 \times 12 \div 6 \times 12 = 12$ quotient.

23. There are two numbers ; the greater of them is 25 times 78, and their difference is 9 times 15 ; their sum and product are required.

Ans. $78 \times 25 = 1950$ the greater, $1950 - 15 \times 9 = 1815$ the less.

$1950 + 1815 = 3765$ the sum, and $1950 \times 1815 = 3539250$ the prod.

24. A Merchant began trade with £.25327 ; for 6 years together, he cleared £.1253 per annum ; the next 5 years, he cleared £.1729 per annum ; but, the last 4 years, had the misfortune to lose £.3019 per annum : What was he worth at the 15 years' end ?

Ans. £.29414.

25. If a man spends £.192 in a year : What is that per calendar month ?

$192 \div 12 = £.16$ *Ans.*

26. If the Federal Debt, which is 42 million dollars, be equally divided between the 13 States : What will be the share of each ?

Ans. $3230769\frac{3}{13}$ dollars.

27. If

* A Number is said to be squared, when it is multiplied into itself.

27. If 9000 men march in a column of 750 deep : How many march abreast ? $9000 \div 750 = 12$ *Ans.*

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the same ? $3072 \div 32 - 32 = 64$ *Ans.*

29. What number is that, which, multiplied by 3589, will produce 92050672 ? $92050672 \div 3589 = 25648$ *Ans.*

30. Suppose the quotient arising from the division of two numbers to be 5379, the divisor 37625 : What is the dividend, if the remainder came out 9357 ?

$$37625 \times 5379 + 9357 = 202394232 \text{ } \textit{Ans.}$$

31. There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted ; and 30 added to the remainder, the half sum shall make 35 : Can you tell me the number ?

$$35 \times 2 - 30 + 20 \times 5 \times 7 \div 3 = 700 \text{ } \textit{Ans.}$$

32. A sheepfold was robbed three nights successively ; the first night, half the sheep were stolen, and half a sheep more ; the second, half the remainder were lost, and half a sheep more ; the last night they took half what were left and half a sheep more ; by which time they were reduced to 30 : How many were there at first ?

Begin with 30, and, reckoning back from the last night to the first, you will find that 31 were stolen the 3d night, 62 the 2d, and 124 the first. *Ans.* 247.

33. Two boys, A and B, had 850 chefnuts between them ; but A had 150 more than B : How many had each ?

$850 \div 2 = 425$ half sum, and $150 \div 2 = 75$ half diff. then $425 + 75 = 500$ A's, and $425 - 75 = 350$ B's.

34. A and B played at marbles, having 14 apiece at the first ; but after playing several games, B, having lost some of his, would play no longer, and it was found that the difference of the squares of the numbers, which each then had, was 336 : Pray, how many did B lose ?

$14 + 14 = 28$ sum, $336 \div 28 = 12$ diff. $28 \div 2 = 14$ half sum, and $12 \div 2 = 6$ half diff. then $14 + 6 = 20$ A retired with, and $14 - 6 = 8$ B had left, therefore B lost $14 - 8 = 6$.

35. Said Harry to Charles, My father gave me 12 more apples than he gave my brother Jack, and the difference of the squares of our separate parcels was 288 : Now, if you are Arithmetician enough to tell how many he gave us, each, you shall have half of mine ?

$288 \div 12 = 24$ the whole : $24 \div 2 = 12$ and $12 \div 2 = 6$, then $12 + 6 = 18 =$ Harry's share, and $12 - 6 = 6 =$ Jack's share.

36. What number added to the 27th part of 6615, will make 570 ?

$$570 - 6615 \div 27 = 325 \text{ } \textit{Ans.}$$

R E D U C T I O N

Teaches to bring, or exchange, numbers of one denomination to others of different denominations, retaining the same value.

It is of *two* sorts, viz. Descending and Ascending; the former of which is performed by multiplication, and the latter, by division.

R E D U C T I O N D E S C E N D I N G.

R U L E.*

Multiply the highest denomination, given, by so many of the next less as make one of that greater, and thus continue until you have brought it down as low as your question requires.

PROOF. Change the order of the question, and divide your last product by the last multiplier, and so on.

E X A M P L E S.

1. In £.27 15s. 9d. 2qrs. how many farthings?

£. s. d. qr.

27 15 9 2

multiplied by 20 = shillings in a pound.

555 = shillings.

— by 12 = pence in a shilling.

6669 = pence.

— by 4 = farthings in a penny.

Ans. = 26678 farthings.

NOTE. In multiplying by 20, I added in the 15s. by 12, the 9d. and by 4, the 2qrs. which must alway be done in like cases.

To prove the above question, change the order of it, and it will stand thus: In 26678 farthings how many pounds?

4)26678

12)6669 2qrs.

2|0)55|5 9d.

Answer, £.27 15 9 2

2. In £.36 12s. 10d. 1qr. how many farthings?

Ans. 35177.

3. In £.95 11s. 5d. 3qr. how many farthings?

Ans. 91751.

4. In £.719 9s. 11d. how many half pence?

Ans. 34535 8.

5. In 29 guineas, at 28s. how many pence?

Ans. 9744.

6. In

* The reason of this Rule is exceedingly obvious; for pounds are brought into shillings by multiplying them by 20; shillings into pence by multiplying them by 12; and pence into farthings by multiplying them by 4; and the contrary by division; and this will be true in the reduction of numbers consisting of any denomination whatever.

6. In 37 pistoles, at 22s. how many shillings, pence and farthings. *Ans.* 814s. 9768d. 39072qrs.

7. In 49 half johannes, at 48s. how many sixpences? *Ans.* 4704.

8. In 473 French crowns, at 6s. 8d. how many threepences? *Ans.* 12613 $\frac{1}{3}$.

9. In 53 moidores, at 36s. how many shillings, pence, and farthings? *Ans.* 1908s. 22896d. 91584qrs.

10. In £.29 how many groats, threepences, pence and farthings? *Ans.* 1740 groats, 2320 threepences, 6960d. 27840qrs.

11. Reduce 47 guineas and one fourth of a guinea into shillings, sixpences, groats, threepences, twopences, pence and farthings. *Ans.* 1323 shill. 2646 sixpences, 3969 groats, 5292 threepences, 7938 twopences, 15876 pence, and 63504 qrs.

REDUCTION ASCENDING.

RULE.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue till you have brought it into that denomination which your question requires.

EXAMPLES.

1. In 547325 farthings how many pence, shillings and pounds?
Farthings in a penny = 4)547325

Pence in a shilling = 12)136831 1 qr.

Shillings in a pound = 20)114012 7d.
Ans. 136831d. 11402s. and £.570 2s. 7d. 1qr.

Note. The remainder is always of the same name as the dividend.

2. Bring 35177 farthings into pounds.

3. Bring 91751 farthings into pence, &c.

4. Bring 345358 half pence into pence, shillings and pounds.

5. Reduce 9744 pence to guineas, at 28s. per guinea.

6. In 39072 farthings how many pistoles, at 22s.?

7. In 4704 sixpences how many half johannes?

8. In 12613 $\frac{1}{3}$ threepences how many French crowns, at 6s. 8d.?

9. In 91584 farthings how many moidores, at 36s.?

10. In 27840 farthings how many pence, threepences, groats, shillings and pounds?

11. In 63504 farthings how many pence, twopences, threepences, groats, sixpences, shillings and guineas?

Note. The preceding questions may serve as proofs to those in Reduction descending.

REDUCTION DESCENDING and ASCENDING.

1. MONEY.

1. In £.97 how many pence and English or French crowns,
at 6s. 8d. ? *Ans.* 23280d. and 291 crowns.
2. In 947 English crowns, at 6s. 8d. how many shillings, and
English guineas ? *Ans.* 6313s. 4d. and 225 guin. 13s. 4d.
3. In 519 English half crowns how many pence and pounds ?
Ans. 20760d. and £.86 10s.
4. In 1259 groats how many farthings, pence, shillings and
guineas ? *Ans.* 20144qrs. 5036d. 419s. 8d. and 14 guin. 27s. 8d.
5. In 75 pistoles how many pounds ? *Ans.* £.82 10s.
6. In 735 French crowns how many shillings and French
guineas, at 26s. 8d. *Ans.* 4900s. and 183 guin. 20s.
7. In 5793 pence how many farthings, pounds and pistoles ?
Ans. 23172qrs. £.24 2s. 9d. and 21 pistoles, 20s. 9d.
8. In £.99 how many shillings, and half johannes, at 48s. ?
Ans. 1980s. and 41 half joes. 12s.
9. In £179 how many guineas ? *Ans.* 127 guin. 24s.
10. In £.345 how many moidores ? *Ans.* 191 moid. 24s.
11. In 59 half joes, 37 moidores, 45 guineas, 63 pistoles, 24 En-
glish crowns, and 19 dollars ; how many pounds, half joes, moi-
dores, guineas, pistoles, English crowns, dollars, shillings, pence
and farthings ?
Ans. £.354 4s. 147 half joes, 28s. 196 moidores, 28s. 253 guin-
eas, 322 pistoles, 1062 Eng. crowns, 4s. 1180 dollars, 4s. 7084 shil-
lings, 85008d. and 340032qrs.

When it is required to know how many sorts of coin, of dif-
ferent values, and of equal number, are contained in any number
of another kind ; reduce the several sorts of coin into the lowest
denomination mentioned, and add them together for a divisor ;
then reduce the money given, into the same denomination for a
dividend, and the quotient, arising from the division, will be the
number required.

Note. Observe the same direction in weights and measures.

1. In 275 half johannes how many moidores, guineas, pistoles,
dollars, shillings and sixpences, of each the like number ?

A moidore is 36s.	}	72 sixpences,	275 half joes.
that is			48 shills. in a johan.

A guinea is 28s.	}	56 ditto.	2200
that is			1100

A pistole is 22s.	}	44 ditto.	13200 shillings.
that is			2 sixp. in a shill.

A dollar is 6s.	}	12 ditto.	Dividend = 26400 sixpences.
that is			

One shilling has	}	2 do. 187	26400 (141 of each, and 33 sixp. or
		1 do.	16s. 6d. over, the answer.

Divisor = 187 sixpences.

2. A Gentleman distributed £.37. 10s. between 4 poor persons, in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third, 10s. and the fourth, 5s. What did each person receive? *Ans. The first man £.15.*

T R O Y W E I G H T.

1. How many grs. in a silver bowl, that weighs 3^{lb} 10oz. 18 pwt.?

^{lb} oz. pwt.

3 10 17

12 ounces in a pound.

—
46 ounces.

20 pennyweights in an ounce.

—
932 pennyweights.

24 grains in one pwt.

—
3728

1864

Proof. 24)22368 grains, answer.

2|0)93|2

12) 46—12 pwt.

^{lb} 3—10 oz.

2. In 487 ozs. how many pwt. & grs.? *Ans.* 9740 pwt. £3233760 gr.

3. In 13 ingots of gold, each weighing 9oz. 5pwt. how many grains? *Ans.* 57720 gr.

4. In 97397 grs. how many pounds? *Ans.* 16^{lb}. 10oz. 18pwt. 5gr.

5. How many rings, each weighing 5pwt. 7gr. may be made of 3^{lb}. 5oz. 16pwt. 2gr. of gold? *Ans.* 158.

3. A V O I R D U P O I S.

Cwt. grs. ^{lb} oz.

1. In 91 3 17 14 how many ounces?

4

367 quarters.

28

2943

735

10293 pounds.

16

61762

10294

164702 ounces.

Proof.

16)164702

28)10293 14oz.

367 17^{lb}.

Cwt. 91 3qrs.

2. In

REDUCTION.

65

2. In 12 tons, 15cwt. 1qr. 19lb 6oz. 12dr. how many drams ?

Anf. 7323500dr.

3. In 24lb 11oz. 9dr. how many drams ?

Anf. 6329dr.

4. In 44800 pounds, how many drams and tons ?

Anf. 11468800dr. and 20 tons.

5. In 28lb Avoirdupois, how many pounds Troy ?

28

7000 grains in 1lb Avoird.

$\left. \begin{array}{l} \text{grs. in} \\ \text{1lb tr.} \end{array} \right\} = 576|0)19600|0(34\text{lb}$

1728

2320

2304

160

12

576|0)192|0(0 oz.

20

576|0)3840|0(6 pwt.

3456

3840

24

1536

768

576|0)9216|0(16 gr.

576

3456

3456

lb oz. pwt. gr.

6. In 47 9 13 17 Troy, how many pounds Avoirdupois ?

47 9 13 17

12

573

20

11473

24

45899

22947

7|000)275|369(39lb

21

65

63

2369

16

14214

2369

7|000)37|904(5 oz.

35

2904

16

17424

2904

7|000)46|464(6 $\frac{4454}{7556}$ dr.

42

4464

4. APOTHECARIES'

4. APOTHECARIES' WEIGHT.

1. How many grains are there in 37lb 6
- $\frac{3}{4}$
- ?

$$\begin{array}{r}
 \text{lb} \quad \frac{3}{4} \\
 37 \quad 6 \\
 12 \\
 \hline
 450 \text{ ounces.} \\
 8 \\
 \hline
 3600 \text{ drams.} \\
 3 \\
 \hline
 10800 \text{ scruples.} \\
 20 \\
 \hline
 \end{array}$$

216000 grains.

$$\begin{array}{r}
 \text{Proof.} \\
 2|0)21600|0 \\
 \hline
 3)10800 \\
 \hline
 8)3600 \\
 \hline
 12)450 \\
 \hline
 37\text{lb} \quad 6\frac{3}{4}
 \end{array}$$

2. In 9lb 8
- $\frac{3}{4}$
- 13 2
- $\frac{1}{2}$
- 19gr. how many grains?

Ans. 55799gr.

3. In 55799 grains how many pounds, &c.

Ans. 9lb 8 $\frac{3}{4}$ 13 2 $\frac{1}{2}$ 19gr.

5. CLOTH MEASURE.

1. In 127 yards how many quarters and nails?

$$\begin{array}{r}
 4 \\
 \hline
 508 \text{ qrs.} \\
 4 \\
 \hline
 \end{array}$$

Ans. 2032 nails.

$$\begin{array}{r}
 \text{Proof.} \\
 4)2032 \\
 \hline
 4)508 \\
 \hline
 \end{array}$$

127 yards.

2. In 9173 nails how many yards?

Ans. 573yds. 1qr. 1n.

3. In 75 ells English how many quarters and nails?

Ans. 375qrs. 1500n.

4. In 56 ells Flemish, how many quarters and nails?

Ans. 168qrs. 672n.

5. In 39 ells French how many quarters and nails?

Ans. 234qrs. 936n.

6. In 7248 nails how many yards, ells Flemish, ells English and ells French?

Ans. 453yds. 604 ells Flem. 362 ells Eng. 2qrs. 302 ells French.

7. In 19 pieces of cloth, each 15 yards, 2 quarters, how many yards, quarters and nails?

Ans. 294yds. 2qrs. 1178qrs. and 4712n.

6. LONG MEASURE.

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles?

<u>43 miles.</u> 8	<i>Proof.</i> 3)8173440
<u>344 furlongs.</u> 40	12)2724480
<u>13760 rods.</u> 5½	3)227040
<u>68800</u> 6880	11)75680
<u>75680 yards.</u> 3	<u>6880</u> 2
<u>227040 feet.</u> 12	4,0)1376,0
<u>2724480 inches.</u> 3	8)344
	<u>43</u> —

Here I divide by 11, and multiply the quotient by 2, because twice $5\frac{1}{2}$ is 11; or I might first have multiplied by 2, and, then, have divided the product by 11.

8173440 *Answer.*

2. How many barley corns will reach round the globe, it being 360 degrees? *Anf.* 4755801600.

3. How many inches from Newburyport to London, it being 2700 miles? *Anf.* 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles? *Anf.* 13440 *times.*

5. In 190080 inches, how many yards and leagues? *Anf.* 5280 *yds.* and 1 *league.*

7. T I M E .

1. In 20 years how many seconds?

<i>d. h.</i>
365 6 in a year
<u>24</u>
1466
<u>730</u>
8766 hours in 1 year.
<u>20</u>
175320 hours in 20 years.
<u>60</u>
10519200 minutes in ditto.
<u>60</u>

<i>Proof.</i>
6 0)63115200 0
<u>6 0)1051920 0</u>
2 0)17532 0
<u>4X6)8766</u>
4)1461
<u>365 d. 6h.</u>

631152000 seconds in ditto.

2. Suppose your age to be 15y. 19d. 21h. 37m. 45s. how many seconds are there in it, allowing 365 days and 6 hours to the year? *Anf.* 475047465.

3. In

3. In 31558154 seconds how many solar years? *Anf. 1 year.*
 4. How many minutes from the first day of January to the 14th day of August, inclusively? *Anf. 325440.*
 5. How many days since the commencement of the christian Era?
 6. How many minutes since the commencement of the American war, which happened on the 19th day of April, 1775?
 7. How many seconds between the commencement of the war, April 19th, 1775, and the independence of the United States of America, which took place the 4th day of July, 1776? *Anf. 38102400.*

8. M O T I O N.

1. In 9 signs,
- $13^{\circ} 25'$
- , how many seconds?

$ \begin{array}{r} 9s \quad 13^{\circ} \quad 25' \\ 30 \\ \hline 283 \text{ degrees.} \\ 60 \\ \hline 17005 \text{ minutes.} \\ 60 \\ \hline 1020300 \text{ seconds.} \end{array} $	<p style="text-align: right;"><i>Proof.</i></p> $ \begin{array}{r} 6 0)102030 0 \\ \hline 6 0)1700 5 \\ \hline 3 0)28 3-25 \\ \hline 9s \quad 13^{\circ} \quad 25' \end{array} $
--	--

9. L A N D or S Q U A R E M E A S U R E.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

<p style="text-align: center;"><i>Acres. R. Poles.</i></p> $ \begin{array}{r} 29 \quad 3 \quad 19 \\ 4 \\ \hline 119 \text{ roods.} \\ 40 \\ \hline \end{array} $	<p style="text-align: right;"><i>Proof.</i></p> $ \begin{array}{r} 4 0)477 9 \\ \hline 4 119-19p. \\ \hline 29ac. \quad 3 \text{ roods.} \end{array} $
--	--

Answer 4779 perches.

2. In 1997 poles how many acres?
- Anf. 12a. 1r. 37p.*

3. In 89763 square yards how many acres, &c.?

Anf. 18a. 2r. 7p. 101ft. 36in.

4. How many square feet, square yards, and square poles, in a square mile?

Anf. 27878400 feet, 3097600 yards, and 102400 poles.

10. S O L I D M E A S U R E.

1. In 15 tons of hewn timber how many solid inches?

15 tons.

15 tons.
50
—
750 feet.
1728
—
6000
1500
5250
750
—

Proof.
5|0
1728)1296000(75|0
12096
—
8640
8640
—
15 tons.

Anf. 1296000 inches.

2. In 9 tons of round timber how many inches ?

Anf. 622080

3. In 25 cords of wood how many inches ?

Anf. 5529600

11. WINE MEASURE.

1. In 9 hhd. 15 gals. 3 qts. of wine how many quarts ?

hhd. gal. qts.
9 15 3
63
—
32
55
—
582 gallons.
4
—

Proof.
4)2331
—
63)582—37ts.
—
9 hhd. — 15 gals.

Anf. 2331 quarts.

2. In 12 pipes of wine how many pints ?

Anf. 12096.

3. In 9758 pints of brandy how many pipes ?

Anf. 9 p. 1 hhd. 22 gal. 3 qts.

5. In 1008 quarts of cyder how many tons ?

Anf. 1 ton.

12. ALE or BEER MEASURE.

1. In 29 hhd. of beer how many pints ?

hhd.
29
54
—
116
145
—
1566 gallons.
4
—
6264 quarts.
2
—

Proof.
2)12528
—
4)6264
—
54(1566
—
29 hhd.

Anf. 12528 pints.

2. In

2. In 47 *bar.* 18 *gal.* of ale how many pints? *Ans.* 13680.
 3. In 36 puncheons of beer how many butts? *Ans.* 24.

13. DRY MEASURE.

1. In 42 chaldrons of coals how many pecks?

Chaldrons.

42
32

—
84

126

—
1344 *busshels.*

4

Proof.

4)5376

32)1344(42
128

—
64

64

—

Ans. 5376 *pecks.*

2. In 75 bushels of corn how many pints? *Ans.* 4800.
 3. In 9376 quarts how many bushels? *Ans.* 293.

VULGAR FRACTIONS.

Fractions, or broken numbers, are expressions for any assignable parts of an unit, or whole number; and are represented by two numbers, placed one above another, with a line drawn between them, thus; $\frac{5}{8}$, $\frac{4}{3}$, &c. signifying five eighths, four thirds, that is, one and one third, &c.

The figure above the line, is called the *numerator*, and that below it, the *denominator*.

The denominator (which is the divisor in division) shews how many parts the integer is divided into; and the numerator (which is the remainder after division) shews how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed. Any whole number may be made an improper fraction by drawing a line under it, and putting unity, or 1 for a denominator, as 9 may be expressed fractionwise, thus $\frac{9}{1}$, and 12 thus $\frac{12}{1}$, &c.

1. A *single, simple, or proper fraction* is, when the numerator is less than the denominator, as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c. and is a *simple* expression for any number of parts of the integer.

2. An *improper fraction* is, when the numerator exceeds the denominator, as, $\frac{5}{3}$, $\frac{8}{5}$, $\frac{12}{4}$, &c.

3. A *compound fraction* is the fraction of a fraction, coupled by the word *of*, thus, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$, &c. which are read thus, two thirds of three fourths; one half of three fifths of seven eighths.

4. A *mixed number* is composed of a whole number and a fraction, as $7\frac{3}{5}$, $35\frac{4}{13}$, &c. that is, seven and three fifths, &c.

5. A

5. A fraction is said to be in its least, or lowest terms, when it is expressed by the least numbers possible.

6. The *common measure* of two, or more numbers, is that number which will divide each of them without a remainder : Thus, 5 is the common measure of 10, 20 and 30 ; and the *greatest* number, which will do this, is called the *greatest common measure*.

7. A number, which can be measured by two, or more, numbers, is called their *common multiple* : And, if it be the least number, which can be so measured, it is called the *least common multiple* : thus, 40, 60, 80, 100, are multiples of 4 and 5 ; but their least common multiple is 20.

8. A *prime number* is that, which can only be measured by itself, or an unit.

9. That number, which is produced by multiplying several numbers together, is called a *composite number*.

10. A *perfect number* is equal to the sum of all its aliquot parts.*

P R O B L E M I.†

To find the greatest common measure of two, or more, numbers.

R U L E.

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain, then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them, as before ; then, of that common measure and one of the other numbers, and so on, through all the numbers, to the last ; then will the greatest common measure, last found, be the answer.

3. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.

E X A M P L E S.

* The following perfect numbers are all which are, at present, known.

6	8589869056
28	137438691328
496	2305843008139952128
8128	2417851639228158837784576
33550336	9903520314282971830448816128

† This and the following problem will be found very useful in the doctrine of fractions, and several other parts of Arithmetic.

The truth of the rule may be shewn from the first example : For, since 108 measures 216, it also measures $216 \div 108$, or 324.

Again, since 108 measures 216 and 324, it also measures $5 \times 324 + 216$, or 1836.

In the same manner it will be found to measure $2 \times 1836 + 324$, or 3996, and so on.

It is also the greatest common measure ; for, suppose there be a greater, then, since the greater measures 1836 and 3996, it also measures the remainder 324 ; and since it measures 324 and 1836, it also measures the remainder 216 ; in the same manner it will be found to measure the remainder 108 ; that is, the greater measures the less, which is absurd ; therefore, 108 is the greatest common measure.

In the same manner, the demonstration may be applied to one or more numbers.

EXAMPLES.

1. What is the greatest common measure of 1836, 3996, and 1044 ?

$$\begin{array}{r} 1836)3996(2 \\ 3672 \\ \hline \end{array}$$

$$\begin{array}{r} 324)1836(5 \\ 1620 \\ \hline \end{array}$$

$$\begin{array}{r} 216)324(1 \\ 216 \\ \hline \end{array}$$

$$\text{Common meas.} = 108)216(2 \\ 216 \\ \hline$$

So 108 is the greatest common measure of 3996 and 1836.

$$\text{Hence } 108)1044(9$$

$$\begin{array}{r} 972 \\ \hline \end{array}$$

$$72)108(1$$

$$\begin{array}{r} 72 \\ \hline \end{array}$$

$$\text{Last greatest com. meas.} = 36)72(2 \\ 72 \\ \hline$$

Therefore, 36 is the answer required.

2. What is the greatest common measure of 1224 and 1080 ?

Anf. 72.

3. What is the greatest common measure of 1440, 672 and 3472 ?

Anf. 16.

PROBLEM II.*

To find the least common multiple of two, or more, numbers.

RULE.

1. Divide by any number that will divide two, or more, of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2. Divide the second line, as before, and so on, till there are no two numbers, that can be divided ; then, the continued product of the divisors and quotients will give the multiple required.

EXAMPLES.

1. What is the least common multiple of 6, 10, 16 and 20 ?

$$\begin{array}{r} *5)6 \quad 10 \quad 16 \quad 20 \\ \hline \end{array}$$

$$\begin{array}{r} *2)6 \quad 2 \quad 16 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} *2)3 \quad 1 \quad 8 \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} *3 \quad 1 \quad *4 \quad 1 \\ \hline \end{array}$$

* * * * *

$$5 \times 2 \times 2 \times 3 \times 4 = 240 \text{ Anf.}$$

I survey my given numbers, and find that five will divide two of them, viz. 10 and 20, which I divide by 5, bringing into a line with the quotients the numbers, which 5 will not measure : Again, I view the numbers in the second line, and find 2 will measure them all, and I get 3, 1, 8, 2 in the third line, and find that two will measure 8 and 2, and in the fourth line get 3, 1, 4, 1, all prime ; I then multiply the prime numbers and the divisors continually into each other,

for the number sought, and find it to be 240.

2. What

* The reason of this rule may also be shewn from the first example : Thus, it is evident that $6 \times 10 \times 16 \times 20 (=19200)$ may be divided by 6, 10, 16 and 20, without a remainder ; but 20 is a multiple of 5 ; therefore, $6 \times 10 \times 16 \times 4$, or 3840, is also divisible by 6, 10, 16 and 20. Also, 16 is a multiple of 4 ; therefore, $6 \times 10 \times 4 \times 4 = 960$, is also divisible by 6, 10, 16 and 20. Also, 10 is a multiple of 2 ; therefore, $6 \times 5 \times 4 \times 4 = 480$, is also divisible by 6, 10, 16 and 20. Also, 6 is a multiple of 3 ; therefore, $3 \times 5 \times 4 \times 4 = 240$, is also divisible by 6, 10, 16 and 20 ; and is evidently the least number that can be so divided.

2. What is the least common multiple of 6 and 8 ? *Ans.* 24.
3. What is the least number that 3, 5, 8 and 10 will measure ? *Ans.* 120.
4. What is the least number which can be divided by the 9 digits, separately, without a remainder ? *Ans.* 2520

REDUCTION of VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

C A S E I.*

To abbreviate, or reduce fractions to their lowest terms.

R U L E.

Divide the terms of the given fraction, by any number, which will divide them without a remainder, and the quotients, again, in the same manner ; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms. *Or,*

Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

E X A M P L E S.

1. Reduce $\frac{288}{480}$ to its lowest terms.

$$8\frac{288}{480} = \frac{(4)36}{(3)160} = \frac{9}{40} = \frac{3}{13\frac{3}{4}} = \frac{3}{5} \text{ the answer.}$$

Or

* That dividing both the terms, that is, both numerator and denominator of the fraction equally by any number whatever, will give another fraction, equal to the former, is evident : And if those divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

- Note 1.* Any number, ending with an even number or cypher, is divisible by 2.
2. Any number, ending with 5 or 0, is divisible by 5.
 3. If the right hand place of any number be 0, the whole is divisible by 10.
 4. If the two right hand figures of any number be divisible by 4, the whole is divisible by 4.
 5. If the three right hand figures of any number be divisible by 8, the whole is divisible by 8.
 6. If the sum of the digits, constituting any number, be divisible by 3 or 9, the whole is divisible by 3 or 9.
 7. If a number cannot be divided by some number less than the square root thereof, that number is a *prime*.
 8. All *prime* numbers, except 2 and 5, have 1, 3, 7 or 9 in the place of units : And all other numbers are *composite*.
 9. When numbers, with the sign of Addition or Subtraction between them, are to be divided by any numbers, each of the numbers must be divided : Thus, $6+9+10=2+3+4=9$.

10. But if the numbers have the sign of Multiplication between them ; then only one of them must be divided : Thus, $\frac{4 \times 6 \times 10}{2 \times 5} = \frac{2 \times 6 \times 10}{1 \times 5} = \frac{2 \times 6 \times 2}{1 \times 1} = \frac{24}{1}$

$$= 24.$$

$$\begin{array}{r} 288)480(1 \\ \underline{288} \\ 192)288(1 \\ \underline{192} \end{array}$$

Com. meas. 96)192(2

2. Reduce $\frac{96}{544}$ to its lowest terms.
3. Reduce $\frac{384}{1152}$ to its lowest terms.
4. Reduce $\frac{576}{2304}$ to its lowest terms.
5. Reduce $\frac{46}{134}$ to its lowest terms.
6. Reduce $\frac{1429}{2858}$ to its lowest terms.

Ans. $\frac{3}{17}$.

Ans. $\frac{1}{3}$.

Ans. $\frac{1}{8}$.

Ans. $\frac{1}{4}$.

Ans. $\frac{1}{2}$.

C A S E II.

To reduce a mixed number to its equivalent improper fraction.

R U L E.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which, subjoin the denominator, and it will form the fraction required.

E X A M P L E S.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ 8+5 \\ \hline \text{Ans. } 293 \\ \hline 8 \end{array}$$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum 293 is the numerator of the fraction sought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

$$\text{Or, } \frac{36 \times 8 + 5}{8} = \frac{293}{8} \text{ Ans. as before.}$$

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction.

Ans. $\frac{2162}{17}$.

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.

Ans. $\frac{12410}{19}$.

C A S E III.†

To reduce a whole number to an equivalent fraction, having a given denominator.

R U L E.

* All fractions represent a division of the numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3 divided by 4 is $\frac{3}{4}$; from whence the rule is manifest; for if any number is multiplied and divided by the same number, it is evident the quotient must be the same as the quantity first given.

† Multiplication and Division are here equally used, and consequently the result is the same as the quantity first proposed.

R U L E.

Multiply the whole number by the given denominator : Place the product over the said denominator, and it will form the fraction required.

E X A M P L E S.

1. Reduce 6 to a fraction, whose denominator shall be 8.
 $6 \times 8 = 48$, and $\frac{48}{8}$ the answer.—Proof $\frac{48}{8} = 48 \div 8 = 6$.
2. Reduce 15 to a fraction, whose denominator shall be 12.
Ans. $\frac{180}{12}$.
3. Reduce 100 to a fraction, whose denominator shall be 70.
Ans. $\frac{7000}{70} = \frac{1000}{1} = 1000$.

C A S E IV.*

To reduce an improper fraction to its equivalent whole, or mixed number.

R U L E

Divide the numerator by the denominator ; the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

E X A M P L E S.

1. Reduce $\frac{293}{8}$ to its equivalent whole, or mixed number.

$8)293(36\frac{5}{8}$ *Answer.*

$$\begin{array}{r} 24 \\ \hline 53 \\ 48 \\ \hline \end{array}$$

Or, $\frac{293}{8} = 293 \div 8 = 36\frac{5}{8}$ as before.

2. Reduce $\frac{127}{17}$ to its equivalent whole, or mixed number.

Ans. $127\frac{4}{17}$.

3. Reduce $\frac{1241}{19}$ to its equivalent whole, or mixed number.

Ans. $653\frac{3}{19}$.

4. Reduce $\frac{45}{5}$ to its equivalent whole number.

Ans. 9.

C A S E V.†

To reduce a compound fraction to an equivalent simple one.

R U L E.

Multiply all the numerators continually together for a new numerator,

* This rule is evidently the reverse of case 2d, and has its reason in the nature of common division.

† That a compound fraction may be represented by a simple one is very evident ; since a part of a part must be equal to some part of the whole. The truth of the rule for this reduction may be shewn as follows.

Let the compound fraction to be reduced, be $\frac{2}{4}$ of $\frac{6}{10}$. Then $\frac{1}{4}$ of $\frac{6}{10} = \frac{6}{40} \div 4 = \frac{6}{160}$, and consequently $\frac{2}{4}$ of $\frac{6}{10} = \frac{6}{10} \times 3 = \frac{18}{10}$ the same as by the rule.

If the compound fraction consists of more numbers than two, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers, and so on.

numerator, and all the denominators, for a new denominator, and they will form the simple fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction, by case 2d, or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged, and the other members continually multiplied, (as by the rule) will produce the fraction required in lower terms.

E X A M P L E S.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

$$\frac{1 \times 2 \times 3 \times 4}{2 \times 3 \times 4 \times 5} = \frac{24}{120} = \frac{1}{5} \text{ the answer.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction.

$$\frac{3 \times 4 \times 5 \times 11}{4 \times 5 \times 6 \times 12} = \frac{660}{1440} = \frac{11}{24} \text{ Ans.}$$

Or, by expunging the equal numerators and denominators it will be $\frac{3 \times 11}{6 \times 12} = \frac{33}{72} = \frac{11}{24}$ as before.

3. Reduce $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{15}{19}$ to a simple fraction. *Ans.* $\frac{225}{532}$.

4. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a simple (or improper) fraction. *Ans.* $\frac{624}{306} = 2\frac{2}{51}$.

5. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple (or improper) fraction. *Ans.* $\frac{75}{64} = 1\frac{11}{64}$.

C A S E VI.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

R U L E I.*

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators into each other, continually, for a common denominator.

E X A M P L E S.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$$1 \times 5 \times 8 = 40 \text{ the new numerator for } \frac{1}{4}.$$

$$2 \times 4 \times 8$$

* By placing the numbers multiplied properly under one another, it will be seen that the numerator and denominator of every fraction are multiplied by the very same number, and consequently their values are not altered. Thus, in the first example.

$$\frac{1}{4} \left| \begin{array}{l} \times 5 \times 8 \\ \times 5 \times 8 \end{array} \right. \quad \frac{2}{5} \left| \begin{array}{l} \times 4 \times 8 \\ \times 4 \times 8 \end{array} \right. \quad \frac{5}{8} \left| \begin{array}{l} \times 4 \times 5 \\ \times 4 \times 5 \end{array} \right.$$

In the second rule, the common denominator is a multiple of all the denominators, and consequently will divide by any of them; Therefore, proper parts may be taken for all the numerators as required.

$$2 \times 4 \times 8 = 64 \text{ the new numerator for } \frac{2}{5}.$$

$$5 \times 4 \times 5 = 100 \text{ ditto for } \frac{5}{8}.$$

$$4 \times 5 \times 8 = 160 \text{ the common denominator.}$$

Therefore, the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$, and $\frac{100}{160}$, the answer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to fractions having a common denominator. *Ans.* $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$ and $\frac{3}{13}$ to a common denominator.

$$\text{Ans. } \frac{936}{1872}, \frac{1040}{1872}, \frac{14508}{1872}, \frac{432}{1872}.$$

4. Reduce $\frac{11}{15}$, $\frac{3}{4}$ of $1\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$ to a common denominator.

$$\text{Ans. } \frac{8448}{11520}, \frac{21600}{11520}, \frac{6720}{11520}, \frac{7200}{11520}.$$

RULE II.

To reduce any given fractions to others, which shall have the least common denominator.

1. By Problem 2, page 72, find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{8}$ to fractions having the least common denominator possible.

$$\begin{array}{r} 4) \overline{3 \quad 4 \quad 8} \\ \underline{3 \quad 1 \quad 2} \end{array}$$

$$4 \times 3 \times 2 = 24 = \text{least common denominator.}$$

$24 \div 3 \times 1 = 8$ the 1st numerator ; $24 \div 4 \times 3 = 18$ the 2d numerator ; $24 \div 8 \times 7 = 21$ the 3d. numerator.

Whence, the required fractions, are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{4}{5}$ to fractions having the least common denominator.

$$\text{Ans. } \frac{30}{60}, \frac{40}{60}, \frac{15}{60} \text{ and } \frac{48}{60}.$$

CASE VII.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULE.*

Reduce the given fraction to a compound one, by comparing it with all the denominations between it and that denomination you would reduce it to ; lastly, reduce this compound fraction to a single one, by case 5th, and you will have a fraction of the required denomination, equal in value to the given fraction.

EXAMPLES.

* The reason of this and the next rule is explained in the rule for reducing compound fractions to simple ones.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of a penny to the fraction of a pound.

By comparing it, it becomes $\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$; which reduced by case 5th, will be $3 \times 1 \times 1 = 3$

$$\text{And } 5 \times 12 \times 20 = 1200 = \frac{1}{400}$$

2. Reduce $\frac{1}{4}$ of a farthing to the fraction of a pound.

$$\text{Ans. } £. \frac{1}{1280}$$

3. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea.

$$\text{Ans. } \frac{5}{2688} \text{ guinea.}$$

4. Reduce $\frac{12}{19}$ of a shilling to the fraction of a moidore.

$$\text{Ans. } \frac{1}{57} \text{ moidore.}$$

5. Reduce $\frac{4}{7}$ of an ounce to the fraction of a lb Avoirdupois.

$$\text{Ans. } \frac{1}{28} \text{ lb.}$$

*6. Reduce 3s. 6d. to the fraction of a pound.

$$\text{Ans. } \frac{7}{8} \text{ £.}$$

7. Reduce 13s. 6d. to the fraction of a pistole.

$$\text{Ans. } \frac{27}{44} \text{ pistole.}$$

+8. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea.

$$\text{Ans. } \frac{4}{7} \text{ guinea.}$$

9. Reduce $\frac{7}{8}$ of a pwt. to the fraction of a pound Troy.

$$\text{Ans. } \frac{7}{1920} \text{ lb.}$$

10. Reduce $\frac{8}{9}$ of a lb Avoirdupois to the fraction of 1 Cwt.

$$\text{Ans. } \frac{1}{126} \text{ Cwt.}$$

11. Express $5\frac{1}{2}$ furlongs in the fraction of a mile.

$$\text{Ans. } \frac{11}{16} \text{ mile.}$$

CASE VIII.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

RULE.

Multiply the given numerator by the parts of the denominations between it and that denomination you would reduce it to, for a new numerator, which place over the given denominator: Or, only invert the parts contained in the integer, and make of them a compound fraction as before; then, reduce it to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny.

By comparing it, the fraction will be $\frac{1}{400}$ of $\frac{20}{1}$ of $\frac{12}{1}$, then

$$\frac{1 \times 20 \times 12}{400 \times 1 \times 1} = \frac{240}{400} = \frac{3}{5} \text{ answer.}$$

2. Reduce

* 3s. 6d. = 42d. and 1 £. = 240d. therefore, $\frac{42}{240} = \frac{7}{40}$ £.

$$+ \frac{4}{3} \text{ £.} = \frac{4}{3} \text{ of } \frac{20}{1} = \frac{4 \times 20}{3 \times 1} = \frac{80}{3} \text{ s. and } \frac{80}{3} \text{ of } \frac{1}{28} = \frac{80 \times 1}{3 \times 28} = \frac{80}{84} = \frac{4}{7} \text{ guinea.}$$

VULGAR FRACTIONS.

2. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing. *Ans.* $\frac{3}{4}$ gr.
3. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny. *Ans.* $\frac{5}{8}$ d.
4. Reduce $\frac{1}{37}$ of a moidore to the fraction of a shilling. *Ans.* $\frac{12}{5}$ s.
5. Reduce $\frac{1}{24}$ of a lb Avoirdupois to the fraction of an ounce. *Ans.* $\frac{4}{10}$ oz.
- *6. Reduce $\frac{1}{7}$ of a guinea to the fraction of a pound. *Ans.* $\frac{4}{5}$ £.
7. Reduce $\frac{7}{1920}$ of a lb Troy to the fraction of a part. *Ans.* $\frac{7}{3}$ pwt.
8. Reduce $\frac{1}{126}$ of cwt. to the fraction of a lb Avoirdupois. *Ans.* $\frac{8}{9}$ lb.

C A S E IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, measure, &c.

R U L E.†

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on, as far as necessary; and the quotients placed after one another, in their order, will be the answer required.

E X A M P L E S.

1. What is the value of $\frac{5}{7}$ of a pound?

$$\begin{array}{r}
 5 \\
 20 \\
 \hline
 7 \overline{)100} \\
 \hline
 14s. - 2 \\
 12 \\
 \hline
 7 \overline{)24} \\
 \hline
 3d. - 3 \\
 4 \\
 \hline
 7 \overline{)12} \\
 \hline
 1\frac{5}{7}qr.
 \end{array}$$

I multiply 5 by 20, the number of shillings in £.1, and the product 100 I divide by the denominator 7, and get the quotient 14s. and 2 remaining, I multiply it by 12, and again dividing the product by 7, find the quotient 3d. and 3 remains, which I multiply by 4, and dividing as before, the quotient is 1qr. and 5 remaining, I place it over the numerator, and find the answer 14s. 3d. $1\frac{5}{7}$ qr.

2. What

$$* \frac{4}{7} \text{ Guin.} = \frac{4}{7} \text{ of } \frac{28}{1} = \frac{4 \times 28}{7 \times 1} = \frac{112}{7} = 112s. \text{ \& } \frac{112}{7} \text{ of } \frac{1}{20} = \frac{112}{140} = \frac{4}{5} \text{ s.}$$

† As the numerator of a fraction may be considered as a remainder, and the denominator as a divisor: This rule therefore has its reason in the nature of division.

VULGAR FRACTIONS.

2. What is the value of $\frac{9}{24}$ of a shilling ? *Ans.* $4\frac{1}{2}d.$
3. What is the value of $\frac{17}{29}$ of a *cwt.* ? *Ans.* $2qr. 9lb. 10oz. 7\frac{2}{3}dr.$
4. What is the value of $\frac{4}{5}$ of a *lb* Avoirdupois ? *Ans.* $12oz. 12\frac{4}{5}dr.$
5. What is the value of $\frac{3}{5}$ of a *lb* Troy ? *Ans.* $7oz. 4pwt.$
6. What is the value of $\frac{3}{13}$ of a ton ? *Ans.* $4cwt. 2qrs. 12lb. 14oz. 12\frac{4}{13}dr.$
7. What is the value of $\frac{6}{9}$ of a yard. *Ans.* $2qrs. 2\frac{2}{3}n.$
8. What is the value of $\frac{7}{8}$ of an ell English ? *Ans.* $4qrs. 1\frac{1}{2}n.$
9. What is the value of $\frac{5}{6}$ of a mile ? *Ans.* $6fur. 26p. 11ft.$
10. What is the value of $\frac{9}{13}$ of a day ? *Ans.* $10h. 36m. 55\frac{5}{13}s.$
11. The value of $\frac{12}{17}$ of a Julian year is required ? *Ans.* $257d. 19h. 45m. 52\frac{6}{17}s.$
12. The value of $\frac{9}{14}$ of a guinea is demanded ? *Ans.* $18s.$
13. What is the value of $\frac{15}{16}$ of a dollar ? *Ans.* $5s. 7\frac{1}{2}d.$
14. What is the value of $\frac{3}{5}$ of a moidore ? *Ans.* $21s. 7\frac{1}{5}d.$
15. What is the value of $\frac{7}{9}$ of an acre ? *Ans.* $3r. 17\frac{1}{9}p.$

C A S E X.

To reduce any given quantity to the fraction of any greater denomination of the same kind.

R U L E.*

Reduce the given quantity to the lowest term mentioned, for a numerator ; then reduce the integral part to the same term, for a denominator ; which will be the fraction required.

E X A M P L E S.

1. Reduce $14s. 3\frac{1}{4}d. \frac{5}{7}$ to the fraction of a pound.

$ \begin{array}{r} 20 \text{ Integral part.} \\ 12 \\ \hline 240 \\ 4 \\ \hline 960 \\ 7 \\ \hline 6720 \text{ denominator.} \end{array} $	$ \begin{array}{r} s. \quad d. \\ 14 \quad 3\frac{1}{4}, \frac{5}{7} \\ \hline 12 \\ \hline 171 \\ 4 \\ \hline 685 \\ 7+5 \\ \hline 4800 \text{ num.} \end{array} $
---	--

2. Reduce $4\frac{1}{2}d.$ to the fraction of a shilling. *Ans.* $\frac{4800}{6720} = \frac{5}{7}f.$
3. Reduce *Ans.* $\frac{3}{8}s.$

* This case is the reverse of the former, therefore proves it.

Note. If there be a fraction given with the said quantity, it must be further reduced to the denominative parts thereof, adding thereto the numerator.

3. Reduce 2qrs. 9lb 10oz. $7\frac{1}{2}$ dr. to the fraction of a Cwt.
Ans. $\frac{1}{2}\frac{1}{2}$ Cwt.
4. Reduce 12oz. $12\frac{1}{2}$ dr. to the fraction of a lb Avoirdupois.
Ans. $\frac{4}{5}$ lb.
5. Reduce 7oz. 4put. to the fraction of a lb Troy.
Ans. $\frac{3}{5}$ lb.
6. Reduce 4cwt. 2qrs. 12lb 14oz. $12\frac{4}{5}$ dr. to the fraction of a ton.
Ans. $\frac{3}{13}$ ton.
7. Reduce 2qrs. $2\frac{2}{3}$ n. to the fraction of a yard.
Ans. $\frac{2}{3}$ yd.
8. Reduce 4qrs. $1\frac{1}{2}$ n. to the fraction of an ell English.
Ans. $\frac{7}{8}$ E. E.
9. Reduce 6fur. 26po. 11ft. to the fraction of a mile.
Ans. $\frac{5}{6}$ mile,
10. Reduce 16h. 36m. $55\frac{5}{13}$ s. to the fraction of a day.
Ans. $\frac{2}{13}$ day.
11. Reduce 257d. 19h. 45m. $52\frac{1}{13}$ s. to the fraction of a Julian year.
Ans. $\frac{1}{13}$ J. Y.
12. Reduce 18s. to the fraction of a guinea.
Ans. $\frac{9}{14}$ guin.
13. Reduce 5s. $7\frac{1}{2}$ d. to the fraction of a dollar.
Ans. $\frac{1}{16}$ dol.
14. Reduce 21s. $7\frac{1}{2}$ d. to the fraction of a moidore.
Ans. $\frac{3}{5}$ moid.
15. Reduce 3r. $17\frac{1}{4}$ p. to the fraction of an acre.
Ans. $\frac{6}{7}$ acre.

ADDITION of VULGAR FRACTIONS.

R U L E.*

Reduce compound fractions to single ones ; mixed numbers to improper fractions ; fractions of different integers to those of the same ; and all of them to a common denominator ; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

E X A M P L E S.

1. Add $7\frac{4}{5}$, $\frac{5}{7}$ of $\frac{3}{8}$ and 7 together.

First. $7\frac{4}{5} = \frac{39}{5}$, $\frac{5}{7}$ of $\frac{3}{8} = \frac{15}{56}$, and $7 = \frac{7}{1}$.

Then the fractions are $\frac{39}{5}$, $\frac{15}{56}$, and $\frac{7}{1}$; *therefore,*

$$39 \times 56 \times 1 = 2184$$

$$15 \times 5 \times 1 = 75$$

$$7 \times 5 \times 56 = 1960$$

$$\frac{4219}{560}$$

$$5 \times 56 \times 1 = 280 \quad \text{Or, thus, } \frac{2184 + 75 + 1960}{280} = 15\frac{19}{280}$$

2. Add

* Fractions, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another ; but when they are reduced to a common denominator, and made parts of the same thing ; their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum or difference of their individuals ; whence the reason of the rules, both for Addition and Subtraction, is manifest.

2. Add $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. *Ans.* $9\frac{101}{105}$.
3. What is the sum of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{1}{4}$, and $8\frac{4}{13}$? *Ans.* $9\frac{2319}{12480}$.
4. What is the sum of $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $9\frac{1}{4}$? *Ans.* $12\frac{59}{80}$.
5. Add $\frac{1}{9}\text{£}$, $\frac{3}{5}\text{s}$, and $\frac{4}{5}\text{d}$. together. *Ans.* 2s , 8d , $\frac{64}{105}$.
6. What is the sum of $\frac{2}{5}$ of 17£ , $9\frac{5}{8}\text{£}$, and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{4}{7}\text{£}$? *Ans.* £ . 16 12s . $3\frac{5}{7}\text{d}$.
7. Add $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{5}{8}$ of a mile together. *Ans.* 1100yds . 2ft . 7inches .
8. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. *Ans.* 2 days , 2 hours , 30 minutes , 45 seconds .

SUBTRACTION of VULGAR FRACTIONS.

R U L E.*

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required. *Note*, a fraction is subtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

E X A M P L E S.

1. From $\frac{7}{4}$ take $\frac{2}{3}$ of $\frac{5}{8}$.
 $\frac{2}{3}$ of $\frac{5}{8} = \frac{10}{24} = \frac{5}{12}$. Then the fractions are $\frac{3}{4}$ and $\frac{5}{12}$.

$$\begin{array}{r} 3 \times 28 = 84 \\ 5 \times 4 = 20 \\ 4 \times 28 = 112 \text{ com. den.} \end{array} \left\{ \begin{array}{l} \frac{3}{4} = \frac{84}{112} \text{ and } \frac{5}{12} = \frac{20}{112}, \text{ therefore,} \\ \frac{84}{112} - \frac{20}{112} = \frac{64}{112} = \frac{4}{7} \text{ Remainder.} \end{array} \right.$$
2. From $4\frac{2}{5}$ take $\frac{5}{9}$. *Ans.* $4\frac{91}{45}$.
3. From $37\frac{1}{4}$ take $19\frac{4}{7}$. *Ans.* $17\frac{13}{28}$.
4. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Ans.* $2\frac{1}{12}$.
5. From $\frac{1}{4}\text{£}$ take $\frac{1}{10}\text{s}$. *Ans.* 4s . $1\frac{1}{2}\text{d}$.
6. From $\frac{5}{2}\text{oz}$. take $\frac{1}{4}\text{pwt}$. *Ans.* 13pwt . $12\frac{5}{8}\text{gr}$.
7. From $\frac{1}{2}$ of a league take $\frac{3}{8}$ of a mile. *Ans.* 1mi . 1fur .
8. From 5 weeks take $19\frac{4}{7}$ days. *Ans.* 15da . 4ho . 48min .

MULTIPLICATION

* In subtracting mixed numbers, when the lower fraction (the subtrahend) is greater than the upper one, (the minuend) you may, without reducing them to improper fractions, subtract the numerator of the subtrahend from the common denominator, and to that difference add the numerator of the minuend, and carry one to the integer of the subtrahend.

E X A M P L E.

From $19\frac{3}{8}$ take $12\frac{7}{8}$. — $19\frac{3}{8} - 12\frac{7}{8} = 6\frac{1}{4}$.

MULTIPLICATION of VULGAR FRACTIONS.

R U L E.*

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required. *Note*, where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

E X A M P L E S.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$$4\frac{1}{3} = \frac{13}{3}, \frac{1}{4} \text{ of } \frac{7}{8} = \frac{1 \times 7}{4 \times 8} = \frac{7}{32}, \text{ and } 6 = \frac{6}{1}.$$

$$\text{Then } \frac{13}{3} \times \frac{1}{5} \times \frac{7}{32} \times \frac{6}{1} = \frac{13 \times 1 \times 7 \times 6}{3 \times 5 \times 32 \times 1} = \frac{646}{480} = 1\frac{11}{80} \text{ the answer.}$$

2. Multiply $\frac{4}{7}$ by $\frac{5}{27}$.

$$\text{Ans. } \frac{20}{459}.$$

3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$.

$$\text{Ans. } \frac{7}{8}.$$

4. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$.

$$\text{Ans. } \frac{5}{14}.$$

5. Multiply $\frac{3}{7}$ of $\frac{5}{9}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{4}$.

$$\text{Ans. } \frac{64}{147}.$$

6. Multiply $9\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{5}$, and $12\frac{4}{7}$ continually together.

$$\text{Ans. } 24\frac{13}{56}.$$

7. What is the continual product of $\frac{3}{4}$ of $\frac{2}{3}$, $5\frac{1}{2}$, 7 and $\frac{1}{3}$ of $\frac{5}{8}$?

$$\text{Ans. } 4\frac{1}{96}.$$

8. What is the continual product of 7, $\frac{1}{2}$, $\frac{5}{7}$ of $\frac{3}{8}$, and $3\frac{1}{3}$?

$$\text{Ans. } 2\frac{1}{12}.$$

Another method for the Multiplication of mixed Quantities.

Case 1. To multiply a whole number by a fraction, or a fraction by a whole number.

Rule. Multiply the whole number by the numerator of the fraction, and divide the product by the denominator: But if the numerator be 1, divide by the denominator only.

	1.	2.	3.	4.	5.	6.	7.
Mult.	8	15	28	36	48	325	259
By	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{12}$
Prod.	2	$7\frac{1}{2}$	$9\frac{1}{3}$	$3(7\frac{2}{3})$	$4(14\frac{1}{2})$	$8(16\frac{1}{4})$	$12(18\frac{1}{3})$

$$\text{Prod. } 24 \quad 36 \quad 203\frac{1}{4} \quad 151\frac{1}{12}$$

Case 2. To multiply a whole number by a mixed one.

Rule. Multiply by the fraction as in case 1st; then multiply by the whole number, and add the two products, as in the examples—
or,

* Multiplication of a fraction implies the taking some part or parts of the multiplicand, and therefore may truly be expressed by a compound fraction. Thus $\frac{4}{5}$ multiplied by $\frac{3}{8}$ is the same as $\frac{4}{5}$ of $\frac{3}{8}$; and as the directions of the rule agree with the method already given, to reduce these fractions to simple ones, it is shewn to be right.

or, to multiply a mixed number by a whole one, change the place of the factors, and proceed as the rule directs.—See example 6.

	1.	2.	3.	4.	5.	6.
Mult.	15	35	68	42	129	$1\frac{7}{5}$
By	$3\frac{1}{2}$	$5\frac{1}{3}$	$7\frac{1}{2}$	$9\frac{3}{7}$	$8\frac{5}{8}$	24
	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
	$7\frac{1}{2}$	$11\frac{2}{3}$	748	126	645	Mult. 24
	45	175	$62\frac{4}{12}$	18	$80\frac{5}{8}$	By $1\frac{7}{5}$
	$52\frac{1}{2}$	$186\frac{2}{3}$	476	378	1032	15)168
			$538\frac{4}{12}$	396	$1112\frac{5}{8}$	$11\frac{3}{5}$
						24
						Prod. $35\frac{3}{5} = \frac{1}{5}$

Case 3. To multiply a mixed number by a mixed number.

Rule. Multiply the integral part of the multiplicand by the denominator of its fractional part, and add thereto its numerator : Then multiply by the mixed multiplier, by Case 2d, and divide the product by the denominator of the fractional part of the multiplicand, as in the following example.

Mult. $42\frac{3}{5}$ { 1st. $42\frac{3}{5} = 213$
By $8\frac{2}{3}$ { which mult. by $8\frac{2}{3}$

3)426	}	After this manner may feet and inches be multiplied, calling 1 inch $\frac{1}{12}$ of a foot, 2 inches $\frac{2}{12}$, 3 inches $\frac{3}{12}$, 4 inches $\frac{4}{12}$, 5 inches $\frac{5}{12}$, 6 inches $\frac{6}{12}$, 7 inches $\frac{7}{12}$, 8 inches $\frac{8}{12}$, 9 inches $\frac{9}{12}$, 10 inches $\frac{10}{12}$, and 11 inches $\frac{11}{12}$ of a foot.
142		
1704		
5)1846		
Product = $369\frac{1}{5}$		

DIVISION of VULGAR FRACTIONS.

R U L E.*

Prepare the fractions as before : Then, invert the divisor and proceed exactly as in Multiplication : The products will be the quotient required.

E X A M P L E S.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$.

$\frac{1}{3}$ of 17 = $\frac{1}{3}$ of $17 = \frac{1 \times 17}{3 \times 1} = \frac{17}{3}$, & $\frac{2}{3}$ of $\frac{6}{8} = \frac{2 \times 6}{3 \times 8} = \frac{12}{24} = \frac{1}{2}$; therefore,

$\frac{17}{3} \div \frac{1}{2} = \frac{17 \times 2}{3 \times 1} = \frac{34}{3} = 11\frac{1}{3}$ the quotient required.

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$.

Ans. $1\frac{4}{21}$.

3. Divide $12\frac{1}{4}$ by $\frac{1}{3}$ of 7.

Ans. $5\frac{8}{35}$.

4. Divide

The reason of the Rule may be shewn thus. Suppose it were required to divide

$\frac{4}{5}$ by $\frac{2}{7}$. Now $\frac{4}{5} \div 2$ is manifestly $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4}{5 \times 2}$; but $\frac{2}{7} = \frac{1}{7}$ of 2; therefore,

$\frac{1}{7}$ of 2, or $\frac{2}{7}$, must be contained 7 times as often in $\frac{4}{5}$ as 2, that is $\frac{4 \times 7}{5 \times 2}$ —the answer, which is according to the rule.

Note. To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

4. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$.
5. Divide $\frac{3}{7}$ by 9.
6. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$.
7. Divide 7 by $\frac{3}{8}$.
8. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112.

Ans. $\frac{41}{62}$.
 Ans. $\frac{1}{21}$.
 Ans. $\frac{1}{8}$.
 Ans. $18\frac{2}{3}$.
 Ans. $425\frac{2}{3}$.

DECIMAL FRACTIONS.

Decimal Fractions are of such a nature, that they vary in the same proportion, and are managed by the same method of operation, as whole numbers are.

On this account, every proper Fraction is supposed to be reducible to another, whose denominator shall be 10, 100, 1000, &c. viz. Unity, with a number of cyphers annexed; and Fractions with such denominators are called *Decimal Fractions*: Such are, $\frac{5}{10}$, $\frac{55}{100}$, $\frac{675}{1000}$, &c.

As the denominator of a decimal fraction is always 10, or 100, or 1000, &c. the denominators need not be expressed: For the numerator only may be made to express the true value: For this purpose it is only required to write the numerator with a point before it at the left hand, to distinguish it from a whole number, when it consists of so many figures as the denominator hath cyphers annexed to unity, or 1; so $\frac{5}{10}$ is written ,5; $\frac{33}{100}$, ,33; $\frac{735}{1000}$, ,735, &c.

Note. The point prefixed is called a Separatrix.

But if the numerator has not so many places as the denominator has cyphers, put so many cyphers before it, viz. at the left hand, as will make up the defect; so write $\frac{5}{100}$ thus ,05; and $\frac{6}{1000}$ thus ,006, &c. And thus do these fractions receive the form of whole numbers.

The 1st, 2d, 3d, 4th, &c. places of decimals, counting from the left hand toward the right, are called primes, seconds, thirds, fourths, &c.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand to 0, as in the following

T A B L E.

C Millions	X Millions	Millions	C Thousands	X Thousands	Thousands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thousandth Parts	X Thousandth Parts	C Thousandth Parts	Millionth Parts	X Millionth Parts	C Millionth Parts
9	8	7	6	5	4	3	2	1	, 2	3	4	5	6	7	8	9

From this table it is evident, that, in decimals, as well as in whole numbers, each figure takes its value by its distance from unit's

unit's place: If it be in the first place after units (or the separating point) it signifies tenths; if in the second, hundredths, &c. decreasing in each place in a tenfold proportion.

Consequently, every single figure expressing a decimal has for its denominator an unit or 1, with so many cyphers as its place is distant from unit's place: Thus, 2 in the decimal part of the table $= \frac{2}{10}$; $3 = \frac{3}{100}$; $4 = \frac{4}{1000}$, &c. And if a decimal be expressed by several figures, the denominator is 1, with so many cyphers as the lowest figure is distant from unit's place. So .357 signifies $\frac{357}{1000}$, and .0053 $= \frac{53}{10000}$, &c.

Cyphers, placed at the right hand of a decimal fraction, do not alter its value, since every significant figure continues to possess the same place: So .5, .50 and .500 are all of the same value, and each equal to $\frac{1}{2}$.

But cyphers, placed at the left hand of a decimal, do alter its value, every cypher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place further from the place of units. So .5, .05, .005, all express different decimals, viz. .5, $\frac{5}{10}$; .05, $\frac{5}{100}$; .005, $\frac{5}{1000}$.

Hence may be observed the contrary effects of cyphers being annexed to whole numbers, and decimals.

It is likewise evident from the table, that, since the places of decimals decrease in a tenfold proportion from units downwards, so they consequently increase in a tenfold proportion from the right hand toward the left, as the places of whole numbers do: For, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $\frac{10}{100} = \frac{1}{10}$; $\frac{10}{10} = 1$, and $1 \times 10 = 10$, which proves that decimals are subject to the same law of Notation, and consequently of operation, as whole numbers are.

Decimal Fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of those, which have fewer places, so many cyphers, as make them equal in places with that which has the most. So these decimals, .5, .06, .455, may be reduced to the decimals .500, .060, and .455, which have, all, 1000 for their denominator.

Of Decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places: Thus, .5 is greater than .459, for if it be reduced to the same denominator with .459, it will be .500.

A mixed number, viz. a whole number, with a decimal annexed, is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number, and the denominator, that of the decimal part. So 45.309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction:

Thus, $45 \times 1000 + .309 = \frac{45309}{1000}$ as above.

ADDITION of DECIMALS.

R U L E.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.
2. Find their sum as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

E X A M P L E S.

1. Find the sum of $19,073 + 2,3597 + 223 + ,0197581 + 3478,1 + 12,358$.

$$\begin{array}{r}
 19,073 \\
 2,3597 \\
 223 \\
 ,0197581 \\
 3478,1 \\
 12,358 \\
 \hline
 3734,9104581 \text{ the sum.}
 \end{array}$$

2. Required the sum of $429 + 21,37 + 355,003 + 1,07 + 1,7$?
Ans. 808,143.
3. Required the sum of $5,3 + 11,973 + 49 + ,9 + 1,7314 + 34,3$?
Ans. 102,2044.
4. Required the sum of $973 + 19 + 1,75 + 93,7164 + ,9501$?
Ans. 1088,4165.

SUBTRACTION of DECIMALS.

R U L E.

Place the numbers according to their value ; then subtract as in whole numbers and point off the decimals as in Addition.

E X A M P L E S.

1. Find the difference of 1793,13 and 817,05693.

$$\begin{array}{r}
 \text{From } 1793,13 \\
 \text{Take } 817,05693 \\
 \hline
 \end{array}$$

Remainder 976,07307

2. From 171,195 take 125,9176. *Ans.* 45,2774.
3. From 219,1384 take 195,91. *Ans.* 23,2284.
4. From 480 take 245,0075. *Ans.* 234,9925.

MULTIPLICATION

MULTIPLICATION of DECIMALS.

C A S E I.

R U L E.

1. Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.
2. Point off so many figures from the product as there are decimal places in both the factors ; and if there be not so many places in the product, supply the defect by prefixing cyphers.

E X A M P L E S.

1. Multiply ,02345
by ,00163

$$\begin{array}{r} 7035 \\ 14070 \\ 2345 \\ \hline \end{array}$$

,0000382235 the product.

2. Multiply 25,238 by 12,17.

Ans. 307,14646.

3. Multiply ,3759 by ,945.

Ans. ,3552255.

4. Multiply ,84179 by ,0385.

Ans. ,032408915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has cyphers.

$$\text{So ,345} \left\{ \begin{array}{l} \text{Multi-} \\ \text{by} \end{array} \left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \end{array} \right\} \text{ makes } \left\{ \begin{array}{l} 3,45 \\ 34,5 \\ 345 \end{array} \right.$$

For ,345 \times 10 is 3,450, &c.

C A S E II.

To contract the operation, so as to retain so many decimal places in the Product as may be thought necessary.

R U L E.

1. Write the unit's place of the multiplier under that figure of the multiplicand, whose place you would reserve in the product ; and dispose of the rest of the figures in a contrary order to what they are usually placed in.
2. In multiplying, reject all the figures which are to the right hand of the multiplying digit, and set down the products, so that their right hand figures may fall in a straight line below each other ; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the preceding figures, when you begin to multiply, and the sum will be the product required.

EXAMPLES.

EXAMPLE.

1. It is required to multiply 56,7534916 by 5,376928, and to retain only five places of decimals in the product.

<i>Contracted way.</i>	<i>Common way.</i>
56,7534916	56,7534916
829673,5	5,376928
<hr/>	<hr/>
28376746 ..	45 40279328
1702605 ...	113 5069832
397274	5107 814244
34052	34052 09496
5108	397274 4412
113	1702604 748
45	28376745 80
<hr/>	<hr/>
305,15943	305,15943 80818048
<hr/>	<hr/>

By the operation in the *common way*, it is evident, that all the figures, which are cut off at the right hand by the perpendicular line, are wholly omitted in the *contracted way*, and, the last product here is the first there ; consequently, the reason of placing the multiplier in a reverse order, must appear very plainly.

DIVISION of DECIMALS.

RULE.*

1. The places of decimal parts in the divisor and quotient counted together must always be equal to those in the dividend ; therefore, divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule require, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness.

EXAMPLES.

* The reason of pointing off so many decimal places in the quotient, as those in the dividend exceed those in the divisor, will easily appear ; for, since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of multiplication : It therefore follows, that the quotient contains so many as the dividend exceeds the divisor.

EXAMPLES.

1. 219)117841075(538037, &c.

1095

834

657

1771

1752

1907

1752

1555

1533

22

In Example 1st, the divisor having no decimals, the quotient must have so many as there are in the dividend. In Example 2, the dividend being an integer must have at least so many cyphers annexed, as there are decimals in the divisor, and so far the quotient will be whole numbers; then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the Rule.

2. 3719)38,0000(102,178, &c.

3719

8100

7438

6620

3719

20010

26033

29770

29752

18

3d. 133)5737(43.1353+

(4th.) 23.7)65321(2756.16+

5th. *72)918.217(12.53+

(6th.) 25.17)315.6293(12.53+

7th. 317)29,417(92+

(8th.) 379)0059374(15+

9th. 375)173948375(463862+

Having a multiplier, to find a divisor which shall give a quotient equal to the product by that multiplier.

RULE.

Divide unity by the given multiplier, and the quotient will be the divisor sought.

What divisor is that, by which dividing 5357, shall give a quotient equal to the product of the same number multiplied by 250?

250)1,000(.004 the answer. And .004)5357,000(1339250.

Proof. $5357 \times 250 = 1339250$.

Having a divisor, to find a multiplier which shall give a product equal to the quotient by that divisor.

RULE.

Divide unity by the given divisor, and the quotient will be the multiplier sought.

What multiplier is that, by which multiplying 5357, shall give a product equal to the quotient of the same number divided by .004?

.004)1,000(250 the answer: Therefore, $5357 \times 250 = 5357 \div .004 = 1339250$.

CASE II.

To contract Division, when there are many decimals in the dividend, and the divisor is large.

RULE.

* The following questions are left unpunctuated in the quotient, to exercise the learner.

R U L E.

1. Whatever place of the dividend corresponds with the unit's place of the divisor, at the first step of the Division, the same place must the first figure of the quotient have.

2. In dividing, reject the last right hand figure of the divisor, at every step (instead of bringing down a figure, as in common) and make the last remainder the dividend for the new divisor at every step : Thus continue the division until the divisor shall be exhausted.

99,5678; 4,6789837568 (,0469931 quotient.

3 982712

99,567)696271

597402

99,56) 98869

89604

99,5) 9265

8955

99) 310

297

9) 13

9

Remainder 4

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers,) it is performed by removing the separatrix, in the dividend, so many places toward the left hand as there are cyphers in the divisor.

E X A M P L E S.

$$\begin{array}{l} 10 \} \\ 100 \} \text{Dividing} \left\{ \begin{array}{l} 7654 \\ 7654 \end{array} \right\} \text{The Quotient is} \left\{ \begin{array}{l} 765,4 \\ 76,54 \\ 7,654 \\ ,7654 \end{array} \right. \\ 1000 \} \\ 10000 \} \end{array}$$

R E D U C T I O N of D E C I M A L S.

C A S E I.

To reduce a Vulgar Fraction to its equivalent Decimal.

R U L E.*

Divide the numerator by the denominator, as in division of decimals, and the quotient will be the decimal required : Or, so many

* Let the given vulgar fraction, whose decimal expression is required, be $\frac{9}{15}$. Now, since every decimal fraction has 10, 100, 1000, &c. for its denominator; and if two fractions be equal, it will be, as the denominator of one is to its numerator; so is the denominator of the other to its numerator, therefore, as 15 : 9 :: 10, &c. : $\frac{9 \times 10}{15}$
 $= \frac{90}{15} = 6$ the numerator of the decimal required; and is the same as by the rule.

many cyphers as you annex to the given numerator, so many places must be pointed off in the quotient, and if there be not so many places of figures in the quotient, the deficiency must be supplied by prefixing so many cyphers before the quotient figures.

E X A M P L E S.

1. Reduce
- $\frac{1}{2}$
- to a decimal.

$$8) \overline{1,000}$$

,125 Answer.

2. Reduce
- $\frac{3}{8}$
- ,
- $\frac{5}{8}$
- and
- $\frac{2}{3}$
- to decimals.
- Answers.*
- ,375. ,625. ,666
- $\frac{1}{3}$
- .

3. Reduce
- $\frac{1}{4}$
- ,
- $\frac{1}{2}$
- ,
- $\frac{3}{4}$
- ,
- $\frac{1}{3}$
- ,
- $\frac{4}{5}$
- ,
- $\frac{5}{6}$
- and
- $\frac{7}{8}$
- to decimals.

Answers. ,25. ,5. ,75. ,333 $\frac{1}{3}$. ,8. ,833 $\frac{1}{3}$. ,875.

4. Reduce
- $\frac{5}{19}$
- ,
- $\frac{27}{39}$
- ,
- $\frac{17}{80}$
- and
- $\frac{9}{36}$
- to decimals.

Answers. ,263 $\frac{1}{3}$. ,692 $\frac{1}{3}$. ,025. ,25.

5. Reduce
- $\frac{7}{75}$
- ,
- $\frac{9}{1125}$
- and
- $\frac{5}{1875}$
- to decimals.

Answers. ,0186 $\frac{1}{3}$. ,00797 $\frac{1}{3}$. ,00266 $\frac{1}{3}$.

C A S E II.

To reduce numbers of different denominations, as of Money, Weight and Measure, to their equivalent decimal values.

R U L E.*

1. Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number for a divisor as will bring it to the next superior denomination, and draw a line perpendicularly between them.

3. Begin with the highest, and write the quotient of each division, as decimal parts, on the right hand of the dividend next below it, and so on, till they are all used, and the last quotient will be the decimal sought.

1. Reduce 17s. 8
- $\frac{1}{2}$
- d. to the decimal of a pound.

$$\begin{array}{r|l} 4 & 3, \\ 12 & 8,75 \\ 20 & 17,729166 \text{ \&c.} \end{array}$$

,880458 &c. the decimal required.

Here, in dividing 3 by 4, I suppose 2 cyphers to be annexed to the 3, which make it 3,00, and ,75 is the quotient, which I write against 8 in the next line; this quotient, viz. 8,75 being pence and decimal parts of a penny, I divide them by 12, which brings them to shillings and decimal parts, I therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce

* The reason of the rule may be explained from the first example: Thus, three farthings are $\frac{3}{4}$ of a penny, which, reduced to a decimal, is ,75; consequently, 8 $\frac{1}{2}$ d. may be expressed, 8,75d. but 8,75 is $\frac{875}{100}$ of a penny = $\frac{875}{1200}$ of a shilling, which, reduced to a decimal, is ,729166 $\frac{1}{3}$. In like manner, 17,729166 $\frac{1}{3}$ are $\frac{17729166}{100000}$ = ,886458 $\frac{1}{3}$ as by the rule.

2. Reduce 1, 2, 3, 4, and so on to 19 shillings, to decimals.

<i>Shillings.</i>	1	2	3	4	5	6	7	8	9	10
<i>Answers.</i>	,05.	,1.	,15.	,2.	,25.	,3.	,35.	,4.	,45.	,5.
<i>Shil.</i>	11	12	13	14	15	16	17	18	19	
<i>Ans.</i>	,55.	,6.	,65.	,7.	,75.	,8.	,85.	,9.	,95.	

Here, when the shillings are even, half the number, with a point prefixed, is their decimal expression; but, if the number be odd, annex a cypher to the shillings, and then halving them, you will have their decimal expression.

* 3. Reduce 1, 2, 3, and so on to 11 pence, to the decimals of a shilling.

<i>Pence.</i>	1	2	3	4	5	6
<i>Answers.</i>	,083+	,166.	,25.	,333+	,416+	,5.
<i>Pence.</i>	7	8	9	10	11	
<i>Answers.</i>	,583+	,666+	,75.	,833+	,916+	

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

<i>Pence.</i>	1	2	3	4	5	
<i>Answers.</i>	,00416+.	,0083+.	,0125.	,01666+.	,0208+.	
<i>Pence.</i>	6	7	8	9	10	11
<i>Ans.</i>	,025.	,02916+.	,0333+.	,0375.	,0416+.	,04583+.

5. Reduce 1, 2 and 3 farthings to the decimals of a penny.

1qr.=,25d. 2qrs.=,5d. and 3qrs.=,75d. Answers.

6. Reduce 1, 2 and 3 farthings to the decimals of a shilling.

Ans. 1qr.=,02083+s. 2qrs.=,04166+s. 3qrs.=,0625s.

7. Reduce 1, 2 and 3 farthings to the decimals of a pound.

Ans. 1qr.=,0010416+£. 2qrs.=,002083+£. 3qrs.=,003125£.

8. Reduce 13s. 5½d. to the decimal of a pound.

Ans. ,6729+.

9. Reduce 7cwt. 3qr. 17lb 10oz. 12dr. to the decimal of a ton.

Ans. ,39538+.

10. Reduce 10oz. 13pwt. 9gr. to the decimal of a pound Troy.

Ans. ,8890625.

11. Reduce 3qrs. 3n. to the decimal of a yard.

Ans. ,9375.

12. Reduce 5fur. 12po. to the decimal of a mile.

Ans. ,6625.

13. Reduce 55m. 37sec. to the decimal of a day.

Ans. ,03862+.

C A S E III.

To find the decimal of any number of shillings, pence and farthings, by Inspection.

RULE.

* The answers to this question are the same as the decimal parts of a foot.

R U L E.*

1. Write *half* the greatest even number of shillings for the first decimal figure.

2. Let the farthings in the given pence and farthings possess the second and third places; observing to increase the *second* place, or place of *hundredths*, by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2, when they exceed 36.

E X A M P L E S.

1. Find the decimal of 13s. 9 $\frac{3}{4}$ d. by Inspection.

$$\begin{array}{rcl} ,6 & . & = \frac{1}{2} \text{ of } 12s. \\ 5 & & \text{for the odd shilling.} \\ 39 & & = \text{the farthings in } 9\frac{3}{4}d. \\ \text{Add } 2 & & \text{for the excess of } 36. \end{array}$$

—
 ,691 = decimal required.

2. Find, by Inspection, the decimal expressions of 18s. 3 $\frac{1}{4}$ d. and 17s. 8 $\frac{1}{2}$ d. *Ans.* £. ,914 and £. ,885.

3. Value the following sums, by Inspection, and find their total, viz. 15s. 3d. + 8s. 11 $\frac{1}{2}$ d. + 10s. 6 $\frac{1}{4}$ d. + 1s. 8 $\frac{1}{2}$ d. + $\frac{1}{2}$ d. + 2 $\frac{3}{4}$ d. *Ans.* £. 1,834 the total.

C A S E IV.

To find the value of any given decimal in the terms of the Integer.

R U L E.

1. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferior denomination, and cut off a remainder as before.

3. Proceed

* The invention of the rule is as follows: As shillings are so many 20ths of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimal; but when they are odd, their half will always consist of two figures, the first of which will be half the even number, next less, and the second a 5: Again, farthings are so many 960ths of a pound, and had it happened that 1000, instead of 960, had made a pound, it is plain any number of farthings would have made so many thousandths, and might have taken their place in the decimal accordingly. But 960 increased by $\frac{1}{24}$ part of itself, is = 1000, consequently, any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal expression for them: Whence, if the number of farthings be more than 12, $\frac{1}{24}$ part is greater than $\frac{1}{2}qr.$ and, therefore, 1 must be added; and when the number of farthings is more than 36, $\frac{1}{24}$ part is greater than $1\frac{1}{2}qr.$ for which 2 must be added.

3 Proceed in this manner through all the parts of the integer, and the several denominations, standing on the left hand, make the answer.

E X A M P L E S.

1. Find the value of ,73968 of a pound.

$$\begin{array}{r} 20 \\ \hline 14.79360 \\ 12 \\ \hline 9.52320 \\ 4 \\ \hline \end{array}$$

2,09280 *Ans.* 14s. 9½d.

2. What is the value of ,679 of a shilling? *Ans.* 8,148.

3. What is the value of ,9999£. ? *Ans.* 19s. 11¾d. ⅓—3r £.1.

4. What is the value of ,617 of a Cwt. ?

Ans. 2qrs. 13lb 10z. 10⅙dr.

5. What is the value of ,8593 of a lb Troy ?

Ans. 10oz. 6pat. 5gr.

6. What is the value of ,397 of a yard ? *Ans.* 1qr. 2n.

7. What is the value of ,8469 of a degree ?

Ans. 58m. 6fur. 35po. 0ft. 11in.

8. What is the value of ,569 of a year ?

Ans. 207da. 16h. 26m. 24sec.

9. What is the value of ,713 of a day ? *Ans.* 17h. 6m. 43sec.

C A S E V.

To find the value of any decimal of a pound by Inspection.

R U L E.

Double the first figures or place of tenths, for shillings, and if the second figure be 5, or more than 5. reckon another shilling; then, after the 5 is deducted, call the figures in the second and third places so many farthings, abating 1 when they are above 12, and 2 when above 36, and the result will be the answer.

Note. When the Decimal has but 2 figures, if any thing remain after the shillings are taken out, a cypher must be annexed to the right hand, or supposed to be so.

E X A M P L E S.

1. Find the value of ,876£. by Inspection.

16s. = double of 8.

1s. for the 5 in the second place, which is to be taken out of 7.

And 6½d. = 26 farthings remain, to be added.

Deduct ¼d. for the excess of 12.

17s. 6½d. the answer.

2. Find

2. Find, by Inspection, the value of ,49£.

8s. - = double of 4.

1s. - for the 5 in the place of hundredths.

10d. = 40 farthings, a 0 being annexed to the remaining 4.

Deduct $\frac{1}{2}$ d. for the excess of 36.

9s. 9 $\frac{1}{2}$ d. the answer.

3. Find the value of ,097£. by Inspection. *Ans.* 1s. 11 $\frac{1}{4}$ d.

4. Value the following decimals by Inspection, and find their sum, viz., ,785£. + ,537£. + ,916£. + ,74£. + ,5£. + ,25£. + ,09£. + ,008£. *Ans.* £.3 16s. 6d.

FEDERAL MONEY.

The pupil being well acquainted with Decimals, it will be proper to introduce here an account of the Federal Money, as settled by Congress, the 8th of August, 1786, when it was “*Resolved*,

“ That the standard of the United States of America, for gold and silver, shall be eleven parts fine and one part alloy.

“ That the *Money Unit* of the United States (being by the Resolve of Congress, of the 6th of July 1785, a Dollar) shall contain, of fine silver, $375\frac{64}{1000}$ grains.

“ That the money of account, to correspond with the division of coins, agreeably to the above Resolve, proceed in a decimal ratio, agreeably to the forms and manner following, viz.

“ Mill, the lowest money of account, of which 1000 shall be equal to the federal dollar, or money unit, - 0,001.

“ Cent, the highest copper piece, of which 100 shall be equal to the federal dollar, - - - 0,010.

“ Dime, the lowest silver coin, of which 10 shall be equal to the dollar, - - - 0,100.

“ Dollar, the highest silver coin, - - - 1,000

“ That, betwixt the dollar and the lowest copper coin, as fixed by the Resolve of Congress of the 6th of July, 1785, there shall be *three* silver coins, and *one* copper coin.

“ That the silver coins shall be as follow : One coin containing $187\frac{82}{1000}$ grains of fine silver, to be called a *Half Dollar* : One coin containing $75\frac{128}{1000}$ grains of fine silver, to be called a *Double Dime* : And one coin containing $37\frac{64}{1000}$ grains of fine silver, to be called a *Dime*.

“ That the two copper coins shall be as follow : One equal to the one hundredth part of the federal dollar, to be called a *Cent* ; and one equal to the two hundredth part of the federal dollar, to be called a *Half Cent*.

“ That 2 $\frac{1}{4}$ Avoirdupois weight of copper, shall constitute 100 Cents.

“ That

"That there shall be two gold coins: One containing $246\frac{268}{1000}$ grains of fine gold, equal to 10 dollars, to be stamped with the impression of the American Eagle, and to be called an *Eagle*: One containing $123\frac{134}{1000}$ grains of fine gold, equal to 5 dollars, to be stamped in like manner, and to be called a *Half Eagle*.

"That the mint price of a pound Troy weight, of uncoined silver, eleven parts fine and one part alloy, shall be 9 dollars, 9 dimes and 2 cents.

"That the mint price of one pound Troy weight of uncoined gold, eleven parts fine and one part alloy, shall be 209 dollars, 7 dimes and 7 cents."

As the money of account proceeds in a decuple, or tenfold proportion, so any number of dollars, dimes, cents and mills, is simply the expression of dollars and decimal parts of a dollar: Thus 9 dollars and 8 dimes are expressed $9.8 = 9\frac{8}{10}$ doll.—12 dollars, 4 dimes and 7 cents thus, $12.47 = 12\frac{47}{100}$ doll.—20 dollars, 3 dimes, 4 cents and 5 mills, thus $20.345 = 20\frac{345}{1000}$ doll.—100 dollars and 9 mills, thus $100.009 = 100\frac{9}{1000}$ doll. and 50 dollars, 5 cents, thus, $50.05 = 50\frac{5}{100}$ doll. wherefore, it is, in all respects, added, subtracted, multiplied and divided, the same as decimals; and, of all coins, it is the most simple.

			Marked.	Mills.	Cent.	Dime.	Dol.	Eag.
Note.	} 10 Mills, 10 Cents, 10 Dimes, 10 Dollars,	} make one	Cent. m. c.	10 =	1			
			Dime. d.	100 =	10 =	1		
			Dollar. D.	1000 =	100 =	10 =	1	
			Eagle. E.	10000 =	1000 =	100 =	10 =	1.

ADDITION of the FEDERAL MONEY.

Add $25\frac{1}{2}$ eagles; 7 dollars, 8 dimes, 3 cents, 4 mills; 125 dollars, 8 cents; 5 eagles, 9 mills; 18 dollars, 7 cents and 4 mills together.*

	Eagles. Dollars. Dimes. Cents. Mills.
1st.	255
2d.	7,834
3d.	125.080
4th.	50.009
5th.	18,074
Sum.	455,997

Note. That the dollars occupy the first place at the left hand of the comma, and eagles, all the places at the left of dollars: But eagles and dollars, reckoned together, express the number of dollars contained in the sum, as 349 is 34 eagles and 9 dollars; equal to 349 dollars, &c.

SUBTRACTION.

* It may be observed that the sum exhibits the particular number of each different piece of money contained in it, viz. $455997\frac{7}{10}$ cents = $4559\frac{97}{100}$

E. D. d. c. m.

dimes = $455\frac{997}{1000}$ dollars = $45\frac{5997}{10000}$ eagles = 4 5 5, 9 9 7.

N

Also,

S U B T R A C T I O N.

	<i>E. d. c. m.</i>	<i>D. d. c. m.</i>	<i>D. d. c. m.</i>
<i>From</i>	13479, 8 1 5	495641, 0 0 1	793,
<i>Take</i>	8985, 9 4 6	218796, 7 9 5	459, 3 7 9
	<hr/>	<hr/>	<hr/>
<i>Rem.</i>	4493, 8 6 9	276844, 2 0 6	338, 6 2 1
	<hr/>	<hr/>	<hr/>
<i>Proof.</i>	13479, 8 1 5		

M U L T I P L I C A T I O N.

When you have pointed off the decimals in the product, according to the rule in Multiplication of Decimals, all beyond the mills, or third place of decimals, are decimal parts of a mill.

D. d. c. m.

Bought 37 horses for shipping, at 48, 5 7 3 per head : What came they to ?

<i>D. d. c. m.</i>	<i>D. d. c. m.</i>
48, 5 7 3	<i>Mult.</i> 45, 8 4 6
37	<i>by</i> 23, 0 9 4
<hr/>	<hr/>
340011	183384
145719	412614
<hr/>	<hr/>
1797, 2 0 1	1375380
<i>D. d. c. m.</i>	91692
	<hr/>
	<i>Anf.</i> 1058,767 ⁵²⁴ / ₁₀₀₀

D I V I S I O N.

1. If 1000 oranges cost 10 dollars, what is that apiece ?

<i>D. d. c. D. d. c.</i>
1000)10,00(0,0 1 <i>Anf.*</i>
10 0 0
<hr/>

2. If 3730 bushels of corn cost 2025,39 dollars : What is that per bushel ?

<i>D. d. c. m.</i>
3730)2025,39(0,5 4 3 <i>Anf.</i>
1865 0
<hr/>

3. Divide 12976 dollars between 7 men.

7)12976

1853,714²/₇ *Anf.* in dol. dim. &c.

160 39
149 20
<hr/>
11 190
11 190
<hr/>

D E C I M A L

Also, the names of the coins, less than a dollar, are significant of their values. For the *mill*, which stands in the 3d place at the right hand of the comma, or place of thousandths, is contracted from *mille*, the Latin for *thousand* : *Cent*, which occupies the second place, or place of hundredths, is an abbreviation of *centum*, the Latin for *hundred* : And *dime*, which is in the first place, or place of tenths, is derived from *disme*, the French for *tenths*.

* And here I would remind the learner, that when he has brought down all his whole numbers, or dollars, in the dividend, he must place a comma in the quotient ; and if, when he has brought down the next figure, he cannot have the divisor once, he must place a cypher at the right hand of the comma, in the place of dimca.

DECIMAL TABLES OF COIN, WEIGHT and MEASURE.

TABLE I. COIN.				Pwts.	Decimals.	Pounds.	Decimals.	Drams.	Decimals.
£. the Integer.				5	,020833	11	,098214	6	,023437
Shil.	dec.	Shil.	dec.	4	,016666	10	,089285	5	,019331
19	,95	9	,45	3	,0125	9	,080357	4	,015625
18	,9	8	,4	2	,008333	8	,071428	3	,011718
17	,85	7	,35	1	,004166	7	,0625	2	,007812
16	,8	6	,3			6	,053571	1	,003906

				Grains.	Decimals.
15	,75	5	,25	12	,002083
14	,7	4	,2	11	,00191
13	,65	3	,15	10	,001736
12	,6	2	,1	9	,001562
11	,55	1	,05	8	,001389
10	,5			7	,001215

Pence.	Decimals.
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farthings	Decimals
3	,003125
2	,0020833
1	,0010416

TABLE II.
COIN & Long Meaf.
1 Shil. & 1 Foot
the Integer.

Pence & Inches.	Decimals.
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Farthings	Decimals
3	,0625
2	,041666
1	,020833

TABLE III.
TROY WEIGHT.
1½ the Integer.

Ounces the same as	Pwts.	Decimals.
TABLE II.	10	,041666
	9	,0375
	8	,033333
	7	,029166
	6	,025

Grains.	Decimals.
12	,002083
11	,00191
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

1 Oz. the Integer.
Pennyweights the
same as Shillings in
the first Table.

Grains.	Decimals.
12	,025
11	,022916
10	,020833
9	,01875
8	,016666
7	,014583
6	,0125
5	,010416
4	,008333
3	,00625
2	,004166
1	,002083

TABLE IV.
Avoirdupois Wt.
1½ the Integer.

Qrs.	Decimals.
3	,75
2	,5
1	,25

Pounds.	Decimals.
27	,241071
26	,232143
25	,223214
24	,214286
23	,205357
22	,196428
21	,1875
20	,178571
19	,169643
18	,160714
17	,151786
16	,142857
15	,133928
14	,125
13	,116071
12	,107143

Pounds.	Decimals.
5	,044643
4	,035714
3	,026736
2	,017857
1	,008928

Ounces.	Decimals.
15	,008370
14	,007812
13	,007254
12	,006696
11	,006138
10	,00558

Qrs. of ozs.	Decim.
9	,005022
8	,004464
7	,003906
6	,003348
5	,00279
4	,002232
3	,001674
2	,001116
1	,000558

TABLE V.
Avoirdupois Weight.
1½ the Integer.

Ounces.	Decimals.
15	,9375
14	,875
13	,8125
12	,75
11	,6875
10	,625
9	,5625
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams.	Decimals.
15	,059493
14	,055587
13	,051681
12	,047775
11	,043868
10	,039962
9	,036056
8	,03215
7	,027343

TABLE VI.
CLOTH MEASURE.
1 Yard the Integer.
Quarters.

Quarters.	Decimals.
3	,75
2	,5
1	,25

TABLE VII.
LONG MEASURE.
1 Mile the Integer.

Yards.	Decimals.
1000	,568182
900	,511364
800	,454545
700	,397727
600	,34
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,034091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001136
1	,000568

Feet.	Decimals.
2	,0003787
1	,0001892

Inches.	Decimals.
6	,0000947
5	,000079
4	,0000632
3	,0000474
2	,0000316
1	,0000158

100 COMPOUND MULTIPLICATION.

TABLE VIII.		Days.	Decimals.	1 Day the Integer.	Hours.	Decimals.
LIQUID MEAS.		90	,246575	Hours.	Decimals.	5
1 Gallon the Integer.		80	,219178	23	,958333	4
Quarts the same as		70	,191781	22	,916666	3
qrs. in TABLE VI.		60	,164383	21	,875	2
1 Pint.		50	,136986	20	,833333	1
3 Gills.		40	,109589	19	,791666	
2		30	,082192	18	,75	
1		20	,054794	17	,708333	
		10	,027397	16	,666666	
TABLE IX.		9	,024657	15	,625	
TIME.		8	,021918	14	,583333	
1 Year the Integer.		7	,019178	13	,541666	
Months the same as		6	,016438	12	,5	
Pence in TABLE II.		5	,013698	11	,458333	
Days.	Decimals.	4	,010959	10	,416666	
365	1.000000	3	,008219	9	,375	
300	,821928	2	,005479	8	,333333	
200	,547945	1	,002739	7	,291666	
100	,273973			6	,25	

COMPOUND MULTIPLICATION*

Is extremely useful in finding the value of Goods, &c. And as in Compound Addition we carry from the lowest denomination to the next higher, so we begin and carry in Compound Multiplication: One general rule being to multiply the price by the quantity.

CASE I.

When the quantity does not exceed 12 yards, pounds, &c. Set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and, in multiplying by it, observe the same rules for carrying from one denomination to another as in Compound Addition.

INTRODUCTORY EXAMPLES.

	1.	2.	3.	4.
	£. s. d.	£. s. d.	£. s. d.	£. s. d.
Multiply	15 17 1	25 12 8	45 9 10	67 18 6½
by	2	3	4	5
	19 6 8¼	13 12 11	31 16 8¼	12 17 10½
	6	7	8	9

9.

* The product of a number, consisting of several parts or denominations, by any simple number whatever, will be expressed by taking the product of that simple number, and each part by itself, as so many distinct questions: Thus £.33 15s. 9d. multiplied by 5, will be £.165 77s. 45d. = (by taking the shillings from the pence, and the pounds from the shillings, and placing them in the shillings and pounds respectively,) £.168 18s. 9d. and this will be true when the multiplicand is any compound number whatever.

COMPOUND MULTIPLICATION. 101

^{9.}	^{10.}	^{11.}	^{12.}
£. s. d.	£. s. d.	£. s. d.	£. s. d.
4 13 4 $\frac{3}{4}$	8 15 11 $\frac{3}{4}$	6 19 1 $\frac{1}{2}$	14 17 18 $\frac{1}{4}$
10	11	12	9

In the last example, I say, 9 times 1 is 9 farthings = 2 $\frac{1}{4}$ d. I set down $\frac{1}{4}$ and carry 2, saying, 9 times 8 is 72, and 2 I carry makes 74 pence = 6s. 2d. I set down 2 in the pence, and carry 6; then, 9 times 7 (the unit figure in the shillings) is 63, and 6 I carry is 69, I set down 9 under the unit figure of the shillings, and carry 6, saying, 9 times 1 is 9, and 6 I carry is 15, then I halve 15, which is 7 and 1 over, which I set in the ten's place in shillings, and that, with the 9, makes 19 shillings; then I carry the 7 as pounds: Lastly, 9 times 4 is 36, and 7 I carry, are 43 pounds. I set down 3, and carry 4, saying, 9 times 1 is 9, and 4 I carry makes 13, which I set down, and the product is £.133 19s. 2 $\frac{1}{4}$ d.

PRACTICAL QUESTIONS.

- What will 9 yards of cloth, at 5s. 4d. per yard, come to?

	£.	s.	d.	
	0	5	4	price of one yard.
Multiplied by			9	yards.
Ans. £.	2	8	0	price of 9 yards.

Questions.	s.	d.	
2d. 3 yards, at	15	4	} per yard.
3d. 6 — at	9	10	
4th. 5 — at	29	6	
5th. 9 — at	13	7 $\frac{1}{2}$	
6th. 7 — at	39	10 $\frac{3}{4}$	

CASE II.

When the multiplier, that is, the quantity, is above 12, you must multiply by two such numbers, as, when multiplied together, will produce the given quantity.

EXAMPLES.

- What will 144 yards of cloth cost, at 3s. 5 $\frac{1}{2}$ d. per yard?

	£.	s.	d.	
	0	3	5 $\frac{1}{2}$	price of one yard.
Multiplied by			12	
Produces	2	1	6	price of 12 yards.
Multiplied by			12	
Answer, £.	24	18	0	price of 144 yards.

Questions.

102 COMPOUND MULTIPLICATION.

Questions.				Answers.			
		s.	d.	£.	s.	d.	
2d.	24 yards at	6	$3\frac{1}{4}$	per yard. =	7	11	6
3d.	27 — at	9	10	— =	13	5	6
4th.	44 — at	12	$4\frac{1}{2}$	— =	27	4	6
5th.	55 — at	8	$3\frac{1}{4}$	— =	22	14	$10\frac{1}{4}$
6th.	72 — at	19	11	— =	71	14	0

C A S E III.

When the quantity is such a number, as that no two numbers in the table will produce it, exactly : Then multiply by two such numbers as come the nearest to it ; and for the number wanting, multiply the given price of one yard by the said number of yards wanting, and add the products together for the answer ; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

E X A M P L E S.

1. What will 47 yards of cloth, at 17s. 9d. per yard, come to ?

	£.	s.	d.	
	0	17	9	price of 1 yard.
Multiplied by			5	
Produces	4	8	9	price of 5 yards.
Multiplied by			9	
Produces	39	18	9	price of 45 yards.
Add	1	15	6	price of 2 yards.
Ans. £.	41	14	3	price of 47 yards.

Note. This may be performed by first finding the value of 48 yards, from which if you subtract the price of 1, the remainder will be the answer as above.

Questions.				Answers.			
Yds.		s.	d.	£.	s.	d.	
2d.	75 at	5	$7\frac{1}{2}$	per yard. =	21	1	$10\frac{1}{2}$
3d.	$67\frac{1}{2}$ —	16	$3\frac{1}{4}$	— =	54	18	$3\frac{1}{4}$
4th.	59 —	9	7	— =	28	5	5
5th.	$135\frac{1}{4}$ —	43	4	— =	293	0	10
6th.	$112\frac{1}{4}$ —	15	$11\frac{1}{4}$	— =	90	1	$7\frac{3}{4}$

C A S E IV.

When the quantity is any number above the Multiplication Table : Multiply the price of 1 yard by 10, which will produce the price of 10 yards : This product, multiplied by 10, will give the price of 100 yards ; then, if the quantity do not exceed hundreds, you must multiply the price of one hundred by the number of hundreds

COMPOUND MULTIPLICATION. 103

dreds in your question ; the price of ten by the number of tens ; and the price of unity, or 1, by the number of units : Lastly, add these several products together, and the sum will be the answer.

EXAMPLES.

1. What will 359 yards of cloth, at 4s. 7½d. per yard, amount to ?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 0 \quad 4 \quad 7\frac{1}{2} \text{ price of one yard.} \\ \hline 10 \end{array}$$

$$\begin{array}{r} 2 \quad 6 \quad 3 \text{ price of 10 yards.} \\ \hline 10 \end{array}$$

$$\begin{array}{r} 23 \quad 2 \quad 6 \text{ price of 100 yards.} \\ \hline 3 \end{array}$$

$$69 \quad 7 \quad 6 \text{ price of 300 yards.}$$

$$5 \text{ times the price of 10 yds.} = 11 \quad 11 \quad 3 \text{ price of 50 yards.}$$

$$9 \text{ times the price of 1 yd.} = 2 \quad 1 \quad 7\frac{1}{2} \text{ price of 9 yards.}$$

$$\text{Answer £.83} \quad 0 \quad 4\frac{1}{2} \text{ price of 359 yards.}$$

Questions.	s.	d.		£.	s.	d.
2d. 297 yards, at	17	3½	per yard. =	256	15	7½
3d. 473 ———	9	11¼	——— =	235	0	5¼
4th. 512 ———	15	10	——— =	405	6	8
5th. 624 ———	12	8	——— =	395	4	0
6th. 765 ———	19	9½	——— =	757	0	7½

CASE V.

When the quantity does not exceed 200, nor the price 12 pence, then, by the pence table, find what it comes to, at one penny per yard, &c. and, multiplying this sum by the number of pence in the price, the product will be the answer.

EXAMPLES.

1. What will 129 yards cost, at 9¼d. per yard ?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 129 \text{ pence} = 0 \quad 10 \quad 9 \text{ the price at 1d. per yd.} \\ \hline 9 \end{array}$$

$$4 \quad 16 \quad 9 \text{ the price at 9d. per yd.}$$

$$\text{Half of 10s. 9d.} = 5 \quad 4\frac{1}{2} \text{ the price at } \frac{1}{2} \text{d. per yd.}$$

$$\text{One fourth of 10s. 9d.} = 2 \quad 8\frac{1}{4} \text{ the price at } \frac{1}{4} \text{d. per yd.}$$

$$\text{Answer £.5} \quad 4 \quad 9\frac{3}{4} \text{ the price at 9¼d. per yd.}$$

Questions.

104 COMPOUND MULTIPLICATION.

Questions.		Answers.		
	Yds.		£.	s. d.
2d.	145 $\frac{1}{2}$	at 11 $\frac{1}{4}$ d. per yard	= 6	19 2 $\frac{1}{2}$
3d.	75	— 7	= 2	3 9
4th.	63 $\frac{1}{2}$	— 3 $\frac{1}{4}$	= 0	19 10
5th.	123 $\frac{1}{4}$	— 2 $\frac{1}{2}$	= 1	5 9 $\frac{1}{4}$
6th.	137	— 4 $\frac{1}{4}$	= 2	8 6 $\frac{1}{4}$

C A S E IV.

To find the value of one hundred weight :—As 112 is the gross hundred, so 112 farthings are = 2s. 4d. and 112 pence = 9s. 4d. therefore, if the price be farthings, or not more than 3d. multiply 2s. 4d. by the farthings in the price of 1lb, or, if the price be pence, multiply 9s. 4d. by the pence in the price of 1lb, and in either case the product will be the answer.

E X A M P L E S.

1. What will 1 Cwt. of chalk come to, at 1 $\frac{1}{2}$ d. per lb ?
 112 farthings = 0 2 4 price of 1 Cwt. at $\frac{1}{4}$ d. per lb.
 1 $\frac{1}{2}$ d. = 6 farthings in the price.

Answer £. 0 14 0 price of 1 Cwt. at 1 $\frac{1}{2}$ d. per lb.

2. 1 Cwt. of tin at 2 $\frac{1}{4}$ d. per lb ? s. d.
 2 4 price of 1 Cwt. at $\frac{1}{4}$ d. per lb.
 9 farthings in the price of 1lb.

Answer £. 1 1 0 price of 1 Cwt. at 2 $\frac{1}{4}$ d. per lb.

3. 1 Cwt. of lead at 7d. per lb ? s. d.
 9 4 price of 1 Cwt. at 1d. per lb.
 7 pence in the price of 1 lb.

£. 3 5 4 price of 1 Cwt. at 7d. per lb.

Questions.		Answers.		
	Cwt.		£.	s. d.
4th.	1 of copper at	0 $\frac{3}{4}$ d. per lb	= 0	7 0
5th.	1 of ———	2 $\frac{1}{2}$ d. $\frac{1}{2}$ q.	= 1	4 6
6th.	1 of ———	4 $\frac{1}{2}$ d.	= 2	2 0

C A S E VII.

To find the value of two, or more, hundreds, by having the price of one pound :—First, find the price of 1 Cwt. by the last Case, and then

COMPOUND MULTIPLICATION. 105

then proceed to find the value of the whole by Case 1st. or 2d. as the question may require.

EXAMPLES.

1. What is the value of $5\frac{1}{4}$ cwt. of sugar, at 6d. per lb ?

$$\begin{array}{r} \text{£. s. d.} \\ 0 \quad 9 \quad 4 \text{ price of 1 cwt. at 1d. per lb.} \\ \hline 6 \end{array}$$

$$\begin{array}{r} 2 \quad 16 \quad 0 \text{ price of ditto at 6d. per lb.} \\ \hline 5 \end{array}$$

$$\begin{array}{r} 14 \quad 0 \quad 0 \text{ price of 5 cwt.} \\ 1 \text{ qr. of a cwt. = } 0 \quad 14 \quad 0 \text{ price of } \frac{1}{4} \text{ cwt.} \\ \hline \end{array}$$

$$\text{Ans. £. } 14 \quad 14 \quad 0 \text{ price of } 5\frac{1}{4} \text{ cwt.}$$

Questions.		Answers.	
Cwt.		£. s. d.	
2d. 4	of sugar at $2\frac{1}{2}$ d. per lb. =	4	13 4
3d. $8\frac{1}{2}$	_____ 5 _____ =	19	16 8
4th. 7	_____ $4\frac{3}{4}$ _____ =	15	10 4
5th. $4\frac{3}{4}$	_____ $3\frac{1}{2}$ _____ =	7	15 2
6th. $9\frac{1}{2}$	_____ 8 _____ =	35	9 4

C A S E VIII.

To find the value of a hundred weight, when the price of 1 lb is any number of pounds and shillings ; or shillings pence and farthings : Multiply the price of 1 lb by 7, its product by 8, and this product by 2 ; which last product will be the answer required.

EXAMPLES.

1. What will 1 cwt. of tobacco cost, at 5s. $7\frac{1}{2}$ d. per lb ?

$$\begin{array}{r} \text{£. s. d.} \\ 0 \quad 5 \quad 7\frac{1}{2} \text{ price of 1 lb.} \\ \hline 7 \end{array}$$

$$\begin{array}{r} 1 \quad 19 \quad 4\frac{1}{2} \text{ price of 7 lb.} \\ \hline 8 \end{array}$$

$$\begin{array}{r} 15 \quad 15 \quad 0 \text{ price of 56 lb or } \frac{1}{2} \text{ cwt.} \\ \hline 2 \end{array}$$

$$\text{Ans. £. } 31 \quad 10 \quad 0 \text{ price of 112 lb or 1 cwt.}$$

0

Questions.

Questions.

Answers.

	s.	d.		£.	s.	d.
2d. 1 Cwt. at	3	10½	per lb =	21	14	0
3d. 1 ditto —	9	6	— =	53	4	0
4th. 1 ditto —	16	11½	— =	94	19	4
5th. 1 ditto —	19	8½	— =	110	5	0
6th. 1 ditto — £.1	7	10	— =	155	17	4

PRACTICAL QUESTIONS in WEIGHTS and MEASURES.

1. What is the weight of 4 hogshheads of sugar, each weighing 7cwt. 3qrs. 19lb? *Ans.* 31cwt. 2qrs. 20lb.
2. What is the weight of 6 chests of tea, each weighing 3cwt. 2qrs. 9lb? *Ans.* 21cwt. 1qr. 26lb.
3. If I am possessed of 1½ dozen of silver spoons, each weighing 3oz. 5pwt.—2 dozen of tea spoons, each weighing 15pwt. 14gr.—3 silver cans, each 9oz. 7pwt.—2 silver tankards, each 21oz. 15pwt. and 6 silver porringers, each 11oz. 18pwt. Pray, what is the weight of the whole? *Ans.* 18lb 4oz. 3pwt.
4. In 35 pieces of cloth, each measuring 27¾ yards, how many yards? *Ans.* 971¼ yards.
5. How much brandy in 9 casks, each containing 45gal. 3qts. 1pt.? *Ans.* 412gal. 3qts. 1pt.
6. If I have 9 fields, each of which contains 12 acres, 2 roods and 25 poles; how many acres are there in the whole? *Ans.* 113A. 3r. 25p.

COMPOUND DIVISION*

Is the dividing of numbers of different denominations: In doing which, always begin at the highest, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and so on, till you have divided the whole, taking care to set down your quotient figures under their respective denominations.

INTRODUCTORY EXAMPLES.

	1.				2.				3.		
	£.	s.	d.		£.	s.	d.		£.	s.	d.
Divide	549	17	9 by 5	3)	197	13	7½	4)	731	5	10½
Quot. £.	109	19	6½								

4.

* To divide a number consisting of several denominations by any simple number whatever, is the same as dividing all the parts or members of which that number is composed by the same number. And this will be true when any of the parts are not an exact multiple of the divisor; for, by conceiving the number, by which it exceeds that multiple, to have its proper value by being placed in the next lower denomination, the dividend will still be divided into parts, and the true quotient found as before: Thus, £.41 17s. 6d. divided by 6, will be the same as £.36 117s. 42d. divided by 6, which is equal to £.6 19s. 7d. as by the rule.

4. £. s. d. 2)97 19 10½	5. £. s. d. 6)37 11 4¾	6. £. s. d. 7)193 15 9½	7. £. s. d. 8)739 12 1½
8. £. s. d. 9)471 18 10½	9. £. s. d. 10)79 13 9½	10. £. s. d. 11)58 19 11½	11. £. s. d. 12)13 17 9½

In the first example, having divided the pounds, the 4, which remains, is 4 pounds, which are equal to 80 shillings, and 17 in the shillings make 97; I then find 5 is contained 19 times in 97, and 2 over: I set down 19 under the shillings, and reduce the 2 shillings, which remain, into pence, and they make 24, and the 9 pence, in the question, added, make 33: Then, how often 5 in 33; I find it 6 times, and 3 over: I set down 6 under the pence, and reduce the 3 pence, which remain, to farthings, and they make 12; then, how often 5 in 12; I find it to be twice: I therefore set down ½d. and the 2 which remains, is 2 of a farthing, which I make no account of.

12. T. cwt. qr. lb. oz. dr. 3)29 13 2 25 12 13	13. T. cwt. qr. lb. 4)6 11 3 19	14. Cwt. qr. lb. 5)14 1 12	15. lb. oz. dr. 6)10 13 9
16. lb. oz. 7)20 13	17. lb. oz. pwt. gr. 8)7 10 15 2	18. lb. oz. pwt. gr. 9)56 9 13 19	19. lb. oz. pwt. gr. 10)849 11 12 14
20. lb. oz. pwt. gr. 12)529 10 7 14	21. lb. 3 3 9 gr. 2)37 10 5 1 12	22. lb. 3 3 9 gr. 5)37 11 6 2 17	
23. lb. 3 3 9 gr. 9)348 9 4 0 19	24. Yds. qr. n. 4)76 3 2	25. E. E. qr. n. 5)97 4 1	26. E. Fl. qr. n. 7)58 2 2
27. E. Fr. qr. n. 12)17 5 1	28. Deg. m. f. p. ft. in. bar. 6)97 55 7 35 4 2 1	29. M. f. p. 7)5 5 19	30. P. ft. in. 8)9 13 10

$$\begin{array}{r} 31. \\ \text{Yds. ft. in.} \\ 9 \overline{) 8 \ 2 \ 11} \end{array}$$

$$\begin{array}{r} 32. \\ \text{Ac. r. p.} \\ 3 \overline{) 76 \ 3 \ 37} \end{array}$$

$$\begin{array}{r} 33. \\ \text{Yr. m. w. d. h. m. s.} \\ 4 \overline{) 25 \ 10 \ 3 \ 5 \ 21 \ 39 \ 45} \end{array}$$

$$\begin{array}{r} 34. \\ \text{Y. m. d.} \\ 5 \overline{) 5 \ 9 \ 25} \end{array}$$

$$\begin{array}{r} 35. \\ \text{M. w. d. h. m.} \\ 6 \overline{) 6 \ 3 \ 5 \ 10 \ 29} \end{array}$$

$$\begin{array}{r} 36. \\ \text{M. d. h. m.} \\ 7 \overline{) 9 \ 21 \ 12 \ 45} \end{array}$$

$$\begin{array}{r} 37. \\ 8 \overline{) 3s \ 25^{\circ} \ 55' \ 25''} \end{array}$$

$$\begin{array}{r} 38. \\ 9 \overline{) 19^{\circ} \ 45' \ 38''} \end{array}$$

$$\begin{array}{r} 39. \\ 12 \overline{) 189^{\circ} \ 37' \ 29''} \end{array}$$

40. Suppose that two places lie east and west of each other, and it is found by observation that it is noon at the former 2 hours, 6' 30" sooner than at the latter; how many degrees are they asunder?

$$4 \overline{) 2h \ 6' \ 30''}$$

Reduce the hours and minutes to minutes, then, minutes divided by 4 give degrees in the quotient.

$$31^{\circ} \ 37' \ 30'' \text{ Anf.}$$

41. The longitude of Cambridge is $4h \ 44' \ 17''$, and that of Philadelphia, $5h \ 0' \ 43''$; how many degrees difference?

$$\begin{array}{r} 5h \ 0' \ 43'' \\ 4 \ 44 \ 17 \\ \hline \end{array}$$

$$4 \overline{) 0 \ 16 \ 26}$$

$$4^{\circ} \ 6' \ 30'' \text{ Anf.}$$

PRACTICAL QUESTIONS.

CASE I.

Having the price of any number of yards, &c. within the pence table, to find the price of unity, or 1 yard. If the quantity do not exceed 12, proceed by setting down the price and dividing it by the quantity; which quotient will be the price of one yard, required; but if the quantity exceed 12, then divide by 2 such numbers, as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

Note. This case proves the first and second cases in Compound Multiplication.

1. If 9 yards of cloth cost £.4 3s. $7\frac{1}{2}d.$ what is it per yard?

$$\begin{array}{r} \text{£. s. d.} \\ 9 \overline{) 4 \ 3 \ 7\frac{1}{2}} \end{array}$$

$$0 \ 9 \ 3\frac{1}{2} \text{ Anf.}$$

2. If

2. If 7 ells cost £.5 17s. 5d. what cost 1 ell ?
3. If 11 sheep cost £.6 5s. 9d. what did each cost ?
4. If 12 gallons of rum cost £.8 11s. 9½d. what is it per gallon ?
5. If 84 cows cost £.253 13s. what is the price of each ?

Ans. £.3. 0s. 4⅓d.

6. If 132 bushels of corn cost £.20 12s. 6d. what is that per bushel ?

Ans. 3s. 1½d.

Note. When the given quantity (or divisor) is large, and not a composite number, the operation may be performed by Long Division.

C A S E II.

Having the price of a hundred weight, to find the price of 1 lb. Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1 lb required.

1. If 1 cwt. of flax cost £.2 7s. 8d. what is that per lb ?

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 8 \overline{) 2 \quad 7 \quad 8} \\
 \hline
 7 \overline{) 0 \quad 5 \quad 11 \frac{1}{2}} \\
 \hline
 2 \overline{) 0 \quad 0 \quad 10 \text{d.} \quad 0 \frac{6}{7} \text{qr.}} \\
 \hline
 \end{array}$$

0 0 5½d. price of one pound.

2. At £.3 10s. per cwt. what cost 1 lb ? *Ans.* 7½d.
3. At £.6 6s. per cwt. what cost 1 lb ? *Ans.* 1s. 1½.
4. At £.42 11s. 8d. per cwt. what cost 1 lb ? *Ans.* 7s. 7¼d.
5. At £.19 5s. per cwt. what cost 1 lb ? *Ans.* 3s. 5¼d.

C A S E III.

Having the price of several hundred weight, to find the price per lb. Divide the whole price by the number of hundreds, which will give the price per cwt. and then proceed as in the last Case.

1. If 5 cwt. of sugar cost £.13 8s. 4d. what is that per lb ?

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 5 \overline{) 13 \quad 8 \quad 4} \\
 \hline
 8 \overline{) 2 \quad 13 \quad 8} \text{ price of 1 cwt.} \\
 \hline
 7 \overline{) 0 \quad 6 \quad 8 \frac{1}{2} \text{d.}} \text{ price of 14 lb or } \frac{1}{5} \text{ cwt.} \\
 \hline
 2 \overline{) 0 \quad 0 \quad 11 \frac{1}{2} \text{d.}} \text{ price of 2 lb or } \frac{1}{5} \text{ cwt.} \\
 \hline
 0 \quad 0 \quad 5 \frac{3}{4} \text{ price of 1 lb.} \\
 \hline
 \end{array}$$

2. If 8 cwt. of cocoa cost £15. 17s. 4d. what is that per lb ? *Ans.* 4½d.
3. If 3¼ cwt. sugar cost £.9 17s. 2d. what is that per lb ? *Ans.* 6½d.

4. If

4. If $1\frac{3}{4}$ cwt. of cotton wool cost £.6 10s. 8d. what is that per lb? *Ans.* 8d.

Note. This Case proves the 7th, in Compound Multiplication.

C A S E IV.

Having the price of any number of yards, &c. to find the price of 1 yard. Divide the price by the quantity, beginning at the highest denomination, and, if any thing remain, reduce it into the next, and every inferior denomination, and, at each reduction, divide as before, remembering, each time, to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed.

1. If $95\frac{1}{2}$ lb of figs cost £.16 13s. $6\frac{3}{4}$ d. what are they per lb?

$\begin{array}{r} \text{Quantity} = 95\frac{1}{2} \\ \text{Mult. by} \quad 4 \\ \hline \end{array}$	$\begin{array}{r} \text{Price} = \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \\ 16 & 13 & 6\frac{3}{4} \end{array} \\ \hline \end{array}$
---	--

Produces 382 for a divisor. *Product.* £.66 14 3 for a dividend.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \quad \text{f.} \quad \text{s.} \quad \text{d.} \\ 382 \overline{) 66 \quad 14 \quad 3 \quad 0} \quad 3 \quad 5\frac{3}{4} \quad 2\frac{5}{8} \text{ per lb.} \\ \underline{20} \end{array}$$

$$\begin{array}{r} 382 \overline{) 1334} 3 \\ \underline{1146} \end{array}$$

$$\begin{array}{r} 188 \\ \underline{12} \end{array}$$

$$\begin{array}{r} 382 \overline{) 2259} 5 \\ \underline{1910} \end{array}$$

$$\begin{array}{r} 349 \\ \underline{4} \end{array}$$

$$\begin{array}{r} 382 \overline{) 1396} 3 \\ \underline{1146} \end{array}$$

$$\begin{array}{r} 250 \end{array}$$

2. If 147 bushels of rye cost £.47 12s. 6d. what is it per bushel?

Ans. 6s. $5\frac{3}{4}$ d.

3. If $23\frac{1}{4}$ yards of baize cost £.25 13s. $9\frac{1}{2}$ d. what is it per yard?

Ans. 15s. $5\frac{1}{4}$ d. $\frac{9}{16}$ d.

4. If $172\frac{3}{4}$ gallons of wine cost £.43 5s. 2d. what is it per gallon?

Ans. $5\frac{5}{8}$ s. 1d.

5. If

5. If a dozen sheep cost £.7 2s. 9d. what are they apiece?
Ans. 11s. 10 $\frac{3}{4}$ d.

Note. This proves the 3d and 4th Cases in Multiplication.

PRACTICAL QUESTIONS in MONEY.

1. Divide £.273 9s. 4d. among 5 men and 4 women, and give the men twice as much as the women.

Men.	Women.		£.	s.	d.	£.	s.	d.	
5 and 4		Divide by 14	273	9	4	19	10	8	= 1 woman's share.
Mult. by 2			14			4			4 women.
			—	—	—	—	—	—	
	10 shares.		133			78	2	8	= women's share.
Add 4 women's shares.			126						
			—	—	—	£.19	10	8	
14 the number of equal shares in the whole			7					2	
		= Divisor.	20						
			—	—	—	£.39	1	4	= 1 man's share.
			14	149	(10				5 men.
			14						
			—	—	—	£.195	6	8	= men's share.
			9			78	2	8	= women's share.
			12						
			—	—	—	£.273	9	4	Proof.
			14	112	(8				
			112						

2. Divide £.120 17s. 4d. among 7 men and 7 women, and give the women 3 times so much as the men.

	£.	s.	d.	
Answer, }	4	6	4	= a man's share.
	12	19	0	= a woman's share.

3. Divide £.39 12s. 5d. among 4 men, 6 women, and 9 boys: Give each man double to a woman, and each woman double to a boy.

	£.	s.	d.	
Answer, }	1	1	5	= a boy's share.
	2	2	10	= a woman's ditto.
	4	5	8	= a man's ditto.

4. Divide 5 guineas among 8 men:—Give A 8d. more than B, and B 8d. more than C, &c. *Ans.* H's share = 15s. 2d.

R U L E S

For reducing the Federal Coin, and the Currencies of the several United States; also English, Irish, Canada, Novascotia, Livres Tournois and Spanish milled Dollars, each to the *par* of all the others.

I. To reduce Newhampshire, Massachusetts, Rhodeisland, Connecticut, and Virginia currency:

1. To Federal Money.

Rule.—Reduce the shillings, pence and farthings, to decimals, by Inspection (Case 3d, Dec.

Frac.) divide the whole by 3, putting the comma one figure further to the right hand in the quotient, than in the pounds of the dividend, and the quotient will be the answer in dollars, cents and mills.

1. Reduce

1. Reduce £.349 19s. 1d. to dollars.

.9 = $\frac{1}{2}$ the shillings.

.05 = odd shilling.

4 = qrs. in 1d.

.954 = decimal.

3)349,954 D. c. m.

1106,513 = 1166, 51, 3 Ans.

2. Reduce 19s. 1 $\frac{1}{4}$ d. to dollars.

.9

.05

.7

.957 = decimal.

3).957 D. c.

3,19 = 3 19

3. Reduce 1s. to cents.

1s. = .05 then

3)0 5 c. m.

0,166 $\frac{2}{3}$ = 16 6 $\frac{2}{3}$

4. Reduce 1d.

1d. = 4 qrs.

3,004 c. m.

001,3 $\frac{1}{3}$ = 1 3 $\frac{1}{3}$

5. Reduce 1qr.

1qr. = .001041 and

3),0 01 041

0,00,347 = 3 $\frac{47}{100}$ mills.

2. To Newyork and Northcarolina currency.

Rule.—Add one third to the Newhampshire, &c. sum, and the sum total will be the Newyork, &c. currency.

Reduce £.100 Newhampshire, &c. to Newyork, &c.

£.

3)100

+ 33 6 8

£.133 6 8 Answer.

3. To Pennsylvania, Newjersey, Delaware and Maryland currency.

Rule.—Add one fourth to the Newhampshire, &c. sum.

Reduce £.100 Newhampshire, &c. to Pennsylvana, &c.

4)100

+ 25

£.125 Answer.

4. To Southcarolina and Georgia currency.

Rule.—Multiply the Newhampshire, &c. sum by 7, and divide the product by 9, and the quotient is the answer.

Reduce £.100 Newhampshire, &c. to Southcarolina, &c.

100

7

9)700

£.77 15 6 $\frac{2}{3}$ Answer.

5. To English Money.

Rule.—Deduct one 4th from the Newhampshire, &c. sum.

Reduce £.100 Newhampshire, &c. to English Money.

4)100

— 25

£.75 Answer.

6. To Irish Money.

Rule.—Multiply the Newhampshire, &c. sum by 13, and divide the product by 16.

Reduce £.100 Newhampshire, &c. to Irish Money.

100

4×3+ the given Sum.

400

3

1200

+ 100

16 = 4×4)1300

4)325

£.81 5 Ans.

7. To

7. To Canada and Nova Scotia currency.

Rule.—Multiply the New-hampshire, &c. sum by 5, and divide the product by 6.

Reduce £.100 Newhampshire, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ \times 5 \\ \hline 500 \\ 6 \overline{) 500} \\ \underline{480} \\ 20 \end{array}$$

£.83 6 8 Answer.

8. To Livres Tournois.

Rule.—Multiply the New-hampshire, &c. pounds by $17\frac{1}{2}$, and the product will be livres : Or, multiply the sum in shillings by 7 : Divide the product by 8, and the quotient will be livres, sous, &c.

Reduce £.100 Newhampshire, &c. to Livres Tournois.

$$\begin{array}{r} 100 \text{ Or, } 100 \\ \times 17\frac{1}{2} \\ \hline 700 \\ 100 \\ \hline 30 \end{array}$$

8)14000

Ans. 1750 Liv. Ans. 1750 livres.

1d. = 1 sou, $5\frac{1}{2}$ den. 1s. = $17\frac{1}{2}$ sous.

1£. = $17\frac{1}{2}$ livres.

9. To Spanish milled Dollars.

Rule 1.—When the sum consists of pounds only : Annex a cypher to the pounds, and divide the whole by 3 : The quotient is dollars.

Reduce £.100 Newhampshire, &c. to dollars.

$$3 \overline{) 1000}$$

Dol. 333 $\frac{1}{3}$ Ans.

Rule 2.—When the sum consists of pounds and shillings : Divide the pounds by 3, and

P

the shillings by 6, not separating them in the quotient, and the quotient will be dollars.

Reduce £.152 15s. 6d. to dollars.

$$\begin{array}{l} \text{£'s, by 3 } \} \text{ £. s. d.} \\ \text{s. by 6. } \} \end{array} \begin{array}{l} 152 \\ 15 \\ 6 \end{array}$$

509 dol. & 1s 6 Ans.

Note. This article may be applied to the Federal dollar, it being of the same value with a Spanish dollar.

11. To reduce Federal Money to Newengland and Virginia currency.

Rule.—Multiply the Federal money by 3, and, if it consist of dollars only, cut off 1 figure, if of cents also, cut off 3, and if of mills, 4 figures at the right hand ; then reduce the figures so cut off to farthings, each time cutting off as at first, and the left hand figures are pounds, shillings, &c.

1. Reduce 1166dolls. 51c. 3m. to Newengland currency, or lawful money.

D. c. m.

$$\begin{array}{r} 1166,513 \\ \hline 3 \end{array}$$

$$\begin{array}{r} \text{£.} 349,9539 \\ \hline 20 \end{array}$$

$$\begin{array}{r} \text{s.} 19,0780 \\ \hline 12 \end{array}$$

,936 = 1d. nearly.

2. Reduce 45 dollars.

D.

$$\begin{array}{r} 45 \\ \hline 3 \end{array}$$

$$\begin{array}{r} \text{£.} 13,5 \\ \hline 20 \end{array}$$

$$\begin{array}{r} \text{s.} 10,0 \end{array}$$

3. Reduce

3. Reduce 12*D.* 7*c.* to lawful money.

$$\begin{array}{r} D. c. \\ 12,07 \\ \underline{3} \\ \pounds.3,621 \\ \underline{20} \\ s.12,420 \\ \underline{12} \\ d.5,040 \end{array}$$

III. To reduce Newjersey, Pennsylvania, Delaware and Maryland currency.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut, and Virginia currency.

Rule.—Deduct one fifth from the Newjersey, &c. sum, and the remainder will be Newhampshire, &c. currency.

Reduce $\pounds.100$ Newjersey, &c. to Newhampshire, &c.

$$\begin{array}{r} 5)100 \\ \underline{20} \end{array}$$

$\pounds.80$ Answer.

2. To Newyork and Northcarolina currency.

Rule.—Add one fifteenth to the Newjersey, &c. sum.

Reduce $\pounds.100$ Newjersey, &c. to Newyork, &c.

$$15=3 \times 5)100$$

$$\begin{array}{r} 3)20 \\ \underline{6} \end{array}$$

$$+ 6 \ 13 \ 4 + \text{giv. sum.}$$

$\pounds.106 \ 13 \ 4$ Answer.

3. To Southcarolina and Georgia currency.

Rule.—Multiply the Newjersey, &c. sum by 28, and divide the product by 45, and the quotient is Southcarolina, &c.

Reduce $\pounds.100$ Newjersey, &c. to Southcarolina, &c.

$$\begin{array}{r} 100 \\ \underline{4 \times 7 = 28} \\ 400 \\ \underline{7} \end{array}$$

$$45=5 \times 9)2800$$

$$\begin{array}{r} 5)311 \ 2 \ 2\frac{2}{3} \\ \underline{1555} \end{array}$$

$\pounds.62 \ 4 \ 5\frac{1}{3}$ Answer.

4. To English Money.

Rule.—Multiply the Newjersey, &c. by 3, and divide the product by 5.

Reduce $\pounds.100$ Newjersey, &c. to English money.

$$\begin{array}{r} 100 \\ \underline{3} \\ 5)300 \end{array}$$

$\pounds.60$ Answer.

5. To Irish Money.

Rule.—Multiply the Newjersey, &c. by 13, and divide the product by 20.

Reduce $\pounds.100$ Newjersey, &c. to Irish.

$$\begin{array}{r} 100 \\ \underline{4 \times 3 + \text{the giv. sum.}} \end{array}$$

$$\begin{array}{r} 400 \\ \underline{3} \end{array}$$

$$1200$$

$$+ 100$$

$$20=4 \times 5)1300$$

$$\begin{array}{r} 4)260 \\ \underline{200} \end{array}$$

$\pounds.65$ Answer.

6. To Canada and Nova Scotia currency.

Rule.—Deduct one third from the Newjersey, &c.

Reduce £.100 Newjersey, &c. to Canada, &c.

$$\begin{array}{r} 3)100 \\ - 33 \quad 6 \quad 8 \\ \hline \end{array}$$

£.66 13 4 Answer.

7. To Livres Tournois.

Rule.—Multiply the Newjersey, &c. pounds by 14, and the product will be Livres Tournois—or multiply the sum in shillings by 7; divide the product by 10, and the quotient will be livres, sous, &c.

Reduce £.100 Newjersey, &c. to Livres Tournois.

$$\begin{array}{r} 100 \quad \text{Or } 100 \quad \left. \begin{array}{l} 1d = 1\frac{1}{8} \text{ sous.} \\ 20 \quad 1s. = 1\frac{1}{4} \text{ sous.} \\ 1\text{£} = 14 \text{ liv.} \end{array} \right\} \\ 14 \quad 20 \\ \hline 400 \quad 2000 \\ 100 \quad 7 \\ \hline \end{array}$$

Ans. 1400 Liv. 10)14000

1400 as before.

8. To Spanish milled dollars.

Rule.—Multiply the Newjersey, &c. pounds by $2\frac{2}{3}$ and the product will be dollars.—Or multiply them by 8: Divide the product by 3, and the quotient will be dollars.—If there be shillings in the given sum, for every 7s. 6d. add 1 dollar to the quotient.

Reduce £.100 10s. Newjersey, &c. to dollars.

$$\begin{array}{r} 100 \quad \text{Or } 100 \\ 8 \quad 2 \\ \hline 3)800 \quad 200 \\ - 266\frac{2}{3} \quad 100 \times \frac{2}{3} = 66\frac{2}{3} \\ 20s. = 1\frac{1}{3} \quad 10s. = 1\frac{1}{3} \\ \hline \end{array}$$

Ans. 268 dol.

268 as before.

IV. To reduce Newyork and Northcarolina currency.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Deduct one fourth from the Newyork, &c.

Reduce £.100 Newyork, &c. to Newhampshire, &c.

$$\begin{array}{r} 4)100 \\ - 25 \\ \hline \end{array}$$

£.75 Answer.

2. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Deduct one sixteenth from the Newyork, &c. sum.

Reduce £.100 Newyork, &c. to Newjersey, &c.

$$\begin{array}{r} 16 = 4 \times 4)100 \\ - \\ 4)25 \\ \hline \end{array}$$

— £.6 5

£.93 15 Answer.

3. To Southcarolina and Georgia currency.

Rule.—Multiply the Newyork, &c. sum by 7, and divide the product by 12: The quotient is Southcarolina, &c.

Reduce £.100 Newyork, &c. to Southcarolina, &c.

$$\begin{array}{r} 100 \\ 7 \\ \hline 12)700 \\ \hline \end{array}$$

£.58 6 8 Answer.

4. To English Money.

Rule.—Multiply the Newyork, &c. sum by 9: Divide the product by 16, and the quotient is English.

Reduce £.100 Newyork, &c. to English money.

116 REDUCTION OF COINS.

$$\begin{array}{r} 100 \\ 9 \\ \hline 16 = 4 \times 4) 900 \\ \hline 4) 225 \\ \hline \end{array}$$

£.56 5 Answer.

5. To Irish Money.

Rule.—Multiply the Newyork, &c. sum by 39 : Divide the product by 64, and the quotient is Irish.

Reduce *£.100* Newyork, &c. to Irish money.

$$\begin{array}{r} 100 \\ \hline 6 \times 6 = \text{thrice the given sum.} \\ \hline 600 \\ 6 \\ \hline 3600 \\ + 300 = 100 \times 3 \\ \hline \end{array}$$

$$64 = 8 \times 8) 3900$$

$$8) 487 \ 10$$

£.60 18 9 Ans.

6. To Canada and Nova Scotia currency.

Rule.—Multiply the Newyork, &c. sum by 5, and divide the product by 8.

Reduce *£.100* Newyork, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ 5 \\ \hline 8) 500 \\ \hline \end{array}$$

£.62 10 Ans.

7. To Livres Tournois.

Rule.—Multiply the Newyork, &c. sum in shillings by 21 : Divide the product by 32, and the quotient will be livres, sous, &c.

Reduce *£.100* Newyork, &c. to Livres Tournois.

$$\begin{array}{r} 100 \text{ Note. } 1d. = 1\frac{3}{4} \text{ sous.} \\ 20 \quad 1s. = 12\frac{1}{2} \text{ sous.} \\ 2000 \quad 1\text{£.} = 13\frac{1}{2} \text{ liv.} \\ 21 \end{array}$$

$$\begin{array}{r} 2000 \\ 4000 \\ \hline \end{array}$$

$$32 = 4 \times 8) 42000$$

$$4) 5250$$

Ans. 1312½ livres.

8. To Spanish milled Dollars.

Rule.—If the Newyork sum be pounds only, annex a cypher to them, then divide by 4, and the quotient is dollars : But if it be pounds and shillings, annex half the shillings to the pounds, and divide as before, and the quotient is dollars.

Reduce *£.100* Newyork, &c. to Dollars.

$$4) 1000$$

250 Doll. Ans.

Reduce *£.100* 8s. to Dollars.

$$4) 1004$$

251 Doll. Ans.

V. To reduce Southcarolina and Georgia currency.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Multiply the Southcarolina, &c. sum by 9, and divide the product by 7.

Reduce *£.100* Southcarolina, &c. to Newhampshire, &c.

$$\begin{array}{r} 100 \\ 9 \\ \hline \end{array}$$

$$7) 900$$

£.128 11 5½ Ans.

2. To

2. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the Southcarolina, &c. sum by 45, and divide the product by 28.

Reduce £.100 Southcarolina, &c. to Newjersey, &c.

$$\begin{array}{r} 100 \\ \hline 9 \times 5 = 45 \\ \hline 900 \\ 5 \\ \hline 28 = 4 \times 7) 4500 \\ \hline 4) 642 \ 17 \ \frac{5}{7} \end{array}$$

£.160 14 3 $\frac{3}{7}$ Ans.

3. To Newyork and Northcarolina currency.

Rule.—Multiply the Southcarolina, &c. sum by 12, and divide the product by 7.

Reduce £.100 Southcarolina, &c. to Newyork, &c.

$$\begin{array}{r} 100 \\ 12 \\ \hline 7) 1200 \\ \hline \end{array}$$

£.171 8 6 $\frac{6}{7}$ Ans.

4. To English Money.

Rule.—From the Southcarolina, &c. sum, deduct one twenty eighth.

Reduce £.100 Southcarolina, &c. to English Money.

$$\begin{array}{r} 28 = 4 \times 7) 100 \\ \hline 4) 14 \ 5 \ 8 \frac{4}{7} \\ \hline - 3 \ 11 \ 5 \frac{1}{7} \text{ from the given sum.} \\ \hline \end{array}$$

£.96 8 6 $\frac{6}{7}$ Ans.

5. To Irish Money.

Rule.—Multiply the Southcarolina, &c. sum by 117, and divide the product by 112.

Reduce £.100 Southcarolina, &c. to Irish.

$$\begin{array}{r} 100 \\ 12 \times 9 = 9 \text{ times the} \\ \hline 1200 \\ 9 \\ \hline 10800 \\ + 100 \times 9 = 900 \\ \hline 112 = 4 \times 4 \times 7) 11700 \\ \hline 4) 1671 \ 8 \ 6 \frac{6}{7} \\ \hline 4) 417 \ 17 \ 1 \ \frac{5}{7} \\ \hline \end{array}$$

£.104 9 3 $\frac{3}{7}$ Answer.

6. To Canada and Novascotia currency.

Rule.—Multiply the Southcarolina, &c. sum by 15, and divide the product by 14.

Reduce £.100 Southcarolina, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ 5 \times 3 \\ \hline 500 \\ 3 \\ \hline 14 = 2 \times 7) 1500 \\ \hline \end{array}$$

$$2) 214 \ 5 \ 8 \frac{4}{7}$$

£.107 2 10 $\frac{2}{7}$ Answer.

7. To Livres Tournois.

Rule.—Multiply the Southcarolina, &c. pounds by 22 $\frac{1}{2}$, and the product will be livres.

Reduce £.100 Southcarolina, &c. to Livres.

$$\begin{array}{r} 100 \\ 22 \frac{1}{2} \\ \hline 200 \\ 200 \\ \hline 50 \end{array}$$

Note 1d. = 1 $\frac{7}{8}$ fous.

1s. = 1 $\frac{1}{2}$ livre.

1£. = 22 $\frac{1}{2}$ livres.

Ans. 2250 livres.

8. To

118 R E D U C T I O N o f C O I N S.

8. *To Spanish milled Dollars.*

Rule.—Multiply the Southcarolina, &c. pounds by 30, and divide the product by 7, and if there be shillings, turn them into dollars and add them.

Reduce £.100 Southcarolina, &c. to Dollars.

$$\begin{array}{r} 100 \\ \hline 10 \times 3 = 30 \end{array}$$

$$\begin{array}{r} 1000 \\ \hline 3 \end{array}$$

$$7)3000$$

Dollars 428 $\frac{2}{3}$. Note, $\frac{2}{3} = 8d$.

VI. *To reduce English Money.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—To the English sum add one third.

Reduce £.100 English to Newhampshire, &c.

$$\begin{array}{r} 3)100 \\ + 33 \ 6 \ 8 \end{array}$$

£.133 6 8 *Answer.*

2. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the English money by 5, and divide the product by 3.

Reduce £.100 English to Newjersey, &c.

$$\begin{array}{r} 100 \\ \hline 5 \end{array}$$

$$3)500$$

£.166 13 4 *Ans.*

3. *To Newyork and Northcarolina currency.*

Rule.—Multiply the English money by 16, and divide the product by 9.

Reduce £.100 English to Newyork, &c.

$$\begin{array}{r} 100 \\ \hline 4 \times 4 \end{array}$$

$$\begin{array}{r} 400 \\ \hline 4 \end{array}$$

$$9)1600$$

£.177 15 6 $\frac{2}{3}$ *Answer.*

4. *To Southcarolina and Georgia currency.*

Rule.—To the English money add one twenty seventh.

Reduce £.100 English to Southcarolina, &c.

$$27 = 3 \times 9)100$$

$$3)11 \ 2 \ 2\frac{2}{3}$$

$$+ 3 \ 14 \ 0\frac{8}{9}$$

£.103 14 0 $\frac{8}{9}$ *Ans.*

5. *To Irish Money.*

Rule.—To the English sum add one twelfth.

Reduce £.100 English money to Irish money.

$$\begin{array}{r} 12)100 \\ + 8 \ 6 \ 8 \end{array}$$

£.108 6 8 *Answer.*

6. *To Canada and Novascotia currency.*

Rule.—To the English sum add one ninth.

Reduce £.100 English to Canada, &c.

$$\begin{array}{r} 9)100 \\ + 11 \ 2 \ 2\frac{2}{3} \end{array}$$

£.111 2 2 $\frac{2}{3}$ *Answer.*

7. *To Livres Tournois.*

Rule.—Multiply the English pounds by 23 $\frac{1}{3}$, and the product will be livres.

Reduce

Reduce £.100 English to Livres Tournois.

$$\begin{array}{r} 100 \text{ Note. } 1d. = 1\frac{1}{8} \text{ sous.} \\ 23\frac{1}{3} \quad 1s. = 1\frac{1}{6} \text{ liure.} \\ \hline 1\text{£.} = 23\frac{1}{3} \text{ livres.} \end{array}$$

$$\begin{array}{r} 300 \\ 200 \\ \hline 33\frac{1}{3} \end{array}$$

Liv. sou. den.
Ans. $2333\frac{1}{3}$ Liv. = 2333 6 8

VII. To reduce Irish Money.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut, and Virginia currency.

Rule.—Multiply the Irish sum by 16, and divide the product by 13.

Reduce £.100 Irish to Newhampshire, &c.

$$\begin{array}{r} 100 \\ \hline 4 \times 4 \end{array}$$

$$\begin{array}{r} 400 \\ \hline 4 \end{array}$$

$$13 \overline{) 1600}$$

£.123 1 6 $\frac{5}{13}$ Answer.

2. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the Irish sum by 20, and divide the product by 13.

Reduce £.100 Irish to Newjersey, &c.

$$\begin{array}{r} 100 \\ \hline 4 \times 5 = 20 \end{array}$$

$$\begin{array}{r} 400 \\ \hline 5 \end{array}$$

$$13 \overline{) 2000} (153 \text{ } 16 \text{ } 11\frac{1}{13} \text{ Answer.}$$

$$\begin{array}{r} 13 \\ \hline 70 \end{array}$$

$$\begin{array}{r} 11 \\ \hline 65 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 13 \end{array}$$

3. To Newyork and Northcarolina currency.

Rule.—Multiply the Irish sum by 64, and divide the product by 39.

Reduce £.100 Irish to Newyork, &c.

$$\begin{array}{r} 100 \\ \hline 8 \times 8 = 64 \end{array}$$

$$\begin{array}{r} 800 \\ \hline 8 \end{array}$$

$$39 \overline{) 6400} (164 \text{ } 2\frac{2}{3} \text{ Answer.}$$

$$\begin{array}{r} 39 \\ \hline 250 \end{array}$$

$$\begin{array}{r} 234 \\ \hline 160 \end{array}$$

$$\begin{array}{r} 156 \\ \hline 4 \end{array}$$

4. To Southcarolina and Georgia currency.

Rule.—Multiply the Irish sum by 112, and divide the product by 117.

Reduce £.100 Irish to Southcarolina, &c.

$$\begin{array}{r} 100 \\ \hline 7 \times 4 \times 4 = 112 \end{array}$$

$$\begin{array}{r} 700 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 2800 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 700 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 2800 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 700 \\ \hline 4 \end{array}$$

$$117 \overline{) 11200} (95 \text{ } 14 \text{ } 6\frac{42}{117} \text{ Ans.}$$

$$\begin{array}{r} 1053 \\ \hline 670 \end{array}$$

$$\begin{array}{r} 585 \\ \hline 85 \end{array}$$

120 REDUCTION OF COINS.

5. To English Money.

Rule.—From the Irish sum deduct one thirteenth.

Reduce £.100 Irish to English money.

$$\begin{array}{r}
 13 \overline{)100(7} \\
 \underline{91} \\
 9 \\
 20 \quad \text{£. s. d.} \\
 \underline{100} \quad 0 \quad 0 \\
 13 \overline{)180(13} \quad -7 \quad 13 \quad 10 \frac{2}{13} \\
 \underline{13} \\
 50 \\
 39 \\
 \underline{11} \\
 12 \\
 13 \overline{)132(10} \\
 \underline{13} \\
 2
 \end{array}$$

6. To Canada and Nova Scotia currency.

Rule.—To the Irish sum add one thirty ninth.

Reduce £.100 Irish to Canada, &c.

$$\begin{array}{r}
 39 \overline{)100(2} \\
 \underline{78} \\
 22 \\
 20 \\
 39 \overline{)440(11} \\
 \underline{39} \quad 100 \\
 \underline{50} \quad +2 \quad 11 \quad 3 \frac{5}{13} \\
 39 \quad \text{£.102 11 3} \frac{5}{13} \text{ Ans.} \\
 \underline{11} \\
 12
 \end{array}$$

$$39 \overline{)132(3}$$

$$\underline{117}$$

$$\underline{15} = 5$$

$$39 \overline{)13}$$

7. To Livres Tournois.

Rule.—Multiply the Irish sum, in pence, by 70; divide that product by 39, and the quo-

tient will be sous, which, divided by 20, will be livres.

Reduce £.100 Irish to Livres Tournois.

$$100 \times 20 \times 12 = 24000d.$$

$$\begin{array}{r}
 70 \\
 \underline{20} \\
 39 \overline{)1680000} 4307,6
 \end{array}$$

$$\text{Ans. Livres. } 2153, 16 \frac{2}{13}$$

$$1d. = 1 \frac{3}{4} \text{ sous. } 1s. = 21 \frac{1}{3} \text{ sous.}$$

$$1\text{£.} = 21 \text{ liv. } 10 \frac{1}{3} \text{ sous.}$$

VIII. To reduce Canada and Nova Scotia currency.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—To the Canada, &c. sum add one fifth.

Reduce £.100 Canada, &c. to Newhampshire, &c.

$$\begin{array}{r}
 5 \overline{)100} \\
 + 20 \\
 \text{£.120 Answer.}
 \end{array}$$

2. To Newyork and Northcarolina currency.

Rule.—Multiply the Canada &c. sum by 8, and divide the product by 5.

Reduce £.100 Canada, &c. to Newyork, &c.

$$\begin{array}{r}
 100 \\
 \times 8 \\
 \hline
 800 \\
 5 \overline{)800} \\
 \text{£.160 Answer.}
 \end{array}$$

3. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—To the Canada, &c. sum add one half.

Reduce £.100 Canada, &c. to Newjersey, &c.

$$\begin{array}{r}
 2 \overline{)100} \\
 + 50
 \end{array}$$

$$\text{£.150 Answer.}$$

4. To Southcarolina and Georgia currency.

Rule.

Rule.—From the Canada, &c. sum deduct one fifteenth.

Reduce £.100 Canada, &c. to Southcarolina, &c.

$$15 = 3 \times 5 \mid 100$$

$$\begin{array}{r} \text{---} \\ 3 \mid 20 \\ \text{---} \end{array}$$

$$\begin{array}{r} \text{---} \\ - 6 \ 13 \ 4 \\ \text{---} \end{array}$$

£.93 6 8 *Answer.*

5. *To English Money.*

Rule.—From the Canada, &c. deduct one tenth.

Reduce £.100 Canada, &c. to English money.

$$10 \mid 100$$

$$\begin{array}{r} \text{---} \\ - 10 \\ \text{---} \end{array}$$

£.90 *Answer.*

6. *To Irish Money.*

Rule.—From the Canada, &c. deduct one fortieth.

Reduce £.100 Canada, &c. to Irish money.

$$40 \mid 100$$

$$\begin{array}{r} \text{---} \\ - 2 \ 10 \\ \text{---} \end{array}$$

£.97 10 *Answer.*

7. *To Livres Tournois.*

Rule.—Multiply the Canada, &c. pounds by 21, and the product will be livres.

Reduce £.100 Canada, &c. to livres Tournois.

$$100$$

$$7 \times 3 = 21$$

$$\begin{array}{r} \text{---} \\ 700 \\ \text{---} \end{array}$$

$$\begin{array}{r} \text{---} \\ 3 \\ \text{---} \end{array}$$

$$1d. = 1\frac{3}{4} \text{ fous.}$$

$$15. = 21 \text{ fous.}$$

$$1\text{£} = 21 \text{ livres.}$$

Ans. 2100

8. *To Spanish milled Dollars.*

Rule.—Reduce the Canada, &c. sum to shillings: Divide them by 5, and the quotient is dollars. *Or,* Multiply the pounds by 4, and the product is dollars:

Q

And if there be shillings, turn them into dollars, and add them to the product:

Reduce £.100 Canada, &c. to dollars.

$$100$$

$$\begin{array}{r} \text{---} \\ 20 \\ \text{---} \end{array}$$

$$5 \mid 2000$$

$$155 \ 15$$

$$\begin{array}{r} \text{---} \\ 4 \\ \text{---} \end{array}$$

$$620$$

$$+ 3 = 155.$$

Doll. 400 *Ans.*

Doll. 623 *Ans.*

IX. *To reduce Livres Tournois.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—Multiply the livres by 2: Divide the product by 35, and the quotient will be pounds. *Or,* Multiply the livres by 8: Divide the product by 7, and the quotient will be shillings.

Reduce 1750 livres to Newhampshire, &c. currency.

$$1750$$

$$\begin{array}{r} \text{---} \\ 2 \\ \text{---} \end{array}$$

$$\text{Or, } 1750$$

$$\begin{array}{r} \text{---} \\ 8 \\ \text{---} \end{array}$$

$$\begin{array}{r} \text{---} \\ \text{£.} \\ \text{---} \end{array}$$

$$35 \mid 3500 (100 \text{ Ans. } 7) 14000$$

$$\begin{array}{r} \text{---} \\ 35 \\ \text{---} \end{array}$$

$$00$$

$$2,0 \mid 200,0$$

£.100 as bef.

2. *To Newyork and Northcarolina currency.*

Rule.—Multiply the livres by 32: Divide the product by 21, and the quotient will be shillings.

Reduce 1312½ livres to Newyork, &c. currency.

$$1312,5$$

$$\begin{array}{r} \text{---} \\ 3 \ 2 \\ \text{---} \end{array}$$

$$26250$$

$$39375$$

$$\begin{array}{r} \text{---} \\ 2,0 \\ \text{---} \end{array}$$

$$21 \mid 42000 (200,0$$

£.100 *Answer.*

3. *To*

3. To Newjersey, Pennsyloania, Delaware and Maryland currency.

Rule.—Divide the livres by 14, and the quotient will be pounds. Or, Multiply the livres by 10 : Divide the product by 7, and the quotient will be shillings.

Reduce 1400 livres to Newjersey, &c. currency.

$$\begin{array}{r}
 1400 \\
 10 \\
 \hline
 7)14000 \quad \text{Or,} \quad 14)1400(\text{£.100.} \\
 \hline
 2,0)200,0 \quad \quad \quad 14 \\
 \hline
 \text{£.100 Answer.} \quad \quad \quad 00
 \end{array}$$

4. To Southcarolina and Georgia currency.

Rule.—Multiply the livres by 2, divide the product by 45, and the quotient will be pounds. Or, deduct one ninth, and the remainder will be shillings.

Reduce 2250 livres to Southcarolina, &c. currency.

$$\begin{array}{r}
 2250 \quad \text{Or,} \\
 2 \quad \quad \quad 9)2250 \\
 \hline
 \text{£.} \quad \quad \quad - 250 \\
 45)4500(100 \text{ Anf.} \quad \hline
 45 \quad \quad \quad 2,0)200,0 \\
 \hline
 00 \quad \quad \quad \text{£.100 as bef.}
 \end{array}$$

5. To English Money.

Rule.—Multiply the livres by 6: Divide the product by 7, and the quotient is shillings: Or, deduct one seventh from the livres, and the remainder will be shillings.

Reduce 2333 $\frac{1}{3}$ livres to English money.

$$\begin{array}{r}
 2333\frac{1}{3} \quad \text{Or,} \\
 6 \quad \quad \quad 7)2333\frac{1}{3} \\
 \hline
 7)14000 \quad \quad \quad - 333\frac{1}{3} \\
 \hline
 2)0)200)0 \quad \quad \quad 2)0)200)0 \\
 \hline
 \text{£.100 as bef.}
 \end{array}$$

Anf. £.100

6. To Irish Money.

Rule. Reduce the livres to sous, then multiply them by 39 : Divide this product by 70, and the quotient will be pence.

Reduce 2153liv. 16 $\frac{2}{3}$ s. to Irish Money. 20

$$\begin{array}{r}
 43076\frac{12}{13} \\
 39 \\
 \hline
 387720 \\
 129228 \\
 \hline
 7,0)168000,0 \\
 \hline
 12)24000 \\
 \hline
 2,0)200,0
 \end{array}$$

£.100 Answer.

7. To Spanish milled Dollars, or to Federal Dollars.

Rule.—Multiply the livres by 4 : Divide the product by 21, and the quotient will be Spanish or Federal Dollars.

Reduce 1000 livres to dollars.

$$\begin{array}{r}
 1000 \quad \text{Or, 1000} \\
 4 \quad \quad \quad 4 \\
 \hline
 21)4000(190 \left\{ \begin{array}{l} \text{Span.} \\ \text{Dol.} \end{array} \right. \quad 21)4000(190 \left\{ \begin{array}{l} \text{Fed.} \\ \text{Dol.} \end{array} \right. \\
 21 \quad \quad \quad 21 \\
 \hline
 190 = 190\frac{10}{21} \quad \quad \quad 190 \\
 189 \text{ Dollars.} \quad \quad \quad 189 \\
 \hline
 10 \quad \quad \quad 10 \\
 6 \quad \quad \quad 10 \\
 \hline
 - s. d. q. \quad \quad \quad - d. c. m. \\
 21)60(2 \ 10 \ 4 \quad \quad \quad 21)100(4 \ 7 \ 6\frac{2}{3} \\
 \hline
 \text{X. 70}
 \end{array}$$

X. To reduce Spanish milled Dollars.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Multiply the Dollars by 3, and double the right hand figure of the product, for shillings; the left hand figures are pounds.

Reduce 529 dollars to Newhampshire, &c.

$$\begin{array}{r} 529 \\ 3 \\ \hline \end{array}$$

£.158 14 Answer.

2. To Newyork and Northcarolina currency.

Rule.—Multiply the number of dollars by 4: Double the right hand figure of the product for shillings, and the left hand figures are pounds.

Reduce 529 dollars to Newyork, &c.

$$\begin{array}{r} 529 \\ 4 \\ \hline \end{array}$$

£.211 12 Answer.

3. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the number of dollars by 3, and divide by 8.

Reduce 529 dollars to Newjersey, &c.

$$\begin{array}{r} 529 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 8)1587(198 \ 7 \ 6 \text{ Answer.} \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 78 \\ 72 \\ \hline \end{array}$$

$$\begin{array}{r} 67 \\ 64 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \end{array}$$

$$\begin{array}{r} \text{Or,} \\ 8)1587 \\ \hline \text{£.198}\frac{3}{4} \text{ Anf.} \end{array}$$

4. To Southcarolina and Georgia currency.

Rule.—Multiply the number of dollars by 7, and divide by 30.

Reduce 529 dollars to Southcarolina, &c.

$$\begin{array}{r} 529 \\ 7 \\ \hline 3,0)370,3 \end{array}$$

£.123 $\frac{1}{10}$ Answer.

*5. To English Money, at 4s. 6d. per Dollar.

Rule.—Multiply the dollars by 9, and divide by 40.

Reduce 529 dollars to English money.

$$\begin{array}{r} 529 \\ 9 \\ \hline 4,0)476,1 \end{array}$$

£.119 $\frac{1}{10}$ Answer.

6. To Canada and Novascotia currency.

Rule.—Divide the dollars by 4.

Reduce 529 dollars to Canada, &c.

$$4)529$$

£.132 $\frac{1}{4}$ Answer.

7. To Livres Tournois.

Rule.—Multiply the dollars by 5 $\frac{1}{4}$, and the product will be livres. Or, Multiply them by 21: Divide by 4, and the quotient will be livres.

Reduce 100 Spanish dollars to livres.

$$\begin{array}{r} 100 \\ 5\frac{1}{4} \\ \hline 500 \\ 100 \times \frac{1}{4} = 25 \\ \hline 525 \end{array}$$

Anf. 525 livres. 525 as bef.

Note,

* Note, That in England dollars are Bullion, that is, they are bought and sold by weight, and their value varies as other articles of merchandize.

$$\text{Note, } \left\{ \begin{array}{l} 1 \text{ Cent} = 1\frac{1}{20} \text{ Sous.} \\ 1 \text{ Dime} = 10\frac{1}{2} \text{ Sous.} \\ 1 \text{ Dollar} = 5\frac{1}{4} \text{ Livres.} \end{array} \right\}$$

$$\begin{array}{l} \text{The Pound} \left\{ \begin{array}{l} \text{Sterling,} \\ \text{Newhamp. \&c.} \\ \text{Newyork, \&c.} \\ \text{Newjersey, \&c.} \\ \text{Southcarol. \&c.} \end{array} \right\} \text{ contains } \left\{ \begin{array}{l} 1718\frac{3}{4} \\ 1289 \\ 966\frac{3}{4} \\ 1031\frac{1}{4} \\ 1657,366 \end{array} \right\} \text{ Grains of} \\ \text{fine Silver.} \left\{ \begin{array}{l} 1858 \\ 1393\frac{1}{2} \\ 1045 \\ 1114\frac{3}{4} \\ 1791,819 \end{array} \right\} \text{ Grains of} \\ \text{Stand. fin.} \left\{ \begin{array}{l} \text{The propor-} \\ \text{tion of alloy} \\ \text{being } \frac{3}{37} \text{ of} \\ \text{the fine silver.} \end{array} \right\} \end{array}$$

$$\begin{array}{l} \text{The } \{ \text{Dollar} \} \text{ contains } \{ 375,64 \} \text{ Grains } \{ \text{Silver} \} 409,78 \text{ grs. of Stand. Silver.} \\ \text{Federal } \{ \text{Eagle} \} \text{ contains } \{ 246,268 \} \text{ of fine } \{ \text{Gold} \} 268,659 \text{ grs. of Stand. Gold.} \end{array}$$

The alloy being $\frac{1}{11}$ of the fine $\left\{ \begin{array}{l} \text{Silver.} \\ \text{Gold.} \end{array} \right\}$ The Subdivisions are in the same proportion.

DUODECIMALS, or CROSS MULTIPLICATION,

Is a Rule made use of by workmen and artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus ; inches or primes ('), seconds ("), thirds (' ' '), fourths (' ' ' '), &c.

This method of multiplying is not confined to *twelves* ; but may be greatly extended : For any number, whether its inferior denominations decrease from the integer in the same ratio, or not, may be multiplied crosswise ; and, for the better understanding of it, the learner must observe, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand ; but if he multiplies by any number of an inferior denomination, the value of an unit in the product will be so much inferior to the value of an unit in the multiplicand as an unit of the multiplier is less than an integer.

Thus, pounds multiplied by pounds are pounds ; pounds multiplied by shillings are shillings, &c. shillings multiplied by shillings are twentieths of a shilling ; shillings multiplied by pence are twentieths of a penny ; pence multiplied by pence are 40ths of a penny, &c.

R U L E.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term, observing, in duodecimals, to carry an unit for every 12, from each lower denomination to its next superior, and for other numbers accordingly.

3. In

3. In the same manner multiply all the multiplicand by the primes or second denomination in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand.

4. Do the same with the seconds in the multiplier, setting the result of each term two places to the right hand of those in the multiplicand.

5. Proceed in like manner with all the rest of the denominations, and their sum will be the answer required.

E X A M P L E S.

1. Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

Or thus.

$$\begin{array}{r} F. \quad ' \\ 2 \quad 6 \\ 2 \quad 6 \\ \hline 5 \quad 0 \\ 1 \quad 3 \quad 0 \\ \hline \end{array}$$

Or thus.

$$\begin{array}{r} 2\frac{1}{2} \\ 2\frac{1}{2} \\ \hline 5 \\ 1\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 2,5 \\ 2,5 \\ \hline 125 \\ 50 \\ \hline \end{array}$$

Ans. 6,25 square feet.

Ans. 6 3

Ans. $6\frac{1}{4}$ square feet = 6ft. 36 in.

So that the 3 is not 3 inches, but 36 inches, or $\frac{1}{4}$ of a square foot.

2. Multiply 9f. 8' 6" by 7f. 9' 3"

$$\begin{array}{r} F. \quad ' \quad '' \\ 9 \quad 8 \quad 6 \\ 7 \quad 9 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 67 \quad 11 \quad 6 \\ 7 \quad 3 \quad 4 \quad 6'' \\ \quad 2 \quad 5 \quad 1 \quad 6''' \\ \hline \end{array}$$

= Product by the feet in the multiplier.

= ditto by the primes.

= ditto by the seconds.

75 5 3 7 6 Answer.

3. How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? Ans. 24f. 10' 11"

4. How many cubic feet in a stick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick? Ans. 35f. 6' 8" 6"

5. How many cubic feet of wood in a load 6 feet 7 inches long, 3 feet 5 inches high, and 3 feet 8 inches wide? Ans. 82f. 5' 8" 4"

6. There is a house with 4 tiers of windows, and 4 windows in a tier; the height of the first tier is 6f. 8'; of the second, 5f. 9'; of the third, 4f. 6'; and of the fourth, 3f. 10'; and the breadth of each is 3f. 5'; how many square feet do they contain in the whole? Ans. 283f. 7'

The two following questions are Sexcessimals.

7. If

7. If two places differ in longitude $2^{\circ} 12'$; what is their difference of time?

Mult. $2^{\circ} 12' 00'' 00'''$

by $3' 59'' 20'''$ the time in which the sun passes through 1 degree.

8' 46'' 32''' Ans.

8. Two places differ in longitude $31^{\circ} 27' 30''$; what is the difference, in time, of the sun's coming to the meridian of those places, the sun passing through 15° in an hour?

$31^{\circ} 37' 30''$

$4' 00''$ In 4' of a solar day, or day of 24 hours, the sun passes 1 deg.

2 6' 30'' 00''' Answer.

9. Mult. £. 3 6 8 by £. 2 5 7.

£.	s.	d.
3	6	8
2	5	7

£ 3 × £ 2 = £ 6	= 6	0	0
6s. × £ 2 = 12s.	= 0	12	0
8d. × £ 2 = 16d.	= 0	1	4
£ 3 × 5s. = 15s.	= 0	15	0
6s. × 5s. = $\frac{30}{20}$ s.	= 0	1	6
8d. × 5s. = $\frac{40}{20}$ d.	= 0	0	2
£ 3 × 7d. = 21d.	= 0	1	9
6s. × 7d. = $\frac{42}{20}$ d.	= 0	0	$2\frac{1}{10}$
8d. × 7d. = $\frac{56}{20}$ d.	= 0	0	$0\frac{7}{10}$

Ans. £. 7 11 11 $\frac{1}{2}$

10. A, B and C bought a drove of sheep in company; A paid £ 14 5s. B, £ 13 10s. and C, £ 11 5s. They agreed to dispose of them at the market; that each man should take 18s. as pay for his time, &c. and that the remainder should be divided in proportion to their several stocks; At the close of the sale, they found themselves possessed of £ 46 5s. What was each man's gain, exclusive of the pay for his time, &c.?

£ 14 5 + £ 13 10 + £ 11 5 = £ 39, and £ 46 5 - £ 39 = £ 7 5,

and £ 7 5 - 18s. × 3 = £ 4 11s. whole gain, and £ 4 11 ÷ 39

= 2s. 4d. gain in the pound.

£ 14 5 0	£ 13 10 0	£ 11 5 0
× 2 4	× 2 4	× 2 4
<u>1 8 6</u>	<u>1 7 0</u>	<u>1 2 6</u>
4 9	4 6	3 9

A. £ 1 13 3 B. £ 1 11 6 C. £ 1 6 3

Proof. $\left\{ \begin{array}{l} \text{£. s. d.} \\ 1 \ 13 \ 3 \\ 1 \ 11 \ 6 \\ 1 \ 6 \ 3 \end{array} \right.$

£ 4 11 0

SINGLE RULE OF THREE DIRECT.

The Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

If more require more, or less require less, the question belongs to the Rule of Three Direct.

But

But if *more* require *less*, or *less* require *more*, it belongs to the Rule of Three Inverse.*

R U L E. †

1. State the question by making that number, which asks ‡ the question, the third term, or putting it in the third place; that, which is of the same name or quality as the demand, the first term; and that, which is of the same name or quality with the answer required, the second term.

2. Multiply the second and third numbers together; divide the product by the first, and the quotient will be the answer to the

* *More* requiring *more*, is when the third term is greater than the first, and requires the fourth term to be greater than the second. And *less* requiring *less*, is when the third term is less than the first, and requires the fourth term to be less than the second.

Also, *more* requiring *less*, is when the third term is greater than the first, and requires the fourth term to be less than the second. And *less* requiring *more*, is when the third term is less than the first, and requires the fourth term to be greater than the second.

† This Rule, on account of its great and extensive usefulness, is sometimes called the *golden rule of Proportion*: For, on a proper application of it and the preceding rules, the whole business of Arithmetic, as well as every mathematical inquiry, depends. The rule itself is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: Thus, the quantity of goods bought, is in proportion to the money laid out; the space, gone over by an uniform motion, is in proportion to the time, &c.

As the idea, annexed to the term, *proportion*, is easily conceived, the truth of the rule, as applied to ordinary inquiries, may be made evident by attending to principles, already explained.

It has been shewn, in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole; and in Division, that the price of the whole, divided by the quantity, is the price of one: Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain that the answer, found by this rule, will be the same as that, found by Multiplication of Money; and, where one is the last term of the proportion, it will be the same as that, found by Division of Money.

In like manner, if the first term be any number whatever, it is plain, that the product of the second and third terms will be greater than the true answer, required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit; consequently, this product, divided by the first term, will give the true answer required.

Direct and Inverse proportion are properly only parts of the same general rule; but I have preserved the common distinction, and given some loose definitions, which, to young persons in general, are more intelligible.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and Compound Division.

2. If the first term, and either the second or third can be divided by any number without a remainder, let them be divided and the quotient used instead of them.

The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.

1. Divide the second term by the first: Multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first; multiply the quotient into the second, and the product will be the answer.

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

‡ Note, The term which asks or moves the question, has generally some words like these before it, viz. What will? What cost? How many? How far? How long? or, How much? &c.

the question, which (as also the remainder) will be in the same denomination you left the second term in, and which may be brought into any other denomination required.

Two, or more, statings, are sometimes necessary, which may always be known from the nature of the question.

The method of proof is by inverting the question.

But, that I may make the method of working this excellent Rule as intelligible as possible to the learner, I shall divide it into the several cases following:

1. The fourth number is always found in the same name in which the second is given, or reduced to; which, if it be not the highest denomination of its kind, reduce to the highest when it can be done.

2. When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name to which the second is reduced, which reduce back to the highest possible.

3. If the first and third be of different names, or one or both of divers denominations, reduce them both to the lowest denomination mentioned in either.

4. When the product of the second and third is divided by the first; if there be a remainder after the division, and the quotient be not the least denomination of its kind; then multiply the remainder by that number, which one of the same denomination with the quotient contains of the next less, and divide this product again by the first number; and thus proceed till the least denomination be found, or till nothing remain.

5. If the first number be greater than the product of the second and third; then bring the second to a lower denomination.

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the lowest name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it most generally happens) put the separate content of each properly under one another, then add them together, and you will have the whole quantity.

1. If 6lb of sugar cost 9s. what will 30lb cost at the same rate?

Here the first clause (if 6lb of sugar cost 9s.) supposes the rate; then follows the question: What will 30lb cost?

30lb, which moves the question, is the 3d term. 6lb, the same kind, is the 1st, and 9 shillings the 2d.

As 6 : 9 :: 30 : the Answer.

$$\begin{array}{r} 9 \\ \hline 6 \overline{)270} \end{array}$$

45s. = £.2 5s. Ans.

Again, By inverting the order of the question, it will be,

2. If

SINGLE RULE OF THREE. 129

2. If 9s. buy 6lb of sugar, how much will £.2 5s. buy at that rate ?

$$\begin{array}{rcl} s. & lb & s. \\ As & 9 : 6 :: 45 : & \text{the Answer.} \end{array}$$

$$\begin{array}{r} 9)270 \\ \hline \end{array}$$

30 lb Answer.

Again, 3. If 30lb of sugar be worth £.2 5s. how much may I buy for 9s. ?

N. B. The three last questions are only the first *varied*, being put purely to shew how any question, in this Rule, may be inverted.

5. If 5yds. cloth cost £.1 10s. what will 20yds. ditto come to ?

$$\begin{array}{rcl} yds. & s. & yds. \\ As & 5 : 30 :: 20 & \\ 30 \div 5 = & 6 & 5)30s. \end{array}$$

$$\begin{array}{r} 2,0)12,0s. = \end{array}$$

Here I divide the 2d term by the 1st, and multiply the quotient into the 3d for the answer.

7. If 20yds. cost £.120, how many yards may I have for £30 ?

$$\begin{array}{rcl} £. & yds. & £. \\ As & 120 : 20 :: 30 & \\ 120 \div 20 = 6 \text{ quot. } \& 30 \div 6 = 5 \text{ yds. } & \text{Ans.} \end{array}$$

Here I divide the 1st term by the 2d, and then, the 3d term by that quotient for the answer.

9. If 1cwt. of tobacco cost £.5 12 9½; what will 8cwt. ditto cost ?

$$\begin{array}{rcl} cwt. & £. & s. & d. & cwt. \\ As & 1 : 5 & 12 & 9\frac{1}{2} :: 8 \end{array}$$

$$\begin{array}{r} \text{Ans. } £.45 \end{array}$$

Here is no need of reducing the middle term, because it can be performed by compound multiplication, the 1st term being an unit.

R

$$\begin{array}{rcl} s. & lb & s. \\ As & 45 : 30 :: 9 : & \text{the Answer.} \end{array}$$

$$\begin{array}{r} 9 \\ \hline 45)270(6lb \text{ the Answer.} \end{array}$$

Again, 4. Suppose £.2 5s. will buy 30lb sugar : What will 6lb of the same sugar cost ?

$$\begin{array}{rcl} lb & s. & lb \\ As & 30 : 45 :: 6 : & \text{the Answer.} \end{array}$$

$$\begin{array}{r} 30)270 \\ \hline \end{array}$$

9s. Answer.

$$\begin{array}{rcl} yds. & s. & yds. \\ Again, 6. & As & 5 : 30 :: 20 \\ 20 \div 5 = & 4 & \end{array}$$

$$120s. = £.6.$$

Here I divide the 3d term by the 1st, and multiply the quotient into the 2d, for an answer.

$$\begin{array}{rcl} £. & yds. & £. \\ Again, 8. & As & 120 : 20 :: 30 \\ 120 \div 30 = 4 \text{ quot. } 20 \div 4 = 5 \text{ yds. } & \text{Ans.} \end{array}$$

Here I divide the 1st term by the 3d, and then, the 2d term by that quotient for the answer.

10. If 8cwt. of tobacco cost £.45 2 4; what is that per cwt. ?

$$\begin{array}{rcl} £. & s. & d. \\ 8)45 & 2 & 4 \end{array}$$

$$\text{Ans. } 5 \ 12 \ 9\frac{1}{2}$$

Here there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

11. If

290 SINGLE RULE OF THREE.

11. If 9cwt. 3qrs. sugar cost £.27 17s. 6d. what will 2cwt. 1qr. 11lb cost ?

4	20	4
39	557	9
28	12	28
312	6690	73
78		19
1092		263

As 1092 $\frac{lb}{d.}$ 6690 :: 263 $\frac{lb}{d.}$ the answer.

2007
4014
1338
12
1092)1759470(1611
1092
2)0)1314 3d.
6674 £.6 14/3 Ans.
6552
1227
1092
1350
1092
258.
4
1092)1032(0qr.

12. If 57yds. cost £69, what will 9yds. cost at that rate ?

yds. £. yds.

As 57 $\frac{yds.}{£.}$ 69 :: 9

9
57)621(10£. Ans.
57
51
20
57)1020(17s.
57
450
399
51
12

57)612(10d.

57
42
4

57)168($2\frac{2}{3}$ qrs.

114

Note 1. If you look at the stating, you will see that the first and third terms are of the same kind, but of different denominations, and therefore are reduced to the same name or denomination, and, that the demand of the question lies on the third term.

2. That the middle term, being given in pounds, shillings and pence, is reduced to pence.

Here all the terms being whole numbers, there is no need of reducing the middle one until after stating.

13. If

SINGLE RULE OF THREE. 131

13. If my income be 109 guineas per annum, I desire to know what I may spend per day, so that I may lay up £.45 at the year's end ?

Ans. £.0 5 10 $\frac{1}{4}$ $\frac{1}{365}$ per day.

Note 1. You must subtract £.45 from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus :

D. Guin. £. D. s. d. qr.

As 365 : 109 — 45 :: 1 : 5 10 $\frac{1}{365}$ the answer.

14. If my salary be £.43 12s. 5d. per annum, what does it amount to per week ?

Ans. £.0 16 8 $\frac{283}{365}$

The Stating.

W. £. s. d. W.

As 52 : 43 12 5 :: 1 : the *Ans.*

Note. As there are 52 weeks and 1 day in a year, you will get the true answer to the above question by the following ratio.

D. £. s. d. D.

As 365 : 43 12 5 :: 7 : the answer.

15. Suppose my income to be 16s. 8 $\frac{283}{365}$ d. per week, what is it per annum ?

Ans. £.43 12 5.

The Stating.

D. s. d. D.

As 7 : 16 8 $\frac{283}{365}$:: 365 : the *Ans.* above.

Note 1. You must first reduce the middle term to pence.

2. You must multiply by 365 (the denominator of the fraction) and add to the product the 283 which remains ; and remember always to do so in similar cases.

3. You must divide by 7, the first term, and the quotient will be the answer in 365ths of a penny, which (in all similar cases) must be first divided by the denominator, and then brought into pounds.

16. If I am to pay 1s. 7d. per week for pasturing a cow ; what must I give per week for 37 cows ?

C. s. d. C. £. s. d.

As 1 : 1 7 :: 37 : 2 18 7 *Ans.*

17. How many yards of cloth may be bought for 195dol. 75c. of which 9 $\frac{1}{2}$ yards cost 11dol. 2c. ?

Dol. c. yds. Dol. c. yds. qrs.

As 11 02 : 9 $\frac{1}{2}$:: 195 75 : 168 3 *Answer.*

18. If I buy 57 yards of cloth for 49 guineas ; what did it cost per ell English ?

yds. guin. yds.

As 57 : 49 :: 1 $\frac{1}{4}$: £.1 10s. 1 $\frac{12}{21}$ d. *Ans.*

19. A merchant, failing in trade, owes in all £.3475, and has in money and effects but £.2316 13 4 : Now, supposing his effects are delivered up, pray what will each creditor receive on the pound ?

£. £. s. d. £.

As 3475 : 2316 13 4 :: 1 : £.0 13s. 4d. *Ans.*

20. A

132 SINGLE RULE OF THREE.

20. A owes B £.3475, but B compounds with him for 13s. 4d. on the pound; pray, what must he receive for his debt?

$$\begin{array}{ccccccc} \text{£.} & \text{s.} & \text{d.} & \text{£.} & \text{£.} & \text{s.} & \text{d.} \\ \text{As } 1 & : & 13 & 4 & :: & 3475 & : & 2316 & 13 & 4. \end{array}$$

21. If the distance from Newburyport to York be 31 miles; I demand how many times a wheel, whose circumference is $15\frac{1}{2}$ feet, will turn round in performing the journey?

$$\text{Feet.} \quad \text{Cir.} \quad \text{M.} \quad \text{Cir.}$$

$$\text{As } 15\frac{1}{2} : 1 :: 31 : 10560 \text{ times, answer.}$$

22. Bought 9 chests of tea, each weighing 3cwt. 2qrs. 21lb, at £.4 9s. per cwt. what came they to?

$$\begin{array}{ccccccc} \text{Cwt.} & \text{£.} & \text{s.} & \text{C. qr. lb} & \text{£.} & \text{s.} & \text{d.} \\ \text{As } 1 & : & 4 & 9 & :: & 3 & 2 & 21 \times 9 & : & 147 & 13 & 8\frac{1}{4}. \end{array}$$

23. What will $37\frac{1}{2}$ grofs of buttons come to at 13 cents per dozen?

$$\text{Doz. c.} \quad \text{Grofs.} \quad \text{D. c.}$$

$$\text{As } 1 : 13 :: 37\frac{1}{2} : 58,50 \text{ Answer.}$$

24. A farm, containing 125 A. 3r. 27p. is rented at £.3 9s. per acre; what is the yearly rent of that farm?

$$\text{A.} \quad \text{£.} \quad \text{s.} \quad \text{A.} \quad \text{R.} \quad \text{P.} \quad \text{£.}$$

$$\text{As } 1 : 3 \quad 9 :: 125 \quad 3 \quad 27 : 434 \quad 8\text{s.} \quad 4\frac{1}{2}\text{d.} \quad 1\frac{1}{2}\text{d.} \quad \text{Answer.}$$

25. If a ship cost £.537, what are $\frac{3}{8}$ of her worth?

$$\text{Eigh.} \quad \text{£.} \quad \text{Eigh.} \quad \text{£.} \quad \text{s.} \quad \text{d.}$$

$$\text{As } 8 : 537 :: 3 : 201 \quad 7 \quad 6 \text{ Answer.}$$

26. If $\frac{7}{16}$ of a ship cost 1163D. what is the whole worth?

$$\text{Sixt.} \quad \text{D.} \quad \text{Sixt.} \quad \text{D.} \quad \text{c.} \quad \text{m.}$$

$$\text{As } 7 : 1163 :: 16 : 2658,28,5 \text{ Answer.}$$

27. Bought a cask of wine at 4s. 7d. per gallon, for 125 dollars: How much did it contain?

$$\text{s.} \quad \text{d.} \quad \text{Gal.} \quad \text{D.} \quad \text{Gal.} \quad \text{qt.} \quad \text{pt.}$$

$$\text{As } 4 \quad 7 : 1 :: 125 : 163 \quad 2 \quad 1\frac{1}{5}\frac{5}{5} \text{ Answer.}$$

28. What comes the insurance of £.537 15s. to, at £.4 $\frac{1}{2}$ per centum?

$$\text{£.} \quad \text{£.} \quad \text{£.} \quad \text{s.} \quad \text{£.} \quad \text{s.} \quad \text{d.}$$

$$\text{As } 100 : 4\frac{1}{2} :: 537 \quad 15 : 24 \quad 3 \quad 11\frac{1}{2} \quad \frac{9}{16} \text{ Anf.}$$

29. What come the commissions of £.785 to at $3\frac{1}{2}$ guineas per cent.?

$$\text{£.} \quad \text{guin.} \quad \text{£.} \quad \text{£.} \quad \text{s.} \quad \text{d.}$$

$$\text{As } 100 : 3\frac{1}{2} :: 785 : 38 \quad 9 \quad 3\frac{1}{2} \quad \frac{4}{16} \text{ Anf.}$$

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards; each package contained 8 parcels, each parcel, 12 pieces, and each piece, 20 yards; what came the whole to, and what per yard?

$$\text{Yds.} \quad \text{guin.} \quad \text{pack.} \quad \text{£.}$$

$$\text{As } 7 : 3 :: 9 : 10368 \text{ Anf. for the whole cost.}$$

$$\text{Yds.} \quad \text{guin.} \quad \text{yd.}$$

$$\text{As } 7 : 3 :: 1 : \text{£.} \quad 0 \quad 12\text{s.} \text{ Anf. per yard.}$$

31. A merchant bought 49 tuns of wine for £.273; freight cost £.27; duties £.12; cellar £.9 10s, other charges £.15, and he

he would gain £.55 10s. by the bargain ; what must I give him for 23 tuns ?

Tuns. £. £. £. £. s. £. £. s. Tuns. £.

As 49 : 273+27+12+9 10+15+55 10 :: 23 : 184 *Ans.*

32. If £.100 gain £.6 in a year, what will £.475 gain in that time ?

£. £. £. £. s.

As 100 : 6 :: 475 : 28 10 *Ans.*

33. The earth being 360 degrees in circumference, turns round on its axis in 24 hours ; how far does it turn in one minute, in the 43d parallel of latitude ; the degree of longitude, in this latitude, being about 51 statute miles ?

H. D. M. M. M.

As 24 : 360 × 51 :: 1 : 12 $\frac{3}{4}$ *Ans.*

34. Shipt for the Westindies 225 quintals of fish, at 15s. 6d. per quintal ; 37000 feet of boards, at 8 $\frac{1}{2}$ dolls. per 1000 ; 12000 shingles, at $\frac{1}{2}$ guin. per 1000 ; 19000 hoops at 1 $\frac{1}{2}$ doll. per 1000, and 53 half joes ; and in return, I have had 3000 gallons of rum at 1s. 3d. per gallon ; 2700 gallons of molasses, at 5 $\frac{1}{2}$ d. per gallon ; 1500lb of coffee, at 8 $\frac{1}{2}$ d. per lb, and 19cwt. of sugar, at 12s. 3d. per cwt. and my charges on the voyage were £ 37 12s. pray, did I gain or lose, and how much, by the voyage ?

Answer, lost £.134 9s. 9d.

35. If a staff, 4 feet long, cast a shade (on level ground) 7 feet ; what is the height of that steeple, whose shade, at the same time, measures 198 feet ?

F. sh. F. hei. F. sh. F. hei.

As 7 : 4 :: 198 : 113 $\frac{1}{2}$ *Ans.*

*36. Suppose a tax of £.755 be laid on a town, and the inventory of all the estates in the town amounts to £.9345, what must A pay, whose estate is £.149 ?

£. £. £. £. s. d.

As 9345 : 755 :: 149 : 12 0 9 $\frac{1095}{9345}$ *Ans.*

37. If

* It may not be amiss to shew the general method of assessing town or parish taxes. First, then, an inventory of the value of all the estates, both real and personal, and the number of polls, for which each person is rateable, must be taken in separate columns : The most concise way is then to make the total value of the inventory the first term, the tax to be assessed, the second, and £.1 the third, and the quotient will shew the value on the pound : 2dly, Make a table, by multiplying the value on the pound by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, or otherwise, before stating to find the value on the pound, you must deduct the sum of the average tax from the whole sum to be assessed ; for which average, you must have a separate column, as well as for the real and personal estates.

E X A M P L E.

Suppose the General Court should grant a tax of £.100000, of which the town of Newburyport is to pay £1321 17s. 6d. and, of which the polls, being 750, are to pay 5s. 3d. each ;—The town's inventory amounts to £.45000 ; what will it be on the

134 SINGLE RULE OF THREE.

37. If 50 gallons of water, in one hour, fall into a cistern, containing 230-gallons, and by a pipe in the cistern 35 gallons run out in an hour, in what time will it be filled?

Gal. gal. h. gal. h.

As 50—35 : 1 :: 230 : 15½ *Ans.*

38. A butcher went with £.416, to buy cattle : Oxen, at £.22 each, cows at £.4, steers at £.3 10s. and calves at £.2 10s. and of each a like number ; how many of each could he purchase with that sum ?

£. £. £. s. £. s. each. £. each.
As 22 + 4 + 3 10 + 2 10 : 1 :: 416 : 13 *Ans.*

39. Said Harry to Dick, my purse and money are worth 3¼ guineas ; but the money is worth eleven times as much as the purse ; pray how much money is there in it ?

Guin. s. d.

As 12 : 1 :: 3¼ : 7 7. then £.4 11s.—7s. 7d.=£.4 3s. 5d. *Ans.*

40. How many dozen pair of gloves, at 13 groats per pair, may I have for 125 dollars ?

Gr. pr. dol. doz. pr.

As 13 : 1 :: 125 : 14 5½ *Ans.*

41. There is a cistern, having four cocks ; the first will empty it in 10 minutes ; the second in 20 minutes ; the third in 40, and

the pound ; and what is A's tax, whose estate (as by the inventory) is as follows, viz. real £.376, personal £.149, and he has 3 polls.

Pol. s. d. Pol. £. s. d.

First, As 1 : 5 3 :: 750 : 196 17 6 the average part of the tax to be deducted from £.1321 17s. 6d. and there will remain £.1125.

Secondly, As 45000 : 1125 :: 1 : 6d. on the pound.

			T A B L E.					
£.	£. s. d.		£.	£. s. d.		£.	£. s. d.	
1	is 0 0 6		20	is 0 10 0		200	is 5 0 0	
2	— 0 1 0		30	— 0 15 0		300	— 7 10 0	
3	— 0 1 6		40	— 1 0 0		400	— 10 0 0	
4	— 0 2 0		50	— 1 5 0		500	— 12 10 0	
5	— 0 2 6		60	— 1 10 0		600	— 15 0 0	
6	— 0 3 0		70	— 1 15 0		700	— 17 10 0	
7	— 0 3 6		80	— 2 0 0		800	— 20 0 0	
8	— 0 4 0		90	— 2 5 0		900	— 22 10 0	
9	— 0 4 6		100	— 2 10 0		1000	— 25 0 0	
10	— 0 5 0							

Now, to find what A's rate will be.

His real estate being £.376, I find, by the table, that £.300 is £.7 10s. that £.70 is - - - 1 15 and that £.6 is - - - 3

Real.	Personal.	Polls.	Total.
£. s. d.	£. s. d.	£. s. d.	£. s. d.
9 8 0	3 14 6	0 15 9	13 18 3

for his real estate - - - £.9 8

In like manner I find his tax for personal estate to be } 3 14 6

His 3 polls, at 5s. 3d. each, are 0 15 9

+ £.9 8s.=£.13 18s. 3d. *Answer.*

and the fourth in 80 minutes, in what time will all four, running together, empty it ?

$$\text{As } \left\{ \begin{array}{l} 10 \\ 20 \\ 40 \\ 80 \end{array} \right\} \text{ Cist. Min. } : 1 :: 60 : \left\{ \begin{array}{l} 6 \\ 3 \\ 1\frac{1}{2} \\ \frac{3}{4} \end{array} \right\} \text{ Cist. Min. } : 1 : 5\frac{1}{3} \text{ Ans.}$$

$11\frac{1}{4}$ Cist.

42. A and B depart from the same place, and travel the same road ; but A goes 5 days before B, at the rate of 20 miles per day ; B follows at the rate of 25 miles per day : In what time and distance will he overtake A ?

M. M. D. M. D. D. D. M. D. M.

As 25—20 : 1 :: 20 × 5 : 20. And, As 1 : 25 :: 20 : 500

43. If the earth revolves 366 times in 365 days, in what time does it perform one revolution ?

Revol. Days. Revol.

As 366 : 365 :: 1 : 23^h 56' 3" 56" + = 1 Sidereal day.*

44. If the earth makes one complete revolution in 23^h 56' 3" +, in what time does it pass through one degree ?

As 366° : 23^h 56' 3" :: 1° : 3' 55" 20" Answer.

45. If the earth performs its diurnal revolution in a solar† day, or 24 hours ; in what time does it move one degree ?

As 360° : 24^h :: 1° : 4' Answer.

46. Sold a cargo of flax seed in Ireland, for £.1795 10s. Irish money ; what does that amount to, in Massachusetts currency, £.81 5s. Irish being equal to £.100 Massachusetts ?

Irish. Mass. Irish. Mass.

As £.81 $\frac{1}{4}$: £.100 :: £.1795 $\frac{1}{2}$: £.2209 16s. 11d. Ans.

Or, As £.13 : £.1795 $\frac{1}{2}$:: £.16 : £.2209 16s. 11d. as before, because £.13 Irish are equal to £.16 Massachusetts.

47. My correspondent in Maryland purchased a cargo of flour for me, for £.437 that currency ; how much Massachusetts money must I remit him, £.125 Maryland being equal to £.100 Massachusetts ?

£. £. £. £. s.

As 125 : 100 :: 437 : 349 12 Ans.

Or, As £. £. s. { Because £.5 Maryland are equal to £.4 Massachusetts.

48. A bill of exchange was accepted at Newburyport for the payment of £.345 10s, for the like value delivered in Newyork, at £.133 $\frac{1}{3}$ Newyork currency for £.100 Massachusetts ditto ; how much money was paid in Newyork, £.75 Massachusetts being equal to £.100 of Newyork ?

As

* A sidereal day is the space of time which happens between the departure of a star from, and its return to the same meridian again.

† The solar day is that space of time which intervenes between the sun's departing from any one meridian, and its return to the same again.

136 SINGLE RULE OF THREE.

Mass. N. Y. Mass. N. Y.

As £.75 : £.100 :: £.345 10s. : £.460 13s. 4d. *Ans.*

49. When the exchange from Massachusetts to Georgia is £.83 $\frac{1}{3}$ Georgia per £.100 Massachusetts, how much Massachusetts money must be paid in Boston to balance £.457 Georgia currency?

Geor. Mass. Geor. Mass.

As £.83 $\frac{1}{3}$: £.100 :: £.457 : £.548 8s. *Ans.*

50. A merchant delivered at Boston £.320 Massachusetts currency, to receive £.400 in Philadelphia; what was the Massachusetts pound valued at?

B. P. B. P.

As £.320 : £.400 :: £.1 : £.1 5s. *Ans.*

£. £. £. £. s. { Because £.80 Massachusetts are equal to £.100 Pennsylvania.

51. If I draw a bill of exchange for £.537 10s. 6d. Massachusetts, to be paid in Ireland, at £.123 $\frac{1}{3}$ Massachusetts, per £.100 Irish; for how much Irish money must I draw the bill?

Mass. Irish. Mass. Irish.

As £.123 $\frac{1}{3}$: £.100 :: £.537 10s. 6d. : £.436 14s. 9 $\frac{1}{2}$ d.

£. £. £. £. { Because £.16 Massachusetts are = £.13 Irish.

52. Suppose a bill is drawn in Ireland, and payable in Boston, for £.673 12s. 6d. Irish; how much Massachusetts money comes it to, the exchange at £.81 $\frac{1}{4}$ Irish per £.100 Massachusetts?

£. £. £. s. d. £. s. d.
As 81 $\frac{1}{4}$: 100 :: 673 12 6 : 829 1 6 $\frac{2}{3}$.

The value of any quantity of silver in any of the currencies of the United States may be found by the following proportion.

As the number of grains, contained in £.1, is to £.1; so are the grains, in any given quantity, to its value.

53. What is the value of 1lb of silver in Massachusetts currency; the pound (or 20 shillings) containing 1393 $\frac{1}{2}$ grains?

£. s. d.
As 1393 $\frac{1}{2}$: 1 :: 5760 : 4 2 8.

All questions in the Rule of Three, whether direct or inverse, may be solved by the following rule.

Let that number, which is of the same name or quality as the number sought, be the third term; then, consider whether the number sought should be more or less than the third; if *more*, let the *greater* of the two other terms be the *middle* term, and the *less*, the *first*; but if the fourth number ought to be *less* than the third, then give the *less* the *second* place, and the *greater*, the first. The question being thus stated, the proportion will be; as the first term is to the second, so is the third to the fourth, or number sought.

Euclid's Elements V. 14.

Note. The first and second terms must always be brought into one name, and the third into the lowest mentioned, then proceed

SINGLE RULE OF THREE. 137

in the common method, by multiplying the second and third terms together, and dividing the product by the first, and the quotient will be the answer, in the same name as the third term was reduced into.

54. If 15 yards of cloth cost £.6, how many yards may I have for £.125?

$$\begin{array}{rcl} \text{£.} & \text{£.} & \text{yds.} \\ \text{As } 6 & : 125 & :: 15 \end{array}$$

15

625

125

6)1875

312½ yards Ans.

55. If 12 men can do a job in 20 days; in what time will 18 men do it?

$$\begin{array}{rcl} \text{M.} & \text{M.} & \text{D.} \\ \text{As } 18 & : 12 & :: 20 \end{array}$$

20

18)240(13⅓ days, Ans.

18

60

54

6

56. If I give 7s. 9d. for 3 yards, how many yards may I have for £.39?

$$\begin{array}{rcl} \text{s.} & \text{d.} & \text{£.} & \text{yds.} & \text{yds.} & \text{qrs.} & \text{n.} \\ \text{As } 7 & 9 & : 39 & :: 3 & : 301 & 3 & 2\frac{2}{3} \text{ Ans.} \end{array}$$

Or thus,

State the question in the usual way, and let the second term keep its proper, or natural place; then, multiply it by the greater or less extreme, that is, by the first or third number, accordingly as the answer ought to be greater or less; divide the product by the other term, and the quotient will be the answer.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

R U L E.*

Having made the necessary preparations, as directed in Multiplication and Division of Vulgar Fractions, state your question as in whole numbers, and invert the first term of the proportion; then multiply the three terms continually together, and the product will be the answer.

1. If ⅔ of a yard cost ⅔ of a £. what will ⅔ of an Ell Eng. cost?

$$\frac{5}{8} \text{ yd.} = \frac{5}{8} \text{ of } \frac{4}{7} \text{ of } \frac{1}{5} = \frac{5 \times 4 \times 1}{8 \times 1 \times 5} = \frac{1}{2} \text{ Ell English.}$$

E. Eng. £. E. Eng.

$$\text{As } \frac{1}{2} : \frac{5}{7} :: \frac{2}{15} \text{ And } \frac{2}{1} \times \frac{5}{7} \times \frac{1}{15} = \frac{2 \times 5 \times 1}{1 \times 7 \times 15} = \frac{10}{105} \text{ £.} = 17\text{s. } 1\frac{1}{2}\text{d. } \frac{3}{32} \text{ Ans.}$$

2. If ⅔ yd. cost ⅔ £. what will 40½ yds. come to?

Ans. £.59 1s. 3d.

3. If 70 bushels of corn cost £.12⅓, what is it per bushel?

Ans. 3s. 7½d.

4. If

* This rule and the next, depend upon the same principle as the Rule of Three in whole numbers.

128 RULE OF THREE DIRECT

4. If $\frac{7}{16}$ of a ship cost £.51, what are $\frac{3}{4}$ of her worth?
Ans. £.10 18s. 6 $\frac{1}{2}$ d. $\frac{3}{4}$.
5. At £.3 $\frac{5}{8}$ per cwt. what will 9 $\frac{2}{3}$ lb come to? *Ans.* 6s. 3d. $\frac{5}{6}$.
6. A person having $\frac{4}{5}$ of a vessel, sells $\frac{2}{3}$ of his share for £.319;
what is the whole vessel worth? *Ans.* £.598 2s. 6d.
7. A merchant sold 5 $\frac{1}{2}$ pieces of cloth, each containing 12 $\frac{2}{3}$ yds.
at 9s. $\frac{1}{2}$ d. per yard; what did the whole amount to?
Ans. £.31 9s. 10d. $\frac{3}{4}$.
8. A buys of B. £.560 $\frac{3}{4}$ bank stock at £.85 $\frac{2}{3}$ per cent. what
comes it to? *Ans.* £.480 7s. 6 $\frac{1}{2}$ d.
9. A merchant makes insurance upon a vessel and cargo, val-
ued at £.3750 16s. at 15 $\frac{1}{2}$ guineas per cent. what does the pre-
mium amount to? *Ans.* 813 18s. 6 $\frac{1}{2}$ d.
10. A merchant in Holland draws a bill upon his correspond-
ent in Boston for 3750 ducats at 8s. 4 $\frac{1}{2}$ d. : How much Massachu-
setts currency must he receive? *Ans.* £.1565 12s. 6d.
11. A gentleman from Boston being in England, where the
price of silver is to that of gold, as 1 to 15 $\frac{1}{4}$, exchanged 158 $\frac{1}{4}$ lb
of silver for gold; on his return to Massachusetts, where the
price of silver is to that of gold, as 1 to 15 $\frac{1}{3}$, a friend, wanting
his gold, gave him the value thereof in silver; what weight of
silver did he gain by the exchange?

£ S.	G.	£ S.	G.	S.	G.	£ S.
As 15 $\frac{1}{4}$: $\frac{1}{4}$:: 158 $\frac{1}{4}$: 10 $\frac{1}{2}$	As $\frac{1}{4}$: 15 $\frac{1}{3}$:: 10 $\frac{1}{2}$: 162 $\frac{3}{6}$

Ans. 4 $\frac{1}{12}$ lb.

12. A merchant bought a number of bales of velvet, each con-
taining 129 $\frac{1}{2}$ yards, at the rate of 7 dollars for 5 yards, and sold
them out at the rate of 11 dollars for 7 yards; and gained 200
dollars by the bargain; how many bales were there?

Yds. Dol.	Yds. Dol.	}	Sold 5 yards for 7 $\frac{6}{7}$ Dollars.
As 7 : 11 :: 5 : 7 $\frac{6}{7}$			Bought 5 yds. for 7 Dollars.
			In 5 yards gained $\frac{6}{7}$ Dollar.

Dol. Yds.	Dol.	Yds.	Yds.	B.	Yds.	B.
As $\frac{6}{7}$: 5 :: 200 : 1166 $\frac{2}{3}$, and,	As 129 $\frac{1}{2}$: $\frac{1}{4}$:: 1166 $\frac{2}{3}$: 9	<i>Ans.</i>				

Although the method before laid down be universally applica-
ble, yet there are other methods more ready and expeditious in
some particular cases.

R U L E I.

If the first and third terms be fractions, and the second a
whole number, reduce the first and third to one common denomi-
nator, then, rejecting the denominators, make the numerator of
the first, the first term, and the numerator of the third, the third
term, and work as in whole numbers.

If $\frac{5}{8}$ yard cost 9s. what cost $\frac{7}{12}$ yard at that rate?
 $\frac{5}{8} = \frac{15}{24}$ and $\frac{7}{12} = \frac{14}{24}$. Now, As 15 : 9s. :: 14 : 8s. 4 $\frac{1}{4}$ d. *Ans.*

R U L E II.

R U L E II.

If, of the first and third terms, one be 1, and the other a fraction ; put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land cost £. 12, what will $\frac{5}{8}$ of an acre cost, at that rate ?

$$\begin{array}{rcll} & \text{Den.} & \text{£.} & \text{Num.} \\ \text{As } 8 & : & 12 & :: 5 : 7 \text{ } 10 \text{ } \text{Ans.} \end{array}$$

R U L E III.

If the second term be a fraction likewise, (that is, if all the terms be fractions) ; having reduced the first and third to one common denominator, multiply the numerator of the first term by the denominator of the second, for a divisor ; and the numerator of the third by the numerator of the second, for a dividend ; divide the last product by the first, and the quotient will be the answer.

If $\frac{1}{2}$ yard of cloth cost $\frac{3}{4}$ £. what cost $\frac{7}{8}$ yard ?

$\frac{3}{4} = \frac{4}{8}$, which reduces it to a common denominator ; then

$$\begin{array}{rcll} \text{As } 4 & : & \frac{3}{4} & :: 7 \\ \underline{4} & & \underline{3} & \\ 16 & & 16)21(1\frac{5}{16} = 26s. \text{ } 3d. \text{ } \text{Ans.} \\ & & \underline{16} & \\ & & 5 & \end{array}$$

To find the value of Gold in Massachusetts Currency.

PROB. I. Given the weight of any quantity of gold, to find its value.

Oz. £. Oz. £. pwt. s. gr. d.
THEOREM 1. As 1 : $5\frac{1}{3}$:: 12 : 64 :: 1 : $5\frac{1}{3}$:: 1 : $2\frac{2}{3}$ (Case 1.) = $\frac{2\frac{2}{3}}{1}$

(Case 2.) = $\frac{5\frac{1}{3}}{2}$ (Case 3.) = $\frac{3}{3}$. Therefore,

Rule 1.—If the given quantity be in grains ; say, As the denominator is to the number of grains ; so is the numerator to their value, in pence.

1. What is the value of 18 grains of gold ?

By Case 1.

By Case 2.

By Case 3.

$$\begin{array}{rcll} & \text{Gr.} & & \\ \text{As } 1 & : & 18 & :: 2\frac{2}{3} \\ & \underline{2\frac{2}{3}} & & \\ & 36 & & \\ & \underline{12} & & \\ & 48 & & \\ 12)48(4s. \text{ } \text{Ans.} & & & \\ & & \text{Gr.} & \\ \text{As } 2 & : & 18 & :: 5\frac{1}{3} \\ & & \underline{5\frac{1}{3}} & \\ & & 90 & \\ & & \underline{6} & \\ & & 96 & \\ 2)96(48d. = 4s. & & & \\ & & \text{Gr.} & \\ \text{As } 3 & : & 18 & :: 8 \\ & & \underline{8} & \\ & & 144 & \\ & & \underline{48d.} & \\ & & 4s. & \end{array}$$

Rule

Rule 2. If the given quantity consist of ounces, pennyweights and grains, halve the grains, and then proceed as in multiplication of pounds, shillings and pence, making the numerator, in Case 2d, the multiplier.

1. What is the value of 7 ^{Oz.} 8 ^{pwt.} 16 ^{gr.} of gold?

$$\begin{array}{r} \text{Gr.} \qquad \text{gr.} \qquad \text{oz. pwt. gr.} \\ 16 \div 2 = 8, \text{ then, } 7 \quad 8 \quad 8 \\ \hline \qquad \qquad \qquad 5\frac{1}{2} \end{array}$$

$$\begin{array}{r} 37 \quad 3 \quad 4 \\ 2 \quad 9 \quad 6\frac{2}{3} \\ \hline \end{array}$$

£.39 12 10 $\frac{2}{3}$ Ans.

Rule 3.—If the given quantity consist of pounds only, multiply by 64, and the product will be the answer; but if it consist of pounds, ounces, &c. it will be most convenient to reduce the pounds to ounces, and proceed by Rule 2.

1. What is the value of 36 $\frac{1}{2}$ of gold, at £.64 per lb?

$$\begin{array}{r} 64 \\ \hline 144 \\ 216 \\ \hline \end{array}$$

£.2304 Ans.

2. What is the value of 15 $\frac{1}{2}$ 9oz. 12 $\frac{1}{2}$ pwt. 18gr. of gold?

$$\begin{array}{r} 12 \\ \hline \text{oz. } 189 \quad 12 \quad 9 = 18 \div 2 \\ \hline \qquad \qquad \qquad 5\frac{1}{2} \end{array}$$

$$\begin{array}{r} 948 \quad 3 \quad 9 \\ 63 \quad 4 \quad 3 \\ \hline \end{array}$$

£.1011 8 0 Ans.

PROB. 2. To ascertain the value of any given quantity of gold in Spanish milled dollars.

THEOREM 2. 1pwt. of gold = 5 $\frac{1}{2}$ s. 1 dollar = 6s. And

$$\frac{5\frac{1}{2}}{6} = \frac{11}{12} = \frac{2}{3}. \text{ Therefore,}$$

Rule. Reduce the given quantity of gold to pennyweights; then, as the denominator is to the given quantity; so is the numerator to the answer in dollars. Or,

Divide by the denominator, and multiply the quotient by the numerator. Or,

Divide

Divide by the denominator and subtract the quotient from the dividend. In either case, you will have the answer.

1. What is the value of 6oz. 6pwt. of gold, in Spanish dollars?

$$\begin{array}{r} \text{As } 9 \frac{1}{4} \text{ } 126 :: 8 \\ \hline 9)1008 \\ \hline \text{Ans. } 112 \text{ dolls.} \end{array}$$

$$\begin{array}{r} 20 \\ \hline 126 \text{ pwt.} \\ \hline \text{Or,} \\ 9)126 \\ \hline 14 \times 8 = 112 \text{ Anf.} \end{array}$$

$$\begin{array}{r} \text{Or,} \\ 9)126 \\ \hline 14 \\ \hline 112 \text{ Anf.} \end{array}$$

2. In 7 ^{oz.} 13 ^{pwt.} 17 ^{gr.} how many dollars? 20

$$\begin{array}{r} 153 \frac{17}{24} \\ \hline 619 \\ 307 \\ \hline 3689 \\ \text{As } \frac{9}{1} : \frac{3689}{24} :: \frac{8}{1} : \frac{29512}{216} \\ 216)29512(136 \text{ doll.} \\ \hline 791 \\ 648 \\ \hline 1432 \\ 1296 \\ \hline 136 \\ 6 \\ \hline 216)816(3s. \\ 648 \\ \hline 168 \\ 12 \\ \hline 216)2016(9d. \\ 1944 \\ \hline 72 \\ 4 \\ \hline 216)288(1gr. \\ 216 \\ \hline 72 \end{array}$$

3. In 9 ^{oz.} 8 ^{pwt.} 6 ^{gr.} how many dollars? 20

$$\begin{array}{r} 188 \\ \hline \text{As } 9 \frac{1}{4} \text{ } 188 \frac{1}{4} :: 8 \\ \hline 9)1506 \end{array}$$

Anf. 167 $\frac{1}{3}$ or 3 eight pences.

Or, suppose it were required to reduce the quantity of gold to dollars, 90ths and 8ths of a 90th?

Find the value for ounces and pennyweights, as in the first Example; the quotient will be dollars, and the remainder, (if any) 9ths of a dollar: Then, as one grain is very nearly $\frac{3}{90} + \frac{2}{9}$ of $\frac{1}{90}$ of a dollar; divide the grains by 3, and the quotient will be 9ths of a dollar: Then multiply this remainder into $\frac{3}{90} + \frac{2}{9}$, and add all to the other work.

4. In

142 RULE OF THREE IN VULGAR FRACTIONS.

In 5oz. 19pwt. 17gr. how many dollars, 90ths and 8ths of a 90th?

$$\begin{array}{r} \text{oz. pwt.} \\ 9 : 5 \quad 19 :: 8 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 119 \\ 8 \end{array}$$

$$9)95\frac{1}{2}$$

$$\begin{array}{r} \text{gr.} \\ 3)17(5 \text{ ninths} = \frac{50}{90} \\ \hline 15 \end{array}$$

$$2 \times \frac{3}{90} + \frac{2}{8} = \frac{6}{90} + \frac{4}{8}$$

$$105\frac{7}{9} = \frac{70}{90}, \text{ then } 105\frac{70}{90} + \frac{50}{90} + \frac{6}{90} + \frac{4}{8} = 106\frac{36}{90}, \frac{4}{8} \text{ Ans.}$$

PROB. 3. To ascertain the weight of gold equivalent to any given sum, currency.

Rule 1. If the given sum be in pence, reverse Rule 1. Theorem 1. that is; As the numerator 8 is to the given sum in pence; so is the denominator 3 to the weight required, in grains.

What weight of gold is equal to 4s.?

$$\begin{array}{r} \text{d.} \\ \text{As } 8 : 48 :: 3 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 48d. \end{array}$$

$$8)144$$

Ans. 18 grains.

Rule 2. If the given sum be in pounds, shillings and pence.

As $5\frac{1}{2}$ is equal to $\frac{16}{8}$; therefore, divide the given sum by 8, and that quotient by 2; add the two quotients together, double the last denomination, and you will have the answer.

What quantity of gold is equivalent to £.45 13s. 4d.?

Mark the pounds, shillings and pence, as oz. pwt. and gr. }

$$\begin{array}{r} \text{oz. pwt. gr.} \\ 8)45 \quad 13 \quad 4 \end{array}$$

$$\begin{array}{r} 2)5 \quad 14 \quad 2 \\ \hline 2 \quad 17 \quad 1 \end{array} \left. \vphantom{\begin{array}{r} 2)5 \quad 14 \quad 2 \\ \hline 2 \quad 17 \quad 1 \end{array}} \right\} \text{Add.}$$

$$\begin{array}{r} 8 \quad 11 \quad 3+3 \end{array}$$

$$\text{Oz. } 8 \quad 11 \quad 6 \text{ Ans.}$$

PROB. 4. To find the weight of gold equivalent to any given number of dollars.

Rule. As the numerator 8 is to the number of dollars; so is the denominator 9 to the answer in pennyweights: Or, divide the dollars by the numerator 8, and add the quotient to the dividend,

Or,

RULE OF THREE DIRECT IN DECIMALS. 143

Or, divide as before, and multiply the quotient by the denominator 9. In either case you will have the answer.

Required the weight of gold equal to 76 dollars.

As 8 : 76 :: 9 Or thus, 8)76 Or, $9\frac{1}{2} \times 9 = 85\frac{1}{2} \text{ pwt.}$

$$\begin{array}{r} 9 \\ \hline 8 \overline{)684} \end{array}$$

$$\begin{array}{r} 8 \overline{)684} \end{array}$$

oz. pwt. gr.

Ans. $85\frac{1}{2} \text{ pwt.} = 4 \ 5 \ 12$

$$\begin{array}{r} 9\frac{1}{2} \\ \hline \end{array}$$

Ans. $85\frac{1}{2} \text{ pwt.}$

RULE OF THREE DIRECT IN DECIMALS.

R U L E.

Having reduced your fractions to decimals, and stated your question as in whole numbers, multiply the second and third together; divide by the first, and the quotient will be the answer.

1. If $\frac{5}{8}$ of a yard cost $\frac{7}{12}$ of a pound; what will $9\frac{2}{3}$ yards come to?

$$\frac{5}{8} = ,625 \quad \frac{7}{12} = ,583 \quad + \quad \text{and} \quad \frac{2}{3} = ,666 \quad +$$

$$\begin{array}{r} \text{Yd.} \quad \text{£.} \quad \text{Yd.} \\ \text{As } ,625 \quad + \quad ,583 :: 9,666 \end{array}$$

$$\begin{array}{r} ,583 \\ \hline 28998 \\ 77328 \\ \hline 48330 \end{array}$$

$$,625 \overline{)5,635278} (9,0164 + = \text{£.9 cs. } 3\frac{1}{2} \text{ d. Ans.}$$

$$\begin{array}{r} 1027 \\ 625 \\ \hline 4028 \\ 3750 \\ \hline 2780 \\ 2500 \\ \hline 280 \end{array}$$

2. If 10z. of silver cost 6s. 8d. what is the price of a bowl, which weighs 1lb 7oz. 13gr. ?

Ans. £.6 6s. 10d.

3. If $9\frac{2}{3}$ yards cost £.3 7s. 6d. what will $\frac{1}{2}$ yard come to?

Ans. 3s. $5\frac{1}{2}$ d.

4. If 1hhd. sugar, weighing 9cwt. 3qrs. 14lb cost £.27 13s. 7d. what will 3cwt. 1qr. 17lb come to?

Ans. £.9 10s. $8\frac{1}{2}$ d.

5. A tobaccoist bought 5hhd. of tobacco, each weighing 8cwt. 2qrs. 19lb, for £.161 16s. 8d. what was it per ounce?

Ans. $\frac{1}{2}$ d.

6. There is a cistern, which has 3 cocks, the first will empty it in $\frac{1}{4}$ of an hour, the second in $\frac{2}{3}$, and the third in $1\frac{1}{2}$ hour; in what time will it be emptied, if all three run together?

As

144 RULE OF THREE DIRECT IN DECIMALS.

$$\begin{array}{l} \text{As } \left\{ \begin{array}{l} .25 : 1 :: 1 : 4 \\ .75 : 1 :: 1 : 1,333+ \\ 1,5 : 1 :: 1 : 0,666+ \end{array} \right. \begin{array}{l} \text{h. Cist.} \\ \text{h. Cist.} \\ \text{Cist. h.} \end{array} \text{Cist. h. Cist.} \\ \text{As } 6 : 1 :: 1 : ,1666+ = 10 \text{ min. Ans.} \\ \hline 6 \text{ Cist.} \end{array}$$

7. A conduit has a cock, which, running into a cistern, will fill it in 12 minutes: This cistern has 3 cocks; the first will empty it in $1\frac{1}{4}$ hour, the second in $37\frac{1}{2}$ minutes, and the third in $\frac{1}{2}$ an hour: In what time will the cistern be filled, if all four run together?

$$\begin{array}{l} \text{As } \left\{ \begin{array}{l} .2 : 1 :: 1 : 5 \\ 1,25 : 1 :: 1 : 0,8 \\ .625 : 1 :: 1 : 1,6 \\ .5 : 1 :: 1 : 2 \end{array} \right. \begin{array}{l} \text{h. Cist.} \\ \text{h. Cist.} \\ \text{the filling Cock.} \\ \text{emptying Cocks.} \end{array} \left\{ \begin{array}{l} 5 \text{ Cist. filled in an hour.} \\ 4,4 \text{ do. emptied in do.} \\ .6 \text{ Cist. difference.} \end{array} \right. \\ \hline 4,4 \end{array}$$

$$\begin{array}{l} \text{Cist. h. Cist. h. h. m.} \\ \text{Then, as } ,6 : 1 :: 1 : 1,66+ = 1 \text{ } 40 \text{ Ans.} \end{array}$$

D. d. c.

8. If 19 yards cost 25, 7 5 what will 435 $\frac{1}{2}$ yards come to?

$$\begin{array}{l} \text{yds. D. d. c. yds.} \\ \text{As } 19 \text{ } \ddagger \text{ } 25, 7 \text{ } 5 :: 435,5 \\ \hline 25,75 \end{array}$$

$$\begin{array}{r} 217 \text{ } 75 \\ 3048 \text{ } 5 \\ 21775 \\ 8710 \\ \hline \text{D. d. c. m.} \\ 19)11214,125(590,2 \text{ } 1 \text{ } 7 \text{ } \frac{2}{19} \text{ Ans.} \end{array}$$

$$\begin{array}{r} 95 \\ \hline 171 \\ 171 \\ \hline \end{array}$$

$$\begin{array}{r} 41 \\ 38 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ 19 \\ \hline \end{array}$$

$$\begin{array}{r} 135 \\ 133 \\ \hline \end{array}$$

2

9. If 345 yards of tape cost 5, 1 7 5, what will one yard cost?

$$\begin{array}{l} \text{yds. D. d. c. m. yd.} \\ \text{As } 345 \text{ } \ddagger \text{ } 5, 1 \text{ } 7 \text{ } 5 :: 1 \end{array}$$

$$\begin{array}{r} \text{D. d. c. m.} \\ 345)5, 1 \text{ } 7 \text{ } 5(0 \text{ } 1 \text{ } 5 \text{ Ans.} \\ \hline 3 \text{ } 4 \text{ } 5 \end{array}$$

$$\begin{array}{r} 1 \text{ } 7 \text{ } 2 \text{ } 5 \\ 1 \text{ } 7 \text{ } 2 \text{ } 5 \\ \hline \end{array}$$

D. d. c. m.

10. If I give 12, 8 2 5 for 675 tops, how many tops will 19 mills buy?

$$\begin{array}{l} \text{D. d. c. m. T. d. c. m.} \\ \text{As } 12, 8 \text{ } 2 \text{ } 5 \text{ } \ddagger \text{ } 675 :: ,0 \text{ } 1 \text{ } 9 \end{array}$$

$$,019$$

$$\begin{array}{r} 6075 \\ 675 \\ \hline \end{array}$$

$$\begin{array}{r} 12,825)12,825(1 \text{ top, Ans.} \\ \hline 12,825 \end{array}$$

RULE

RULE OF THREE INVERSE, OR RECIPROCAL PROPORTION,

Teaches, by having three numbers given, to find a fourth, which shall have the same proportion to the second, as the first has to the third.

Therefore, the greater the third term is, in respect to the first, the less will the fourth term be, in respect to the second; or, the less the third term is in proportion to the first, the greater the fourth must be in proportion to the second; and this is called *reciprocal, inverted or indirect* Proportion.

The principal difficulty, that will embarrass the learner, will be, to distinguish when the proportion is direct, and when indirect. This is done by an attentive consideration of the sense and tenor of the question proposed: For if thereby it appears that, when the third term of the stating is less than the first, the answer must be less than the second; or when the third is greater than the first, the answer must be greater than the second; then the proportion is direct: But, if the third be less than the first, and yet the sense of the question requires the fourth to be greater than the second; or if the third being greater than the first, the answer must be less than the second, the proportion is inverse.

R U L E.*

State and reduce the terms as in the Rule of Three Direct; then, multiply the first and second terms together, and divide the product by the third; the quotient will be the answer in the same denomination as the middle term was reduced into.

If there be fractions in your question, they must be stated as before directed, and if they be vulgar, invert the third term: Then multiply the three terms continually together, and the product will be the answer.

E X A M P L E S.

1. How much shalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{1}{4}$ yards of cloth which is $1\frac{1}{4}$ yard wide?

$$\begin{array}{rcl} & yd. & yds. & qrs. \\ \text{As } 1\frac{1}{4} & : & 6\frac{1}{4} & :: 3 \\ \hline 4 & & 4 & \\ \hline 5 & & 27 & \end{array}$$

$$\begin{array}{rcl} & qrs. & qrs. & qrs. \\ \text{As } 5 & : & 27 & :: 3 \\ & & 5 & \\ \hline & & 3)135 & \\ \hline & & 4)45 & \\ \hline & & & \end{array}$$

$11\frac{1}{4}$ yards, Anf.

The

* The reason of this rule may be explained from the principles of Compound Multiplication and Compound Division, in the same manner as the direct rule.—For example, If 4 men can do a piece of work in 12 days, in what time will 8 men do it?

The same by Vulgar Fractions.

First, $1\frac{1}{4} = \frac{5}{4}$, $6\frac{3}{4} = \frac{27}{4}$, and $3\text{ qrs.} = \frac{3}{4}$. Then,

As $\frac{5}{4} \text{ yds.} : \frac{27}{4} \text{ yds.} :: \frac{3}{4} \text{ yds.}$ And $\frac{5}{4} \times \frac{27}{4} \times \frac{4}{3} = \frac{5 \times 27 \times 4}{4 \times 4 \times 3} = \frac{540}{12} = 45 = 11\frac{1}{4} \text{ yds.}$ Ans.

The same by Decimal Fractions.

$1\frac{1}{4} = 1,25$. $6\frac{3}{4} = 6,75$. And $3\text{ qrs.} = ,75$. Then,

As $1,25 \text{ yds.} : 6,75 \text{ yds.} :: ,75 \text{ yds.}$

$1,25$
 $\frac{8375}{1350}$
 675
 $,75 \text{ } 8,4375 (11,25 \text{ yds. Ans.}$

75

93

75

187

150

375

375

3. How many yards of carpet, $2\frac{3}{4}$ feet wide, will cover a floor, which is 18 feet long and 16 feet wide?

$\text{ft. ft. ft. ft. yds.}$ { Note, I multiply $2\frac{3}{4}$ by 3, because 3 feet = 1 yard.
 As $16 \text{ ft.} : 18 \text{ ft.} :: 2\frac{3}{4} \text{ ft.} \times 3 : 34\frac{1}{2} \text{ yds.}$ Ans.

4. Suppose I lend a friend £.350 for 5 months, he promising the like kindness; but, when requested, can spare but £.125, how long may I keep it to balance the favour?

£. Mo. £. Mo.

As $350 \text{ £.} : 5 \text{ Mo.} :: 125 \text{ £.} : 14 \text{ Mo.}$ Ans.

5. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain 9 months?

Mo. M. Mo. M. M.

As $5 \text{ Mo.} : 450 \text{ M.} :: 9 \text{ Mo.} : 250$, and $450 - 250 = 200 \text{ men,}$ Ans.

6. If a man perform a journey in 15 days, when the day is 12 hours long, in how many will he do it, when the day is but 10 hours?

h. d. h. d.

As $12 \text{ h.} : 15 \text{ d.} :: 10 \text{ h.} : 18 \text{ d.}$ Ans.

7. If

As 4 men $\text{ } : 12 \text{ days} :: 8 \text{ men} \text{ } : \frac{4 \times 12}{8} = 6 \text{ days, the Answer.}$

And here the product of the first and second terms, that is, 4 times 12, or 48, is evidently the time in which one man would perform the work. Therefore, 8 men will do it in one eighth part of the time, or 6 days.

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide must it be, when it is but 19 rods long, to make an acre?

Leng. Br. Leng. Br. ft. in.

As 40 : 4 :: 19 : 8 6 11 $\frac{7}{19}$ *Anf.*

8. If, when wheat is 6s. per bushel, the twopenny loaf weigh 9,6oz. what ought it to weigh, when wheat is 7s. 6d. per bushel?

s. oz. s. d. oz. pwt. gr.

As 6 : 9,6 :: 7 6 : 7 13 14,4 *Anf.*

9. If a piece of board be 30 inches in length, what breadth will make 1 $\frac{1}{2}$ square foot?

sq. ft. in. in. in.

As 1,5 : 1 :: 30 : 7,2 *Anf.*

10. If 9 men can build a house in 5 months, by working 14 hours per day, in what time will the same number of men do it, when they work only 10 hours per day?

h. mo. h. mo.

As 14 : 5 :: 10 : 7 *Anf.*

11. A wall, which was to be built 24 feet high, was raised 8 feet by 6 men, in 12 days : How many men must be employed, to finish the wall in 4 days?

ft. m. ft. m.

As 8 : 6 :: 24—8 : 12 to finish it in 12 days. And,
d. m. d. m.

As 12 : 12 :: 4 : 36 to finish it in 4 days.

12. There is a cistern having a pipe, which will empty it in 6 hours : How many pipes, of the same capacity, will empty it in 20 minutes?

h. pi. min. pi.

As 6 : 1 :: 20 : 18 *Anf.*

13. What number of men must be employed to finish in 9 days, what 15 men would be 30 days about?

da. m. da. m.

As 30 : 15 :: 9 : 50 *Anf.*

14. If a field will feed 6 cows 91 days, how long will it feed 21 cows?

c. d. c. d.

As 6 : 91 :: 21 : 26 *Anf.*

15. How much in length, that is 8 $\frac{5}{7}$ inches broad, will make a foot square?

in. in. in. in.

As $8\frac{5}{7}$: $\frac{1}{1}$:: $8\frac{5}{7}$: $16\frac{32}{49}$ *Anf.*

16. How much in length, that is 13 $\frac{7}{8}$ poles in breadth, will make a square acre?

po. po. po. po.

As $16\frac{0}{1}$: $\frac{1}{1}$:: $13\frac{7}{8}$: $11\frac{59}{112}$ *Anf.*

17. A regiment of soldiers, consisting of 745 men, is to be clothed, each suit to contain 3 $\frac{1}{2}$ yards of cloth, which is 1 $\frac{3}{8}$ yard wide,

148 COMPOUND PROPORTION.

wide, and lined with shalloon $\frac{7}{8}$ yard wide ; how many yards of shalloon will line them ?

As $745 \times 3\frac{1}{2} \div 1\frac{3}{8} :: \frac{7}{8} \div 4097\frac{1}{2}$ yards, *Anf.*

18. If a suit of clothes can be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide ; how many yards of coating $\frac{7}{8}$ of a yard wide will it require for the same person ?

yd. yds. yd. yds. qr. n.
As $1\frac{3}{8} : 4\frac{1}{8} :: \frac{7}{8} : 6 \quad 1 \quad 3\frac{5}{7}$ *Anf.*

ABBREVIATIONS.

To know whether a fraction, when abbreviated, be equivalent in all respects to the original given fraction.

RULE.

As the numerator of the fraction, in its lowest terms, is to its denominator ; so will the numerator of the original fraction be to its own denominator.

Or, as one numerator is to the other ; so will one denominator be to the other, &c.

A owes B £.75 13s. 6d. now £.100 of A's money is equal to £.140 of B's ; what must A pay to satisfy the said debt ?

$$\frac{100}{140} = \frac{5}{7}, \text{ therefore, } \begin{array}{r} \text{£. s. d.} \\ 75 \quad 13 \quad 6 \\ \quad \quad 5 \\ \hline 7)378 \quad 7 \quad 6 \\ \hline \text{£.54 . 1 0}\frac{6}{7} \text{ } \textit{Ans.} \end{array}$$

Now, to prove whether $\frac{5}{7}$ be equal to $\frac{100}{140}$.

Num. Den. Num. Den. Num. Num. Den. Den.
As 5 : 7 :: 100 : 140 — Or, as 5 : 100 :: 7 : 140.

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE,

Teaches to resolve such questions as require two, or more, statings by simple proportion ; and that, whether direct or inverse : It is composed (commonly) of 5 numbers to find a sixth, which, if the proportion be direct, must bear such proportion to the 4th and 5th as the 3d bears to the 1st and 2d ; but if inverse, the 6th number must bear such proportion to the 4th and 5th, as the first bears to the 2d and 3d.

FIRST

FIRST METHOD.*

By two, or more, proportions in the Single Rule of Three.

R U L E.

1. Let either of the two numbers, of which the question is raised, be put in the third place, and the correspondent number, of the same name or kind, in the first; the second will be that, which has no correspondent number given.

2. Three of the five given numbers being thus stated, find a fourth proportional.

3. Put this fourth number for a second number of a second stating, the remaining number of which the question is raised, the third, and its correspondent number of the same name, the first, then will the fourth number resulting be the answer.

If a principal of £.100 gain £.6 interest in a year; what will a principal of £.400 gain in 9 months?

Here of the five given numbers, £.100 principal, £.6 interest, and a year or 12 months, are conjoined in form of a supposition, and thereupon a question is raised concerning £.400 for 9 months; wherefore, let either of the two numbers, £.400 or 9 months, be put for the third number of the first stating, and its corresponding term, £.100 or 12 months, for the first.

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ \text{As } 100 : 6 :: 400 \\ \hline 6 \end{array}$$

$$1,00 \overline{) 24,00}$$

24 Ans.

$$\begin{array}{r} \text{Mo.} \quad \text{£.} \quad \text{Mo.} \\ \text{As } 12 : 24 :: 9 \\ \hline 9 \end{array}$$

$$12 \overline{) 216}$$

£.18 Ans.

Or thus,

$$\begin{array}{r} \text{Mo.} \quad \text{£.} \quad \text{Mo.} \quad \text{£.} \\ \text{As } 12 : 6 :: 9 : 4\frac{1}{2} \end{array}$$

$$\text{And, As } 100 : 4\frac{1}{2} :: 400 : 18.$$

Such questions as, when stated, are found to have both statings direct, may be solved more readily by one compound stating, thus: Place the two terms, of which the question is raised, under one another in the third place, their correspondent terms under each other in the first, and the remaining term in the middle: Then multiply both these first terms together, and the third terms together, and so the double stating is reduced to a simple one of the *Rule of Three Direct*; viz. the product of the two

* The reason of this rule may be shewn from the nature of direct and inverse proportion:—For in this rule, every row is a particular stating in one of those rules; and, therefore, if all the separate dividends be collected into one dividend, and all the divisors into one divisor, their quotients must be the answer sought: Thus, in example 1st.

$$\text{£.} \quad \text{£.} \quad \text{£.}$$

$$\text{Mo.} \quad \text{£.} \quad \text{£.} \quad \text{Mo.}$$

$$\text{As } 100 : 6 :: 400 : \frac{400 \times 6}{100}, \text{ and as } 12 : \frac{400 \times 6}{100} :: 9 : \frac{400 \times 6 \times 9}{100 \times 12} \text{ by the}$$

Rule of Three Direct.

150 COMPOUND PROPORTION.

two first terms is the first of a simple stating ; the second term is the second, and the product of the two third terms is the third, to find a fourth proportional—Thus,

$$\text{As } \left\{ \begin{smallmatrix} 100 \\ 12 \end{smallmatrix} \right\} : 6 :: \left\{ \begin{smallmatrix} 400 \\ 9 \end{smallmatrix} \right\}$$

So the first example will stand thus.

$$\left\{ \begin{smallmatrix} \text{£. } 100 \\ \text{Mo. } 12 \end{smallmatrix} \right\} : \text{£. } 6 :: \left\{ \begin{smallmatrix} 400 \\ 9 \end{smallmatrix} \right\} \text{£. } \left\{ \begin{smallmatrix} 216 \\ 9 \end{smallmatrix} \right\}$$

12|00

36|00

6

12)216

£.18 Ans.

SECOND METHOD.

Always place the three conditional terms in this order : That number which is the principal cause of gain, loss or action, possesses the first place ; that which denotes the space of time or distance of place, the second ; and that, which is the gain, loss or action, the third : This being done, place the other two terms, which move the question, under those of the same name, and if the blank place, or term sought, fall under the third place, then the question is in direct proportion ; therefore,

Rule 1.—Multiply the three last terms together, for a dividend, and the two first for a divisor :—But if the blank fall under the first or second place ; then, the proportion is inverse ; therefore,

Rule 2.—Multiply the first, second and last terms together for a dividend, and the other two for a divisor, and the quotient will be the answer.

1. If £.100 gain £.6 in a year ; what will £.400 gain in 9 months ?
£. P. Mo. £. Int.

100 : 12 :: 6 Terms in the supposition, or conditional terms.

400 : 9 Terms which move the question.

Here, the blank falling under the third place, the question is in direct proportion, and the answer must be found by the first Rule ; therefore,

$$400 \times 9 \times 6 = 21600 \text{ For the dividend, and } 100 \times 12 = 1200 \text{ For the divisor.}$$

See the work at large.

£. Pr.	Mo.	£. Int.
100	12	6
400	9	
9		
100	3600	
12	6	
12,00	216,00	

12,00)216,00(18£. Ans.

12

96

96

COMPOUND PROPORTION. 151

2. If £.100 will gain £.6 in a year ; in what time will £.400 gain £.18 ? £. Mo. £.

100 : 12 :: 6 *Terms in the supposition.*

400 : :: 18 *Terms which move the question.*

Here, the blank falling under the 2d place, the question is in reciprocal or inverse Proportion, and the answer must be sought by the second Rule ; therefore,

$$100 \times 12 \times 18 = 21600 \text{ For the dividend.}$$

$$400 \times 6 = 2400 \text{ For the divisor.}$$

£. Pr. Mo. £. Int.

100 : 12 :: 6

400 : :: 18

6 12

2400

216

100

24|00)216|00(9 months, Anf.

216

3. If £.400 gain £.18 in 9 months ; what is the rate per cent. per annum ?

Pr. Mo. Int.

400 : 9 :: 18

100 : 12

18

96

12

400 216

9 100

36|00)216|00(£.6 Anf.

216

4. What principal, at 6 per cent. per ann. will gain £.18 in 9 months ?

Pr. Mo. Int.

100 : 12 :: 6

9 :: 18

12

9 16

6 100

54|21600(£.400 Anf.

216

00

5. If 8 men spend £.32 in 13 weeks ; what will 24 men spend in 52 weeks ? M. W. £.

8 : 13 :: 32

24 : 52

£.384 Anf.

6. If the freight of 9hhds. of sugar, each weighing 12cwt. 20 leagues, cost £.16 ; what must be paid for the freight of 50 tierces ditto, each weighing 2½cwt. 100 leagues ?

hhds. leag. £.

9 : 20 :: 16

50 : 100

£. s. d.

92 11 10½ Anf.

7. There

7. There was a certain edifice completed in a year by 20 workmen; but the same being demolished, it is necessary that just such an one should be built in 5 months. I demand the number of men to be employed about it?

m. mo. ed.

20 : 12 :: 1

5 :: 1 -- 48 men, Answer.

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high and 6 feet thick?

m. da. ft.

6 : 16 :: $\frac{20 \times 6 \times 4}{24}$

24 : :: $200 \times 8 \times 6$ 80 days, Answer.

THIRD METHOD.

That number, which is of the same name as the number sought, must be the last term, on the right hand; then, take any two of the other numbers, which are of one kind, and if, when compared with the last number, *more* be required; set the greater in the second place, (next to the last term, with four dots between) and the less in the first, (at the left hand, with two dots between :) But, if *less* be required, let the less stand in the second place, and the greater in the first: When these three numbers are properly stated, take any two others, of one kind, which remain in the question, and compare them with the last number, as before, to find whether they require a greater or a less answer, and set them accordingly, immediately to the left hand of those, already stated, with dots, as before directed; thus proceed with every two remaining numbers, until all stand in one continued line.

Place A over the first, third, fifth, &c. numbers, omitting the last, and call them *antecedents*; and C over the second, fourth, sixth, &c. and call them *consequents*. This being done, multiply all the antecedents continually together, for your first term; and all the consequents continually together, for the second. Then will the proportion be, As the product of the antecedents is to the product of the consequents; so will the last number be to the answer.

Euclid's Elements, V. 18.

Take the 8th question in the second method.

If 6 men build a wall 20 feet long, 6 high and 4 thick, in 16 days, in what time will 24 men build one 200 feet long, 8 high, and 6 thick?

COMPARISON OF WEIGHTS, &c. 153

A	C	A	C	A	C	A	C
ft. th.	ft. th.	ft. hi.	ft. hi.	ft. lon.	ft. lon.	men.	men.
4	: 6	:: 6	: 8	:: 20	: 200	:: 24	: 6
							:: 16
				24		200	
				80		1200	
				40		8	
				480		9600	
				6		6	
				2880		57600	
				4			
				11520			

As 11520 : 57600 :: 16
16

345600
57600

1152|0)92160|0(80 days, Answer.
9216

0

COMPARISON OF WEIGHTS AND MEASURES.

EXAMPLES.

1. If 78 pence Massachusetts be worth 1 French crown, How many Massachusetts pence are worth 320 French crowns?

Fr. cr. d. Fr. cr.
As 1 : 78 :: 320
78

2560
2240

24960 Answer.

2. If 24 yards at Boston make 16 ells at Paris, How many ells at Paris will make 128 yards at Boston?

Bost. Par. Bost. Par.
As 24 yds. : 16 ells :: 128 yds. : 85 $\frac{1}{3}$ ells, Ans.

3. If 60 lb at Boston make 56 lb at Amsterdam, How many lb at Boston will be equal to 350 at Amsterdam?

Ans. Bost. Ans. Bost.
As 56 lb : 60 lb :: 350 lb : 375 lb Ans.

4. If

U

154 CONJOINED PROPORTION.

4. If 95^{lb} Flemish make 100^{lb} American, how many American^{lbs} are equal to 550^{lb} Flemish?

Flem. Amer. Flem. Amer.
As 95^{lb} : 100^{lb} :: 550^{lb} : 57⁸⁹⁰₉₅ *Ans.*

CONJOINED PROPORTION

Is when the coins, weights or measures of several countries are compared in the same question; or, in other words, it is joining many proportions together, and by the relation, which several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble, and it may be proved by as many statings in the Single Rule of Three as the nature of the question may require.

C A S E I.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

R U L E.

Place the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right, and let the last number stand on the left hand; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

E X A M P L E S.

1. Suppose 100 yards of America = 100 yards of England, and 100 yards of England = 50 canes of Thoulouse, and 100 canes of Thoulouse = 160 ells of Geneva, and 100 ells of Geneva = 200 ells of Hamburgh: How many yards of America are equal to 379 ells of Hamburgh?

<i>Antecedents.</i>	<i>Consequents.</i>	<i>Abridged.</i>
100 of America =	100 of England.	<i>Ant.</i> <i>Con.</i>
100 of England =	50 of Thoulouse.	5 8
100 of Thoulouse =	160 of Geneva.	379
100 of Geneva =	200 of Hamburgh.	
379 of Hamburgh ?		

Therefore, $\frac{379 \times 5}{8} = 236\frac{7}{8}$ yds. of America = 379 ells of Hamburgh.

I L L U S T R A T I O N.

The two 100's on both sides cancel each other. Let the last cyphers of the three next antecedents and consequents be cancelled, which is dividing by 10. Then divide the second antecedent and consequent

consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents ; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8. 10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2. Now, there being 2 left on each side, they cancel each other, and as there is no farther room for abridging by reason of the odd number 379, the operation is finished, and the answer found, as before.

2. If 20lb at Boston make 23lb at Antwerp, and 155 at Antwerp make 180 at Leghorn; How many at Boston are equal to 144 at Leghorn?

Antecedents. Consequents.

15 15

20 of Boston = 23 of Antwerp. $20 \times 155 \times 144 = 446400$ divid.

155 of Antwerp = 180 of Leghorn. $23 \times 180 = 4140$ divisor.

144 of Leghorn. $414|0)44640|0(107\frac{19}{23}$ Answer.

Or, abridged $\frac{155 \times 144}{23 \times 9} = 107\frac{2}{3}$

3. If 12 fl at Boston make 10 at Amsterdam, 100 fl at Amsterdam 120 fl at Paris, How many fl at Boston are equal to 80 fl at Paris? *Ans.* 80 fl .

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American yards, How many Venetian braces are equal to 32 American yards? *Ans.* $52\frac{4}{11}$.

5. If 40^l at Newburyport make 36 at Amsterdam, and 90^l at Amsterdam make 116 at Dantzick, How many ^l at Newburyport are equal to 260^l at Dantzick? *Ans.* 224 $\frac{4}{28}$ ^l.

C A S E II.

When it is required to find how many of the last sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the first.

R U L E.

Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand ; then multiply the first row for a divisor, and the second for a dividend.

E X A M P L E S.

1. If 12^{lb} at Boston make 10^{lb} at Amsterdam, and 100^{lb} at Amsterdam 120^{lb} at Paris, How many at Paris are equal to 80^{lb} at Boston? *Left. Right.*

Boston	12	10	$10 \times 120 \times 80 = 96000$	
Amsterdam	100	120		$\frac{\quad}{\quad} = 80 \text{ Ans.}$
		80	$12 \times 100 = 1200$	

2. If 40*lb* at Newburyport make 36 at Amsterdam, and 90*lb* at Amsterdam make 116 at Dantzick, How many *lb* at Dantzick are equal to 244 at Newburyport? *Ans.* 283 $\frac{2}{3}$ *lb*.

ARBITRATION

ARBITRATION OF EXCHANGES.

By this term is understood how to choose, or determine the best way of remitting money from abroad with advantage ; which is performed by conjoined proportion : Thus,

Suppose a merchant has effects at Amsterdam to the amount of 3530 dollars, which he can remit by way of Lisbon at 840 rees per dollar, and thence to Boston, at 8s. 1d. per milree (or 1000 rees :) Or, by way of Nantz, at 5 $\frac{2}{5}$ livres per dollar, and thence to Boston at 6s. 8d. per crown : It is required to arbitrate these exchanges, that is, to choose that which is most advantageous ?

1 dollar at Amsterdam = 840 rees at Lisbon.

1000 rees at Lisbon = 97d. at Boston.

3530 dollars at Amsterdam.

$$\frac{840 \times 97 \times 3530}{1000 \times 1} = \text{£.}1198 \text{ 8s. } 8\frac{4}{10}\text{d. by way of Lisbon.}$$

1 dollar at Amsterdam = 5 $\frac{2}{5}$ livres at Nantz.

6 livres at Nantz = 80 pence at Boston.

3530 dollars at Amsterdam.

$$\frac{5\frac{2}{5} \times 80 \times 3530}{1 \times 6} = \text{£.}1059 \text{ by way of Nantz.}$$

Here it may be observed that the difference is £.139 8s. 8 $\frac{4}{10}$ d. in favour of remitting by way of Lisbon rather than by Nantz, which depends on the *course* of exchange, at that time ; but the *course* may vary so, that, in a short time, by way of Nantz may be better ; hence appears the necessity and advantage of an extensive correspondence, to acquire a thorough knowledge in the *courses* of exchange, to make this kind of remittance.

F E L L O W S H I P.

The Rules of Fellowship are those by which the accompts of several merchants, or other persons, trading in partnership, are so adjusted, that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the joint stock, together with the time of its continuance in trade.

S I N G L E F E L L O W S H I P

Is, when the stocks are employed for any certain equal time.

R U L E.*

As the whole stock is to the whole gain or loss, so is each man's particular stock, to his particular share of the gain, or loss,

PROOF.

* That their gain or loss, in this rule, is in proportion to their stocks, is evident : For, as the times, in which the stocks are in trade, are equal, if I put in $\frac{1}{2}$ of the whole stock, I ought to have $\frac{1}{2}$ of the gain ; if my part of the stock be $\frac{1}{3}$, my share of the gain or loss ought to be $\frac{1}{3}$ also. And generally the same ratio that the whole stock has to the whole gain or loss, must each person's particular stock have to his respective gain or loss.

PROOF. Add all the particular shares of the gain or loss together, and, if it be right, the sum will be equal to the whole gain or loss.

E X A M P L E S.

1. Divide the number 360 into 4 such parts, which shall be to each other as 3, 4, 5 and 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{ \begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array} \right\} \text{ Answer.}$$

360 *Proof.*

2. A, B, C and D companied ;—A put in £.145 ; B, £.219 ; C, £.378, and D, £.417, with which they gained £.569 : What was the share of each ?

<i>Whole stock.</i>	<i>Gain.</i>	<i>£. s. d.</i>	
As 145+219+378+417 : 569 ::	$\left\{ \begin{array}{l} 145 : 71 \text{ } 3 \text{ } 8\frac{1}{2} \\ 219 : 107 \text{ } 10 \text{ } 3\frac{3}{4} \\ 378 : 185 \text{ } 11 \text{ } 6 \\ 417 : 204 \text{ } 14 \text{ } 5\frac{1}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 105\frac{3}{4} \\ 37\frac{5}{8} \\ 55\frac{2}{3} \\ 33\frac{3}{8} \end{array} \right.$	<i>A's sha.</i> <i>B's dit.</i> <i>C's dit.</i> <i>D's dit.</i>

£.569 — — — *Proof.*

3. A, B, C and D are concerned in a joint stock of £.168 2s. 6d. of which A's part is £.25 10s. ; B's £.37 15s. ; C's £.49, and D's £.55 17 6d.—Upon the adjustment of their accompts, they have lost £.73 13s. 4d. What is the loss of each ?

Ans. A's loss £.11 3s. 5½d. B's £.16 10s. 9¾d. C's £.21 9s. 4¾d. & D's £.24 9s. 7¾d.

4. A and B companied ; A put in £.45, and took $\frac{3}{5}$ of the gain ; What did B put in ? $5-3=2$. Then, As 3 : 45 :: 2 : 30 *Ans.*

5. A, B and C freighted a ship with 68900 feet of boards : A put in 16520 feet ; B 28750 ; and C the rest ; but, in a storm, the Captain threw overboard 26450 feet ; How much must each sustain of the loss ? *Ans.* A, 6341½ feet. B, 11036¾ & C, 9071½ do.

6. A gentleman died, leaving three sons and a daughter, to whom he bequeathed his estate in the following manner : To the eldest son he gave 312 moidores, to the second 312 guineas, to the third 312 pistoles, and to the daughter 312 dollars ; but when his debts were paid, there were but 312 half joes left ; What must each have in proportion to the legacies which had been bequeathed them ?

Ans. 1st son £.293 0s. 2d.—2d son £.227 17s. 10¾d.—3d son £.179 1s. 2½d. and the daughter £.48 16s. 8¾d.

7. A ship, worth £.780, being lost at sea, of which $\frac{1}{6}$ belonged to A, $\frac{1}{2}$ to B, and the rest to C ; What loss will each sustain, supposing £.450 to have been insured upon her ?

$$780-450=330, \text{ Then } \frac{1}{6}330 \quad \frac{1}{2}330 \quad \frac{1}{3}330$$

£.55=A's £.165=B's £.110=C's share.

8. A

8. A and B venturing equal sums of money, cleared by joint trade £.140 :—By agreement, as A executed the business, he was to have 8 per cent. and B was to have 5 per cent. What was A allowed for his trouble ?

$$\begin{array}{c} \text{£. £. £.} \quad \text{£. £.} \quad \text{£. £. £.} \quad \text{£. £.} \\ \text{As } 8+5 : 140 :: 8 : 86\frac{2}{3} \quad \text{And, as } 8+5 : 140 :: 5 : 53\frac{1}{3}. \\ \text{Ans. £. } 32 \text{ 6s. } 1\frac{2}{3}d. \end{array}$$

9. A bankrupt is indebted to A £.120, to B £.230, to C £.340, and to D £.450, and his whole estate amounts only to £.560 : How must it be divided among the creditors ?

$$\text{Ans. A, £. } 58 \text{ 18s. } 11\frac{1}{4}d. \quad \text{B, £. } 112 \text{ 19s. } 7\frac{1}{4}d. \quad \text{C, £. } 167 \text{ 0s. } 4d. \\ \text{and D, £. } 221 \text{ 1s. } 0\frac{1}{2}d.$$

10. A, B and C put their money into a joint stock ; A put in £.40 : B and C together, £.170 : They gained £.126, of which B took £.42 ; What did A and C gain, and B and C put in respectively ?

As £.210 the whole stock : £.126 the whole gain :: £.40 A's stock : £.24 A's gain.

As £.24 A's gain : £.40 A's stock :: £.42 B's gain : £.70 B's stock. Then £.170—£.70=£.100 C's stock ; and the whole gain £.126—£.66 A's and B's gain=£.60 C's gain.

11. A, B and C companied ;—A put in £.40 ; B £.60, and C, a sum unknown : They gained £.72 ; of which C took £.32 for his share : What did A and B gain, and C put in ?

The whole gain £.72—C's gain £.32=£.40, A's and B's gain : Then, As £.100, A's and B's stock : £.40 their gain :: £.40 A's stock : £.16, his gain. Again, As £.10 A's gain : £.40, his stock :: £.32, C's gain : £.80, his stock.

12. A, B and C put in £.720, and gained £.540, of which, so often as A took up £.3, B took 5, and C 7 ; What did each put in and gain ?

$$\begin{array}{c} \text{£. £. £.} \quad \text{£.} \quad \text{£.} \\ \text{As } 3+5+7 : 540 :: \left\{ \begin{array}{l} 3 : 108 \text{ A's gain.} \\ 5 : 180 \text{ B's ditto.} \\ 7 : 252 \text{ C's ditto.} \end{array} \right. \end{array}$$

$$\text{And, as } 3+5+7 : 720 :: \left\{ \begin{array}{l} 3 : 144 \text{ A's Stock.} \\ 5 : 240 \text{ B's ditto.} \\ 7 : 336 \text{ C's ditto.} \end{array} \right.$$

Or, you may find a common multiplier to multiply the proportions by, or multiplicand to be multiplied by the given proportions, thus, 15)720(48 multiplicand to find the stocks.—And 15)540(36 multiplicand to find the gains.

$$\begin{array}{c} \text{£.} \\ 48 \times 3 = 144 \text{ A's stock} \\ 48 \times 5 = 240 \text{ B's ditto.} \\ 48 \times 7 = 336 \text{ C's ditto.} \end{array} \quad \text{And } \left\{ \begin{array}{l} 36 \times 3 = 108 \text{ A's gain,} \\ 36 \times 5 = 180 \text{ B's ditto.} \\ 36 \times 7 = 252 \text{ C's ditto, as before.} \end{array} \right.$$

13. A, B, C and D companied ; and gained a sum of money, of which A, B and C took £.120, B, C and D, £.180, C, D, and A, £.160, and D, A and B, £.140 ; What distinct gain had each ?

The sum of these 4 numbers is £.600, and a seach man's money is named 3 times, therefore $\frac{1}{3}$, viz. £.200 is the whole gain—Therefore £.200—£.120 A's, B's and C's gain=£.80 D's gain ;—And £.200—£.180 B's, C's and D's gain=£.20 A's gain.—£.200—£.160 C's, D's and A's gain=£.40 B's gain.—And £.200—£.140 D's, A's and B's gain=£.60 C's gain.

14. Two merchants companied ; A put in £.40, and B 288 ducats. They gained £.135, of which A took £.60. What was the value of a ducat ?

As £.60, A's gain : £.40, his stock :: £.135, the whole gain—£.60, A's gain : £.50, B's gain.

Duc. £. Duc. s. d.

And, as 288 : 50 :: 1 : 3 $5\frac{2}{3}$ Ans.

15. Four men spent, at a reckoning, 20 shillings, of which they agreed that A should pay $\frac{1}{4}$, B, $\frac{1}{2}$, C, $\frac{1}{4}$, and D, $\frac{1}{8}$. What must each pay in that proportion ?

$$\begin{array}{c} \text{£.} \quad \text{£.} \quad \text{£.} \quad \text{£.} \quad \text{s.} \\ \text{As } \frac{1}{4} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} : 20 :: \left\{ \begin{array}{l} 15 \text{ } 0 : 9 \text{ } 2\frac{10}{13} \\ 10 \text{ } 0 : 6 \text{ } 1\frac{11}{13} \\ 5 \text{ } 0 : 3 \text{ } 0\frac{12}{13} \\ 2 \text{ } 6 : 1 \text{ } 6\frac{6}{13} \end{array} \right\} \text{Answer.} \end{array}$$

DOUBLE FELLOWSHIP,*

Or *Fellowship with Time*, is occasioned by the shares of partners being continued unequal times.

R U L E.

Multiply each man's stock, or share, by the time it was continued in trade. Then,

As the whole sum of the products, is to the whole gain or loss, so is each man's particular product, to his particular share of the gain or loss.

E X A M P L E S.

1. A, B and C hold a pasture in common, for which they pay £.40 per annum. A put in 9 oxen for 5 weeks ; B, 12 oxen for 7 weeks, and C 8 oxen for 16 weeks. What must each pay of the rent ?

$$9 \times 5 = 45. \quad 12 \times 7 = 84, \text{ and } 8 \times 16 = 128, \text{ then } 128 + 84 + 45 = 257 \text{ As}$$

* When the times are equal, the shares of the gain or loss are evidently as the stocks, as in Single Fellowship ; and when the stocks are equal, the shares are as the times ; wherefore, when neither are equal, the shares must be as their products.

As 257 : 40 :: 45

$$\begin{array}{r}
 45 \\
 \hline
 200 \\
 160 \\
 \hline
 257)1800(7 \\
 \underline{1799} \\
 1 \\
 20 \\
 \hline
 257)200(0 \\
 \underline{12} \\
 257)240(0 \\
 \underline{4} \\
 257)260(3 \\
 \underline{771} \\
 189
 \end{array}$$

As 257 : 40 :: 84

$$\begin{array}{r}
 84 \\
 \hline
 160 \\
 320 \\
 \hline
 257)3360(13 \\
 \underline{257} \\
 790 \\
 771 \\
 \hline
 19 \\
 20 \\
 \hline
 257)380(1 \\
 \underline{257} \\
 123 \\
 12 \\
 \hline
 257)476(5 \\
 \underline{1285} \\
 191 \\
 4 \\
 \hline
 257)764(2 \\
 \underline{514} \\
 250
 \end{array}$$

As 257 : 40 :: 128

$$\begin{array}{r}
 128 \\
 40 \\
 \hline
 257)5120(19 \\
 \underline{257} \\
 2550 \\
 2313 \\
 \hline
 237 \\
 20 \\
 \hline
 257)4740(18 \\
 \underline{257} \\
 2170 \\
 2056 \\
 \hline
 114 \\
 12 \\
 \hline
 257)1368(5 \\
 \underline{1285} \\
 83 \\
 4 \\
 \hline
 257)332(1 \\
 \underline{257} \\
 75
 \end{array}$$

2. Four merchants traded in company, A put in £.100 for five months, B, £.150 for 7 months, C, 220 for 8 months, and D, £.310 for 9 months ; but by misfortunes at sea, they lost £.145. What must each man sustain of the loss ?

Answer, { A, £.11 17s. 8½ 54. C, £.41 16s. 8½ 38. }
 { B, £.24 19s. 2 40. D, £.66 6s. 4½ 54. }

3. A, with a capital of £.100 began trade January 1st 1787, and meeting with success in his business, he took in B as a partner, on the 1st day of March following, with a capital of £.150. Three months after that, they admit C as a third partner, who brought into stock £.180, and after trading together until the 1st of January 1788, they found there had been gained since A's commencing business, £.177 13s. How must this be divided among the partners ?

Ans. A, £.53 16s. 8d. B, £.67 5s. 10d. C, £.56 10s. 6d.

4. Two merchants entered into partnership for 18 months ; A, at first, put into stock £.100, and at the end of 8 months he put in £.50 more ; B, at first, put in £.275, and at 4 months' end took out £.70. Now, at the expiration of the time, they found they had gained £.263. What is each man's just share ?

Ans. A, £.96 9s. 6d. B, £.166 10s. 6d.

5. A and B companied ; A put in, the 1st of January, £.150 ; but B could not put in any until the first of May ; What did he then put in, to have an equal share with A at the year's end ?

As

$$\begin{array}{c} M. \quad \text{£.} \quad M. \\ \text{As } 12 : 150 :: 8 : \frac{150 \times 12}{8} = \text{£ } 225 \text{ Ans.} \end{array}$$

6. A, B and C companied; A put in the first of March, 30*l*. B, the 1st of May, put in 80 yards of broadcloth; and on the 1st of June C put in 120 dollars. On the first of January following, they reckoned their gains, of which A and B took 228*l*. B and C 215*l*. 10*s*. and C and A 187*l*. 10*s*. What was the whole gain, and the gain of each? What did they value a yard of cloth at? and, What was C's dollar worth?

228*l*. + 215*l*. 10*s*. + 187*l*. 10*s*. = 631*l*. and $631 \div 2 = 315*l*. 10*s*.$ the whole gain; then, 315*l*. 10*s*. - 228 = 87*l*. 10*s*. C's gain. 315*l*. 10*s*. - 215*l*. 10*s*. = 100*l*. A's gain, and 315*l*. 10*s*. - 187*l*. 10*s*. = 128*l*. B's gain. To find the value of one yard of cloth, say, As 100*l*. A's gain : 30*l*. his stock :: 128*l*. B's gain : 38*l*. 8*s*. then, inversely, As 10 months : 38*l*. 8*s*. :: 8 months : 48*l*. the value of the whole cloth.

As 80 yds. : 48*l*. :: 1 yd. : 12*s*. answer. Now, to find the value of a dollar. As 100*l*. A's gain : 30*l*. his stock :: 87*l*. 10*s*. C's gain : 26*l*. 5*s*. then, inversely, As 10 months : 26*l*. 5*s*. :: 7 months : 37*l*. 10*s*. = 120 dollars. Lastly: As 120 dollars : 37*l*. 10*s*. :: 1 dollar : 6*s*. 3*d*. answer.

7. A, B and C companied and put in together 191*l*. A's money was in 3 months, B's 5 months, and C's 7 months; they gained 117*l*. which was so divided, as that the $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's, and $\frac{1}{4}$ of C's gain; What did each gain and put in?

Suppose A's gain was 2*l*. then must B have 3*l*. and C 4*l*. by the question :

$$\text{Then, as } 2+3+4 : 117 \text{ the whole gain} :: \begin{cases} 2 : 26 \text{ A's gain.} \\ 3 : 39 \text{ B's ditto.} \\ 4 : 52 \text{ C's ditto.} \end{cases} \text{£.}$$

Then, divide each man's gain by his time; and as the sum of the quotients is to each particular quotient, so is the whole stock, to each man's particular stock.

A's stock, 693*l*. 2*s*. 2*d*. B's stock 623*l*. 15*s*. - 11½*d*. C's stock 594*l*. 1*s*. 10¼*d*. Ansf.

FELLOWSHIP BY DECIMALS.

R U L E.*

Divide the whole gain, or loss, by the whole stock, and the quotient multiplied severally by each man's stock, will give the gain, or loss, of each.

E X A M P L E S.

1. A, B and C companied, A put in 40*l*. 5*s*.; B, 80*l*. 10*s*. and C, 161*l*. with which they gained 120*l*.; What is each man's share of the gain?

* This is no more than Division of Decimals.

A's

A's Stock =	40,25	
B's ditto =	80,5	
C's ditto =	161	
Sum total =	281,75	120,000000(,4259 +
4259	4259	4259
40,25	80,5	161
<hr/>	<hr/>	<hr/>
21295	21295	4259
8518	340720	25554
170366	<hr/>	4259
<hr/>	£ 34,28495	<hr/>
£ 17,142475	20	£ 68,5699
20	<hr/>	20
<hr/>	5,69900	<hr/>
2,849,00	12	11,3980
12	<hr/>	12
<hr/>	8,388	<hr/>
10,194000	4	4,776
4	<hr/>	4
<hr/>	1,552	<hr/>
0,776000		3,104

Proof. A's gain 17*l.* 2*s.* 10*d.* + B's gain 34*l.* 5*s.* 8½*d.* + C's gain 68*l.* 11*s.* 4¼*d.* = 119*l.* 19*s.* 11*d.*

2. A, B and C companied ; A put in 200*l.* B, 150*l.* and C 50*l.* with which they gained 800*l.* ; What is the share of each ?

$800 \div 400 = 2$; then $200 \times 2 = 400$ *l.* = A's gain, $150 \times 2 = 300$ *l.* = B's gain, and $50 \times 2 = 100$ *l.* = C's gain.

3. A, B, C and D trade, and gain 200*l.* which is to be divided in the following manner, viz. so often as A has 6*l.* B must have 10*l.* C, 14*l.* and D, 20*l.* What is the share of each ?

$6 + 10 + 14 + 20 = 50$, and $\frac{200}{50} = 4$, quotient ; then $6 \times 4 = 24$ *l.* A's gain ; $10 \times 4 = 40$ *l.* B's gain ; $14 \times 4 = 56$ *l.* C's ; and $20 \times 4 = 80$ *l.* D's gain.

P R A C T I C E,*

Is a contraction of the Rule of Three dire&, when the first term happens to be an unit, or one ; and has its name from its daily use among merchants and tradesmen, being an easy and concise method of working most questions which occur in trade and business.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

Before

*GENERAL RULE.

1. Suppose the price of the given quantity to be 1*l.* or 1*s.* &c. then will the quantity itself be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each will be the true answer required.

EXAMPLES.

[For the remainder of this note, see page 164.]

Before the questions, hereafter given, can be wrought, the following Tables must be perfectly gotten by heart.

T A B L E S.

Aliquot, or even Parts of Money.

Pts. of a Shil. of a £.			Parts of a Pound.		
d.	s.	£.	s.	d.	£.
6	= $\frac{1}{2}$	= $\frac{1}{40}$	10	0	is $\frac{1}{2}$
4	= $\frac{1}{3}$	= $\frac{1}{60}$	6	8	= $\frac{1}{3}$
3	= $\frac{1}{4}$	= $\frac{1}{80}$	5	0	= $\frac{1}{4}$
2	= $\frac{1}{6}$	= $\frac{1}{120}$	4	0	= $\frac{1}{5}$
$1\frac{1}{2}$	= $\frac{1}{8}$	= $\frac{1}{160}$	3	4	= $\frac{1}{6}$
1	= $\frac{1}{12}$	= $\frac{1}{240}$	2	6	= $\frac{1}{8}$
$\frac{3}{4}$	= $\frac{1}{16}$	= $\frac{1}{320}$	1	8	= $\frac{1}{12}$
$\frac{1}{2}$	= $\frac{1}{24}$	= $\frac{1}{480}$	1	4	= $\frac{1}{15}$
$\frac{1}{4}$	= $\frac{1}{48}$	= $\frac{1}{960}$	1	3	= $\frac{1}{16}$

Parts of 2 Shill.

d.	2s.
1	is $\frac{1}{24}$
$1\frac{1}{2}$	= $\frac{1}{16}$
2	= $\frac{1}{12}$
3	= $\frac{1}{8}$
4	= $\frac{1}{6}$
6	= $\frac{1}{4}$
8	= $\frac{1}{3}$

Aliquot, or even Parts of Weight.

Parts of a Cwt.			Parts of a Ton.		
Qrs.	lb.	Cwt.	Cwt.	qr.	T.
2	0	is $\frac{1}{2}$	10	0	is $\frac{1}{2}$
1	0	= $\frac{1}{4}$	5	0	= $\frac{1}{4}$
0	16	= $\frac{1}{7}$	4	0	= $\frac{1}{5}$
0	14	= $\frac{1}{8}$	2	2	= $\frac{1}{8}$
0	8	= $\frac{1}{14}$	2	0	= $\frac{1}{10}$
0	7	= $\frac{1}{16}$	1	1	= $\frac{1}{16}$
0	4	= $\frac{1}{28}$	1	0	= $\frac{1}{20}$

Pts. of $\frac{1}{2}$ Cwt.

lb.	$\frac{1}{2}$ Cwt.
28	is $\frac{1}{2}$
14	= $\frac{1}{4}$
8	= $\frac{1}{7}$
7	= $\frac{1}{8}$
4	= $\frac{1}{14}$

Parts of 240.

180	is	$\frac{3}{4}$
120	=	$\frac{1}{2}$
80	=	$\frac{1}{3}$
60	=	$\frac{1}{4}$
40	=	$\frac{1}{6}$

Pts. of $\frac{1}{4}$ Cwt.

lb.	$\frac{1}{4}$ Cwt.
14	is $\frac{1}{2}$
7	= $\frac{1}{4}$
4	= $\frac{1}{7}$
2	= $\frac{1}{14}$

Parts of 480.

360	is	$\frac{3}{4}$
160	=	$\frac{1}{3}$

Another T A B L E of Aliquot parts of Money.

Parts of a Shill.			Parts of a Pound.			Parts of a Pound.		
d.		s.	s.	d.	£.	s.	d.	£.
10	is	$\frac{5}{6}$	18	0	is $\frac{9}{10}$	13	4	is $\frac{2}{3}$
9	=	$\frac{3}{4}$	17	6	= $\frac{7}{8}$	12	6	= $\frac{5}{8}$
8	=	$\frac{2}{3}$	16	8	= $\frac{5}{6}$	12	0	= $\frac{6}{10}$
$7\frac{1}{2}$	=	$\frac{5}{8}$	16	0	= $\frac{4}{5}$	8	0	= $\frac{4}{10}$
$4\frac{1}{2}$	=	$\frac{2}{3}$	15	0	= $\frac{3}{4}$	7	6	= $\frac{3}{8}$
			14	0	= $\frac{7}{10}$	6	0	= $\frac{3}{10}$

A T A B L E of DISCOUNT per cent.

£.			£.			£.		
s.	d.		s.	d.		s.	d.	
$1\frac{1}{4}$	per cent.	is 0 3	$8\frac{1}{4}$	per cent.	is 1 9	$22\frac{1}{2}$	per cent.	is 4 6
$2\frac{1}{2}$	per cent.	= 0 6	10	per cent.	= 2 0	25	per cent.	= 5 0
$3\frac{1}{4}$	per cent.	= 0 9	$12\frac{1}{2}$	per cent.	= 2 6	30	per cent.	= 6 0
5	per cent.	= 1 0	15	per cent.	= 3 0	35	per cent.	= 7 0
$6\frac{1}{4}$	per cent.	= 1 3	$17\frac{1}{2}$	per cent.	= 3 6	40	per cent.	= 8 0
$7\frac{1}{2}$	per cent.	= 1 6	20	per cent.	= 4 0	45	per cent.	= 9 0
						50	per cent.	= 10 0

C A S E I.

When the price of 1 yd. lb , &c. is an even part of one shilling :— Find the value of the given quantity at 1s. per yard, lb , &c. then draw a line underneath, and divide it by that even part, and the quotient will be the answer in shillings, which must always be brought into pounds.

E X A M P L E S.

1st. What will $354\frac{1}{2}$ yards cost, at $\frac{1}{4}d$. per yard ?

$\begin{array}{r} s. \quad d. \\ | \frac{1}{4}d. | \frac{1}{4} | 354 \quad 6 \end{array}$ value of $354\frac{1}{2}$ yards, at 1s. per yard.

Ans. £0 7. $4\frac{1}{2}$ value of $354\frac{1}{2}$ yards, at $\frac{1}{4}d$. per yard.

Or thus. Or divide by 8 and 6, thus, 8)354 6

$\begin{array}{r} \text{£. } s. \quad d. \quad s. \quad d. \\ 8)17 \quad 14 \quad 6 = 354 \quad 6 \end{array}$

6)2 4 $3\frac{3}{4}$

6)44 $3\frac{3}{4}$

7 $4\frac{1}{2}$ Ans. as before.

7 $4\frac{1}{2}$ Ans. as before.

2d. What will $759\frac{1}{4}$ yards come to, at $3d$. per yard ?

$\begin{array}{r} s. \quad d. \\ | 3d. | \frac{1}{4} | 759 \quad 9 \end{array}$ value at 1s. per yard.

$\begin{array}{r} \text{£. } s. \quad d. \\ 2|0|18|9 \quad 11\frac{1}{4} \end{array}$ Or thus, $| 3d. | \frac{1}{4} | 37 \quad 19 \quad 9$ value at 1s. per yard.

Ans. £9 9 $11\frac{1}{4}$ value at $3d$. Ans. £9 9 $11\frac{1}{4}$ value of $759\frac{1}{4}$ yards at $3d$. per yard.

Questions.

Answers.

Questions.

Answers.

Yds.	£. s. d.	Yds.	£. s. d.
3d. 642 at $\frac{1}{4}d$. per yd.	0 13 $4\frac{1}{2}$	7th. 685 $\frac{1}{4}$ at 2d.	5 14 $3\frac{1}{2}$
4th. 918 $\frac{1}{4}$ — $\frac{1}{2}d$.	1 18 $3\frac{1}{8}$	8th. 475 $\frac{1}{4}$ — 4d.	7 18 5
5th. 739 $\frac{1}{2}$ — 1d.	3 1 $7\frac{1}{2}$	9th. 913 $\frac{1}{2}$ — 6d.	22 16 9
6th. 567 $\frac{1}{2}$ — $1\frac{1}{2}d$.	3 10 $11\frac{1}{4}$		

CASE II.

[Conclusion of the note begun page 152.]

EXAMPLE. What is the value of 468 yards, at 2s. 9 $\frac{1}{2}d$. per yard ?

£468 s. d. Answer at £1 s. d.

2s. 6d. is $\frac{1}{8}$ = 58 10 0 ditto at 0 2 6

3d. is $\frac{1}{10}$ = 5 17 0 ditto at 0 0 3

$\frac{1}{4}d$. is $\frac{1}{12}$ = 0 9 9 ditto at 0 0 0 $\frac{1}{2}$

The full price = £64 16 9 0 2 9 $\frac{1}{2}$

In this example it is plain that the quantity 468 is the answer at 1l.; consequently as 2s. 6d. is $\frac{1}{8}$ of a pound, $\frac{1}{8}$ part of that quantity, or 58l. 10s. is the price at 2s. 6d.; in like manner, as 3d. is the $\frac{1}{10}$ part of 2s. 6d. so $\frac{1}{10}$ part of 58l. 10s. or 5l. 17s. is the answer at 3d. and as $\frac{1}{4}d$. is $\frac{1}{12}$ of 3d. so $\frac{1}{12}$ of 5l. 17s. or 9s. 9d. is the answer as $\frac{1}{4}d$.—Now, the sum of all these parts is equal to the whole price (2s. 9 $\frac{1}{2}d$.) so the sum of the answers belonging to each price will be the answer at the full price required, and the same will be true in any example whatever.

C A S E II.

When the price is pence, and no even part of a shilling ;—Find the value of the given quantity at 1s. per yard ; divide the pence into aliquot parts, for divisors, and the sum of the quotients arising from them, will be the answer.

E X A M P L E S.

1st. What will $48\frac{1}{2}$ yards come to at 5d. per yard ?

	£.	s.	d.	
3d. $\left \frac{1}{4} \right $	24	7	6	value of $48\frac{1}{2}$ yards, at 1s. per yard.
2d. $\left \frac{1}{6} \right $	6	1	$10\frac{1}{2}$	value of ditto, at 3d. per yard.
	4	1	3	value of ditto, at 2d. per yard.

Ans. £. 10 3 $1\frac{1}{2}$ value of ditto, at 5d. per yard.

Questions.	Answers.	Questions.	Answers.
Yds.	£. s. d.	Yds.	£. s. d.
2d. $568\frac{1}{4}$ at 7d. —	16 11 $5\frac{3}{4}$	5th. $649\frac{1}{4}$ at 10d. —	27 1 $0\frac{5}{8}$
3d. $683\frac{3}{4}$ — 8d. —	22 15 10	6th. $745\frac{1}{4}$ — 11d. —	34 3 $7\frac{1}{4}$
4th. $912\frac{1}{2}$ — 9d. —	34 4 $4\frac{1}{2}$		

C A S E III.

When the price is pence or farthings, and an even part of a pound, Cut off the right hand figure of the given quantity, and the cypher, in the aliquot part, (if it has one) and divide by the remaining figure or figures. When you come to the remainder, double it, and divide as before. The answer will be pounds, shillings, &c. If there be no cypher in the divisor, then none should be cut off from the dividend.

$$\begin{array}{l} d. \\ \left| \frac{1}{4} \right| \frac{1}{96.0} \left| 3795^{\text{lb}} \text{ at } \frac{1}{4}d. \right. \\ 96 = 8 \times 12) 379 | 5 \end{array}$$

$$8) 31 \ 12 \ 6$$

$$\text{Ans. } £ 3 \ 19 \ 0\frac{3}{4}$$

$$\begin{array}{l} d. \\ \left| \frac{1}{2} \right| \frac{1}{48.0} \left| 3795 \text{ yds. at } \frac{1}{2}d. \right. \\ 48 = 6 \times 8) 379 | 5 \end{array}$$

$$6) 47 \ 8 \ 9$$

$$\text{Ans. } £ 7 \ 18 \ 1\frac{1}{2}$$

$$\begin{array}{l} d. \\ \left| \frac{1}{4} \right| \frac{1}{32.0} \left| 3795 \text{ yds. at } \frac{1}{4}d. \right. \\ 32 = 4 \times 8) 379 | 5 \end{array}$$

$$4) 47 \ 8 \ 9$$

$$\text{Ans. } £ 11 \ 17 \ 2\frac{3}{4}$$

$$\begin{array}{l} d. \\ \left| 1 \right| \frac{1}{24.0} \left| 3795 \text{ yds. at } 1d. \right. \\ 24 = 4 \times 6) 379 | 5 \end{array}$$

$$4) 63 \ 5$$

$$\text{Ans. } £ 15 \ 16 \ 3$$

$$d. \\ \left| 1\frac{1}{2} \right|$$

d.

$$| 1\frac{1}{2} | 16\frac{1}{8} | 3795 \text{ yds. at } 1\frac{1}{2}d.$$

$$16 = 4 \times 4) 379 | 5$$

$$4) 94 \ 17 \ 6$$

$$\underline{\underline{\pounds 23 \ 14 \ 4\frac{1}{2} \text{ Anf.}}}$$

d.

$$| 4 | 8\frac{1}{8} | 379 | 5 \text{ yds. at } 4d.$$

$$\underline{\underline{\pounds 63 \ 5 \text{ Anf.}}}$$

d.

$$| 8 | 3\frac{1}{8} | 379 | 5 \text{ yds. at } 8d.$$

$$\underline{\underline{\pounds 126 \ 10 \text{ Anf.}}}$$

d.

$$| 2 | 12\frac{1}{8} | 379 | 5 \text{ yds. at } 2d.$$

$$\underline{\underline{\pounds 31 \ 12 \ 6 \text{ Anf.}}}$$

d.

$$| 3 | 8\frac{1}{8} | 379 | 5 \text{ yds. at } 3d.$$

$$\underline{\underline{\pounds 47 \ 8 \ 9 \text{ Anf.}}}$$

d.

$$| 6 | 4\frac{1}{8} | 379 | 5 \text{ yds. at } 6d.$$

$$\underline{\underline{\pounds 94 \ 17 \ 6 \text{ Anf.}}}$$

d.

$$| 10 | 2\frac{1}{4} | 3795 \text{ yds.}$$

$$24 = 4 \times 6) 3795$$

$$4) 632 \ 10$$

$$\underline{\underline{\pounds 158 \ 2 \ 6 \text{ Anf.}}}$$

C A S E IV.

When the price is between one and two shillings ;—Find the value of the quantity at 1s. per yard, &c. which value being divided by those even parts which the pence are of 1s. and the quotient or quotients, arising therefrom, added thereto ; the sum will be the answer.

E X A M P L E S.

1st. What will $758\frac{1}{2}$ yards, at 1s. 9d. per yard, come to ?

	$\pounds.$	$s.$	$d.$	
6d.	37	18	6	value at 1s. per yard.
3d.	18	19	3	value at 6d. per yard.
	9	9	$7\frac{1}{2}$	value at 3d. per yard.

$$\text{Anf. } \underline{\underline{\pounds 66 \ 7 \ 4\frac{1}{2} \text{ value of } 758\frac{1}{2} \text{ yds. at } 1s. 9d. per yard.}}$$

Questions.	Answers.	Questions.	Answers.
Yds.	$\pounds. \ s. \ d.$	Yds.	$\pounds. \ s. \ d.$
2d. $987\frac{1}{2}$ at $12\frac{1}{2}d.$	$51 \ 8 \ 7\frac{3}{4}$	9th. $647\frac{3}{4}$ at $1s. 5d.$	$45 \ 17 \ 7\frac{3}{4}$
3d. 793 — $12\frac{3}{4}d.$	$42 \ 2 \ 6\frac{3}{4}$	10th. $896\frac{1}{4}$ — $1s. 6d.$	$67 \ 4 \ 4\frac{1}{2}$
4th. $847\frac{1}{2}$ — $1s. 1d.$	$45 \ 18 \ 1\frac{1}{2}$	11th. 458 — $1s. 7d.$	$36 \ 5 \ 2$
5th. $686\frac{3}{4}$ — $1s. 1\frac{1}{2}d.$	$38 \ 12 \ 7$	12th. 964 — $1s. 8d.$	$80 \ 6 \ 8$
6th. $591\frac{1}{4}$ — $1s. 2d.$	$34 \ 9 \ 9\frac{1}{2}$	13th. $752\frac{1}{2}$ — $1s. 10d.$	$68 \ 19 \ 7$
7th. $573\frac{1}{2}$ — $1s. 3d.$	$35 \ 16 \ 10\frac{1}{2}$	14th. $649\frac{3}{4}$ — $1s. 11d.$	$62 \ 5 \ 4\frac{1}{4}$
8th. $846\frac{1}{2}$ — $1s. 4d.$	$56 \ 8 \ 8$		

CASE V.

CASE V.

When the price is any even number of shillings under 40;—Multiply the given quantity by half the price, and double the first figure of the product for shillings. The rest of the product will be pounds.

N. B. If the price be 2s. you need only double the unit figure for shillings. The other figures will be pounds.

EXAMPLES.

1st. What will 746 yards cost at 2s. per yard?

746

Ans. £74 12 value at 2s. per yard.

Note. The above is done, by saying, twice 6 (the unit figure) is 12. The other figures, viz. 74, are pounds.

2d. What will $567\frac{3}{4}$ yds. at 2s. per yard come to? Ans. £56 15s. 6d.

N. B. Before I double the unit figure, viz. 7, I consider that $\frac{3}{4}$ of a yard, at 2s. per yard, will amount to 1s. 6d. Then I double 7, which makes 14s. and 1s. 6d. added, makes 15s. 6d. The other figures are pounds.

Questions.		Answers.		
Yds.		£.	s.	d.
3d.	$129\frac{1}{2}$ at 4s. per yard.	25	18	0
4th.	697 — 6s. ———	209	2	0
5th.	845 — 8s. ———	338	0	0
6th.	$917\frac{1}{4}$ — 10s. ———	458	12	6
7th.	$528\frac{1}{2}$ — 12s. ———	317	2	0
8th.	$646\frac{3}{4}$ — 14s. ———	452	14	6
9th.	591 — 16s. ———	472	10	0
10th.	$845\frac{1}{2}$ — 18s. ———	760	19	0
11th.	$645\frac{1}{4}$ — 24s. ———	774	6	0

CASE VI.

When the price wants an even part of 2s.—First find the value of the whole quantity at 2s. per lb, yard, &c. then divide it by that even part which is wanting, and subtract this quotient from the value at 2s. The remainder will be the answer

EXAMPLES.

1. What will $95\frac{1}{2}$ yards cost at 22d. per yard?

	£.	s.	d.	
2d.	$\frac{1}{12}$	9	11	0 value at 2s. per yard.
		0	15	11 value at 2d. per yard.

Ans. £8 15 1 value at 1s. 10d. per yard.

Questions.		Answers.			Questions.		Answers.		
Yds.		£.	s.	d.	Yds.		£.	s.	d.
2d.	64 at 23d. per yd.	6	2	8	5th.	$375\frac{1}{4}$ at 20d. per yd.	31	5	5
3d.	128 — 22 $\frac{1}{2}$ d. ———	12	0	0	6th.	486 — 18d. ———	36	9	0
4th.	$246\frac{1}{2}$ — 21d. ———	21	11	$4\frac{1}{2}$	7th.	754 — 16d. ———	50	5	4

CASE VII.

C A S E VII.

When the price is between 2s. and 3s.—First find the value of the quantity at 2s. per yard, &c. which value being divided by *those even parts*, which the pence are of 2s. and those quotients added thereto, the sum will be the answer.

E X A M P L E S.

1st. What will $148\frac{1}{2}$ yards come to at 2s. 7d. per yard?

	£.	s.	d.	
4d. $\frac{1}{6}$	14	17	0	value at 2s. per yard.
3d. $\frac{1}{8}$	2	9	6	ditto at 4d. per yard.
	1	17	$1\frac{1}{2}$	ditto at 3d. per yard.

Ans. £19 3 $7\frac{1}{2}$ value at 2s. 7d. per yard.

Questions.

Yds.

2d. $266\frac{1}{4}$ at 2s. 1d. per yard.3d. 344 — 2s. $1\frac{1}{2}$ d. ———4th. $543\frac{1}{2}$ — 2s. 2d. ———5th. $655\frac{3}{4}$ — 2s. 3d. ———

6th. 716 — 2s. 4d. ———

7th. 813 — 2s. 5d. ———

Answers.

£. s. d.

27 14 $8\frac{1}{4}$

36 11 0

58 17 7

73 15 $5\frac{1}{4}$

83 10 8

98 4 9

C A S E VIII.

When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20; First find the value of the quantity at the shillings per yard, &c. according to Case 5th; then suppose the quantity to stand as shillings per yard; divide it by *that even part*, which the pence are of 1s. and this quotient being added to the value before found, the sum will be the answer.

E X A M P L E S.

1st. What will $156\frac{1}{2}$ yards come to, at 6s. 4d. per yard?

Yds.

 $156\frac{1}{2}$

3

	s.	d.
4d. $\frac{1}{2}$	156	6

£46 19 0 value of $156\frac{1}{2}$ yards at 6s. per yard.

52s. 2d. = 2 12 2 value of ditto at 4d. per yard.

Ans. £49 11 2 value of ditto at 6s. 4d. per yard.

Questions.

Questions.				Answers.			
	Yds.	s.	d.	£.	s.	d.	
2d.	17½	at	4 ½	per yard.	3	10	8½
3d.	59¼	—	6 ¾	—	18	2	2¾
4th.	68¼	—	8 1	—	27	11	8¼
5th.	96	—	10 1½	—	48	12	0
6th.	67½	—	12 2	—	41	1	3

C A S E IX.

When the price is any odd number of shillings under 40; Find the value of the greatest even number contained in the price, according to Case 5th, and add thereto the value of the quantity at 1s. per yard, &c. which sum will be the answer: Or, Multiply the quantity by the price, according to the 1st or 2d Case in Simple Multiplication, and divide the product by 20, the quotient will be the answer: Or, lastly, if the price be under 13s. find the value of the quantity at 1s. per yard, &c. and multiply it by the number of shillings in the price of 1 yard; the product will be the answer.

E X A M P L E S.

1st. What will 186 yards cost, at 3s. per yard?

£.	s.		Or thus.
18	12	value at 2s. per yd.	£. s.
9	6	ditto at 1s. per yd.	9 6 value at 1s. per yard.
<hr/>			3
£ 27	18	Ans.	Product £ 27 18 Ans.

2d. What will 647 yards cost, at 17s. per yard?

£ 517	12	value at 16s. per yard.
32	7	ditto at 1s. per yard.

Ans. £ 549 19 ditto at 17s. per yard.

Questions.		Answers.		Questions.		Answers.	
Yds.	s.	£.	s. d.	Yds.	s.	£.	s. d.
3d.	169¼ at 5 per yd.	42	6 3	5th.	139 at 9 —	62	11 0
4th.	248¼ — 7 —	87	1 3	6th.	782 — 25 —	977	10 0

C A S E X.

When the price is an even part of a pound; Find the value of the given quantity, at one pound per yard, &c. then draw a line underneath, and divide it by that part; the quotient will be the answer.

E X A M P L E S.

1st. What will $156\frac{3}{4}$ yards of cloth come to, at 3s. 4d. per yard?

s. d. £. s. d.
 $134 | \frac{1}{2} | 156 \ 15 \ 0$ price at 1l. per yard.

Ans. £26 2 6 price at 3s. 4d. per yard.

Questions.	Answers.	Questions.	Answers.
<i>Yds. s. d.</i>	<i>£. s. d.</i>	<i>Yds. s. d.</i>	<i>£. s. d.</i>
2d. $516\frac{3}{4}$ at 1 0 per yd.	25 16 9	7th. $429\frac{1}{4}$ — 4 0 —	85 17 0
3d. 624 — 1 3 —	39 0 0	8th. $687\frac{1}{2}$ — 5 0 —	171 17 6
4th. $710\frac{1}{2}$ — 1 4 —	47 19 4	9th. 843 — 6 8 —	281 0 0
5th. 648 — 1 8 —	54 0 0	10th. $486\frac{3}{4}$ — 10 0 —	243 7 6
6th. $419\frac{3}{4}$ — 2 6 —	52 9 $4\frac{1}{2}$		

C A S E X I.

When the price wants an even part of a pound; First find the value of the given quantity at 1l. per yard, &c. then divide it by that even part which is wanting, and subtract this quotient therefrom; the remainder will be the answer.

E X A M P L E S.

1st. What will $167\frac{1}{2}$ yards cost, at 17s. 6d. per yard?

s. d. £. s. d.
 $126 | \frac{1}{2} | 167 \ 10 \ 0$ value at 1l. per yard.
 20 18 9 ditto at 2s. 6d. per yard.

Ans. £146 11 3 value at 17s. 6d. per yard.

Questions.	Answers.	Questions.	Answers.
<i>Yds. s. d.</i>	<i>£. s. d.</i>	<i>Yds. s. d.</i>	<i>£. s. d.</i>
2d. $347\frac{1}{2}$ at 13 4 per yd.	231 13 4	4th. 614 — 16 0 —	491 4 0
3d. $485\frac{3}{4}$ — 15 0 —	364 6 3	5th. $912\frac{1}{4}$ — 17 6 —	798 4 $4\frac{1}{2}$

C A S E X I I.

When the price is shillings, pence and farthings, and not an even part of a pound; Multiply the given quantity by the shillings in the price of 1 yard, &c. and take parts of parts from the quantity for the pence, &c. then add them together, and their sum will be the answer, in shillings, &c. Or, you may let the given quantity stand as pounds per yard, &c. then draw a line underneath, and take parts of parts therefrom; which add together, and their sum will be the answer.

N. B. I advise the learner to work the following examples both ways, by which means he will be able to discover the most concise method of performing such questions, in business, as may fall under this case.

1. What

1st. What will $248\frac{1}{2}$ yards, at 7s. 6d. per yard, come to ?

$$\begin{array}{r} \text{d.} \quad \text{s.} \quad \text{d.} \\ 16 \mid \frac{1}{2} \mid 248 \quad 6 \text{ value of } 248\frac{1}{2} \text{ yards, at 1s. per yard.} \\ 7 \end{array}$$

1739 6 value of ditto at 7s. per yard.

124 3 value of ditto at 6d. per yard.

$$2 \mid 0 \mid 186 \mid 3 \quad 9$$

Ans. £93 3 9 value of ditto at 7s. 6d. per yard.

Or thus,

$$\begin{array}{r} \text{d.} \quad \text{£.} \quad \text{s.} \quad \text{d.} \\ 16 \mid \frac{1}{2} \mid 12 \quad 8 \quad 6 \text{ value of } 248\frac{1}{2} \text{ yards, at 1s. per yard.} \\ \text{Multiply by} \quad 7 \end{array}$$

86 19 6 value of ditto at 7s. per yard.

6 4 3 value of ditto, at 6d. per yard.

$$\text{Ans. £93} \quad 3 \quad 9$$

By the latter part of this case,

$$\begin{array}{r} \text{s.} \quad \text{d.} \quad \text{£.} \quad \text{s.} \quad \text{d.} \\ 5 \quad 0 \quad \frac{1}{4} \mid 248 \quad 10 \quad 0 \text{ value of } 248\frac{1}{2} \text{ yards, at 1l. per yard.} \\ 2 \quad 6 \quad \frac{1}{2} \mid 62 \quad 2 \quad 6 \text{ value of ditto, at 5s. per yard.} \\ 31 \quad 1 \quad 3 \text{ value of ditto, at 2s. 6d. per yard.} \end{array}$$

Ans. £93 3 9 value of ditto, at 7s. 6d. per yard.

Questions.	Answers.	Questions.	Answers.
Yds. s. d.	£. s. d.	Yds. s. d.	£. s. d.
2d. $68\frac{1}{2}$ at 4 6 per yd.	15 8 3	5th. $218\frac{1}{2}$ at 12 6	136 11 3
3d. 124 — 5 8	35 2 8	6th. 645 — 4 $1\frac{1}{2}$	133 0 $7\frac{1}{2}$
4th. 146 — 14 9	107 13 6		

C A S E XIII.

When the price of the yard, lb, &c. is pounds, shillings and pence :—
First, multiply the quantity by the pounds, and if the shillings and pence be an even part of a pound, divide the given quantity by that part, and add the quotient to the product for the answer. But if they be not an even part of a pound, you must take parts of parts, and add them together as before. Or, reduce the pounds and shillings into shillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their sum will be the answer in shillings, &c.

N. B. The learner should work the following questions both ways.

EXAMPLES.

E X A M P L E S.

1 ft. What will 156 yards of broadcloth come to, at 3*l*. 6*s*. 8*d*. per yard ? Or thus.

£. s. d.	d.	s.
16/8 1/3 156 0 0 value at 1 <i>l</i> .	4 1/3 156 value at 1 <i>s</i> . per yard.	
3 per yard.	4 1/3 66 shillings in the price of 1 yd.	

468 0 0
52 0 0

936
936

Ans. £520 0 0

10296 value at 3*l*. 6*s*. per yard.

52

52

2|0)1040|0

£520 0 0 Ans.

Questions.			Answers.			Questions.			Answers.		
Yds.	£. s. d.		£. s. d.			Yds.	£. s. d.		£. s. d.		
2d. 345 1/2	at 6 5 0	per yd.	21	59	7 6	6th. 59	— 6 7 6	—	376	2	6
3d. 59 1/2	— 3 6 8	—	199	3	4	7th. 112 1/2	— 3 8 8	—	386	5	0
4th. 75	— 5 3 4	—	387	10	0	8th. 125	— 4 9 7	—	559	17	11
5th. 68	— 4 6 0	—	292	8	0						

C A S E XIV.

When the quantity is any number less than 1000, and the price not more than 12*d*. per yard, &c.—Find the value of the whole quantity at 1*d*. per yard, which may be done by dividing it by 12, mentally, setting down the quotient only in pounds, or shillings, or both. Then multiply this sum by the pence in the price of 1 yard, and the product will be the answer.

1. What will 759 1/2 yards cost, at 7*d*. per yard ?

£. s. d.
0 63 3 1/2 value at 1*d*. per yard.

Or, 3 3 3 1/2 value at 1*d*. per yard.
Mult. by 7

Ans. £22 3 0 1/2 value of 759 1/2 yards, at 7*d*. per yard.

Questions.			Answers.			Questions.			Answers.		
Yds.	d.		£. s. d.			Yds.	d.		£. s. d.		
2d. 975 1/2	at 2	per yd.	8	2	7	5th. 684	— 5 1/2	—	15	13	6
3d. 846	— 3 1/2	—	12	6	9	6th. 984 1/2	— 6 1/4	—	27	13	9 1/4
4th. 793 3/4	— 4 1/4	—	15	14	2 1/4	7th. 440 1/2	— 9	—	16	10	4 1/2

C A S E XV.

When the price is such a number of shillings and pence, as, when reduced into pence, may be produced by any two numbers in the multiplication

tion table, and when the quantity does not exceed 1000 ;—First find the value of the whole at 1*d.* per yard, &c. according to the last case ; then multiply this sum by the component parts of the pence in the price, and the last product will be the answer.

E X A M P L E S.

1st. What will 439½ yards cost, at 6*s.* 9*d.* per yard ?

£. s. d.
1 16 7½ value at 1*d.* per yard.
9

16 9 7½
9

Ans. £ 148 6 7½ value at 81*d.* or 6*s.* 9*d.* per yard.

N. B. In 6*s.* 9*d.* there being 81 pence, I multiply by 9 twice, because 9 times 9 is 81.

Questions.	Answers.	Questions.	Answers.
Yds. s. d.	£. s. d.	Yds. s. d.	£. s. d.
2d. 984½ at 1 2 per yd.	57 8 7	4th. 657 — 3 6	114 19 6
3d. 849½ — 2 8	113 6 0	5th. 593 — 4 8	138 7 4

C A S E XVI.

When the quantity is 240 ; As many pence as there are in the price of 1 yard, &c. so many pounds will the quantity amount to.

N. B. One farthing per yard will come to 5*s.* at a halfpenny per yard to 10*s.* and at three farthings, to 15*s.*

E X A M P L E S.

1st. What will 240 yards come to, at 2*s.* 7½*d.* per yard ?

Ans. £ 31 10*s.*

N. B. The price is 2*s.* 7½*d.* per yard. Now, as in 2*s.* 7*d.* there are 31 pence, so the quantity being 240 comes to 31 pounds. Then according to the same rule, the halfpenny per yard comes to 10*s.* Therefore the answer to the question is 31*l.* 10*s.*

Questions.	Answers.	Questions.	Answers.
Yds. s. d.	£. s. d.	Yds. s. d.	£. s. d.
2d. 240 at 1 7½ per yd.	19 15 0	5th. 240 at 7 8½	92 10 0
3d. 240 — 2 9	33 00 0	6th. 240 — 8 3¼	99 15 0
4th. 240 — 3 4	40 00 0		

C A S E XVII.

When the quantity is not less than 228, nor more than 252 ;—First find the value of 240 yards, &c. by the last Case ; then multiply the price of 1 yard by the number above or under 240, and add or subtract

subtract this product to or from the value of 240 yards, as the question may require; and the sum or remainder will be the answer.

E X A M P L E S.

1st. What will 248 yards come to, at 16s. 5½d. per yard?

	£.	s.	d.	
	197	10	0	value of 240 yards,
16s. 5½d. multiplied by 8	= 6	11	8	value of 8 yards.

Ans. £204 1 8 value of 248 yards.

2d. What will 229 yards cost, at 5s. 9¼d. per yard?

	£.	s.	d.	
	69	15	0	value of 240 yards.
5s. 9¼d. mult. by 11	= 3	3	11¼	value of 11 yards.

Ans. £66 11 0¼ value of 229 yards.

	Yds.		s.	d.		£.	s.	d.	
Questions.	3d.	228	at	6	10½	per	78	7	6
	4th.	252	—	4	6	—	56	14	0
	5th.	230	—	9	8¾	—	111	17	8½
	6th.	251½	—	3	10½	—	48	14	6¾
	7th.	231¼	—	8	2	—	94	8	6½
	8th.	250	—	5	9¼	—	72	2	8½
Answers.									

C A S E XVIII.

When the quantity is 480;—Find the value of 240 yards, &c. by Case the 16th, and multiply this sum by 2. The product will be the answer.

N. B. If the quantity be 12 over, or under 480, proceed according to the directions given in the last case.

E X A M P L E S.

1st. What will 480 yards cost at 2s. 9½d. per yard?

	£.	s.	d.	
	33	10	0	value of 240 yards.
Multipled by				2

Ans. £67 0 0 value of 480 yards.

2d. What will 468 yards come to at 5s. 8¼d. per yard?

	£.	s.	d.	
	68	10	0	value of 240 yards.
				2

	137	00	0	value of 480 yards.
Subtract	3	8	6	value of 12 yards

Ans. £123 11 6 value of 468 yards.

3d. What

3d. What will 492 yards come to, at 3s. 8 $\frac{1}{4}$ d. per yard?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 44 \quad 15 \quad 0 \text{ value of 240 yards.} \\ \hline 2 \end{array}$$

$$\begin{array}{r} 89 \quad 10 \quad 0 \text{ value of 480 yards.} \\ \text{Add } 2 \quad 4 \quad 9 \text{ value of 12 yards.} \\ \hline \end{array}$$

Ans. £91 14 9 value of 492 yards.

N. B. Any person, who is expert in figures, may find the value, *mentally*, of 480 yards, almost as easily as 240, it being nothing more than doubling the amount of 240.

	Yds.	s.	d.	£.	s.	d.	
Questions.	4th. 469 at 6	9 $\frac{1}{2}$	per yd.	159	5	3 $\frac{1}{2}$	Answers.
	5th. 470 — 3	4 $\frac{3}{4}$	—	79	16	0 $\frac{1}{2}$	
	6th. 471 — 5	3 $\frac{1}{2}$	—	124	12	4 $\frac{1}{2}$	
	7th. 472 — 4	9	—	112	2	0	
	8th. 483 — 8	10 $\frac{3}{4}$	—	214	16	8 $\frac{1}{4}$	

C A S E XIX.

When the quantity is 160;—Find the value of 480 yards, and divide it by 3. The quotient will be the answer.

Note. If there be 12 yards over, or under, 160, proceed as before directed.

E X A M P L E S.

1st. What will 160 yards come to, at 3s. 4 $\frac{1}{2}$ d. per yard?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{Divide by 3) } 81 \quad 0 \quad 0 \text{ value of 480 yards.} \\ \hline \end{array}$$

Ans. £27 0 0 value of 160 yards.

2d. What cost 148 yards, at 4s. 2d. per yard?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{Divide by 3) } 100 \quad 0 \quad 0 \text{ value of 480 yards.} \\ \hline \end{array}$$

$$\begin{array}{r} 33 \quad 6 \quad 8 \text{ value of 160 yards.} \\ \text{Subtract } 2 \quad 10 \quad 0 \text{ value of 12 yards.} \\ \hline \end{array}$$

Ans. £30 16 8 value of 148 yards.

3d. What will 172 yards amount to, at 5s. 7 $\frac{3}{4}$ d. per yard?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{Divide by 3) } 135 \quad 10 \quad 0 \text{ value of 480 yards.} \\ \hline \end{array}$$

$$\begin{array}{r} 45 \quad 3 \quad 4 \text{ value of 160 yards.} \\ \text{Add } 3 \quad 7 \quad 9 \text{ value of 12 yards.} \\ \hline \end{array}$$

Ans. £48 11 1 value of 172 yards.

Questions.

	Yds.	s.	d.		£.	s.	d.	
Questions.	4th.	149	at	12 6½	per yard.	93	8	8½
	5th.	150	—	3 8	_____	27	10	0
	6th.	166	—	12 8	_____	105	2	8
	7th.	152	—	5 2¾	_____	39	14	10
	8th.	153	—	6 3	_____	47	16	3
	9th.	171	—	7 9	_____	66	5	3
Answers.								

C A S E XX.

When the quantity is 120; First find the value of 240 yards, &c. then divide it by 2, and the quotient will be the answer.

Note. If there be 12 over, or under, 120, proceed as before directed.

E X A M P L E S.

1st. What will 120 yards cost, at 3s. 7½d. per yard?

£. s. d.
Divide by 2) 43 10 0 value of 240 yards.

Ans. £21 15 0 ditto of 120 yards.

2d. What will 108 yards come to, at 4s. 7d. per yard?

£. s. d.
Divide by 2) 55 0 0 value of 240 yards.

27 10 0 value of 120 yards.
Subtract 2 15 0 ditto of 12 yards.

Ans. £24 15 0 value of 108 yards.

3d. What is the value of 132 yards, at 4s. 3½ per yard?

£. s. d.
Divide by 2) 51 10 0 value of 240 yards.

25 15 0 value of 120 yards.
Add 2 11 6 value of 12 yards.

Ans. £28 6 6 value of 132 yards.

	Yds.	s.	d.		£.	s.	d.	
Questions.	4th.	109	at	3 9½	per yard.	20	13	3½
	5th.	110	—	4 3	_____	23	7	0
	6th.	125	—	15 7	_____	97	7	11
	7th.	112	—	4 9	_____	26	12	0
	8th.	113	—	5 6	_____	31	1	6
	9th.	130	—	6 4	_____	41	3	4
Answers.								

C A S E XXI.

When the quantity is 80 yards, &c. One third part of the value of 240 will be the answer.

N. B. If

N. B. If there be 12 over, or under, proceed as before directed; except when the quantity is found in the Multiplication Table; for, then, Case the 2d. of Compound Multiplication will be more concise; or, when the price is an even part of a pound, Case 10th of Practice, is to be preferred.

E X A M P L E S.

1st. What cost 80 yards of cloth, at 7s. 9½d. per yard?

$\begin{array}{r} \text{£. s. d.} \\ \text{Divide by } 3 \overline{)93 \ 15 \ 0} \end{array}$ value of 240 yards.

Ans. £31 5 0 ditto of 80 yards.

2d. What will 68 yards cost, at 4s. 9½d. per yard?

$\begin{array}{r} \text{£. s. d.} \\ \text{Divide by } 3 \overline{)57 \ 15 \ 0} \end{array}$ value of 240 yards.

$\begin{array}{r} 19 \ 5 \ 0 \text{ value of } 80 \text{ yards.} \\ \text{Subtract } 2 \ 17 \ 9 \text{ value of } 12 \text{ yards.} \\ \hline \end{array}$

Ans. £16 7 3 value of 68 yards.

3d. What will 92 yards cost, at 6s. 4d. per yard?

$\begin{array}{r} \text{£. s. d.} \\ \text{Divide by } 3 \overline{)76 \ 0 \ 0} \end{array}$ value of 240 yards.

$\begin{array}{r} 25 \ 6 \ 8 \text{ value of } 80 \text{ yards.} \\ \text{Add } 3 \ 16 \ 0 \text{ value of } 12 \text{ yards.} \\ \hline \end{array}$

Ans. £29 2 8 value of 92 yards.

	Yds.	s.	d.		£.	s.	d.	
Questions.	4th.	69	at 10	7½ per yard.	36	13	1½	Answers.
	5th.	70	— 3	7¼ ———	12	15	2½	
	6th.	71	— 4	5 ———	15	13	7	
	7th.	84	— 16	10 ———	70	14	0	
	8th.	86	— 14	9 ———	63	8	6	
	9th.	90	— 2	9 ———	12	7	6	

C A S E XXII.

When the quantity is 60 yards, &c. One fourth of the value of 240 will be the answer.

Note. If there be 12 over, or under, proceed as before directed, and observe the exception made in the last case.

E X A M P L E S.

1st. What will 60 yards of cloth cost, at 2s. 9½d. per yard?

$\begin{array}{r} \text{£. s. d.} \\ \text{Divide by } 4 \overline{)45 \ 15 \ 0} \end{array}$ value of 240 yards.

Ans. £11 8 9 value of 60 yards.

Z

2d. What

2d. What will 48 yards, at 4s. $5\frac{1}{2}d.$ per yard, come to ?

£. s. d.
Divide by 4)53 10 0 value of 240 yards.

13 7 6 value of 60 yards.
Subtract 2 13 6 value of 12 yards.

Ans. £10 14 0 value of 48 yards.

3d. What will 72 yards come to, at 5s. $4\frac{1}{2}d.$ per yard ?

£. s. d.
Divide by 4)64 10 0 value of 240 yards.

16 2 6 value of 60 yards.
Add 3 4 6 value of 12 yards.

Ans. £19 7 0 value of 72 yards.

	Yds.	s.	d.		£.	s.	d.	
Questions.	4th.	49	at	6	$10\frac{1}{2}$	per	yard.	16 16 $10\frac{1}{2}$
	5th.	50	—	9	4	—	—	23 6 8
	6th.	51	—	10	3	—	—	26 2 9
	7th.	66	—	6	7	—	—	21 14 6
	8th.	69	—	17	5	—	—	60 1 9
	9th.	70	—	9	3	—	—	32 7 6
} Answers.								

C A S E XXIII.

When the quantity is 180 ; Three fourths of the value of 240 will be the answer.

E X A M P L E S.

1st. What will 180 yards cost, at 5s. $5\frac{1}{2}d.$ per yard ?

£. s. d.
Divide by 2)65 10 0 value of 240 yards.

Divide by 2)32 15 0 value of 120 yards.
16 7 6 value of 60 yards.

Ans. £49 2 6 value of 180 yards.

	Yds.	s.	d.		£.	s.	d.	
Questions.	2d.	180	at	3	$9\frac{1}{4}$	per	yard.	33 18 9
	3d.	180	—	4	$9\frac{1}{2}$	—	—	43 2 6
	4th.	180	—	6	$4\frac{1}{4}$	—	—	57 11 3
	5th.	180	—	7	3	—	—	65 5 0
	6th.	180	—	8	$9\frac{1}{4}$	—	—	79 2 6
	7th.	180	—	13	$9\frac{1}{2}$	—	—	124 2 6
} Answers.								

C A S E XXIV.

When the price of one hundred weight is of several denominations, and the quantity likewise : Multiply the price by the integers, (that is the

the whole numbers) and take parts for the rest from the price of an integer ; which, added together, will be the answer.

EXAMPLES.

1ft. What will 9 Cwt. 3qrs. 14^{lb} of sugar come to, at 4^l. 17s. 4d. per Cwt. ?

grs. lb	£. s. d.	
2 0	4 17	4 price of 1 Cwt.
1 0	9	
0 14		
		Cwt. gr. lb
	43 16 0	price of 9 0 0
	2 8 8	price of 0 2 0
	1 4 4	price of 0 1 0
	0 12 2	price of 0 0 14

Ans. £48 1 2 price of 9 3 14

Cwt. qrs. lb.				£.	s.	d.		£.	s.	d.		
Questions.	2d.	8	1	16	Tobacco, at	5	17	9	per cwt.	49	8	2 $\frac{1}{4}$
	3d.	7	3	19	_____	7	12	8	_____	60	9	0 $\frac{1}{2}$
	4th.	12	1	24	_____	3	18	10	_____	49	2	7 $\frac{1}{4}$
	5th.	16	2	17	_____	2	15	11	_____	46	11	1
	6th.	72	3	27	_____	8	11	5	_____	625	11	10 $\frac{3}{4}$
	7th.	59	1	14	Sugar, at	1	8	7	_____	84	17	1 $\frac{1}{2}$
		lb	oz.									
	8th.	27	10		Coffee, at	0	1	4	per lb	1	16	10
		lb	oz.	pwt. gr.								
	9th.	13	10	12	8 Silv. at	4	7	6	per lb	60	14	11 $\frac{1}{2}$
	oz.	pwt.	gr.									
10th.	17	6	16	Gold, at	3	16	8	per oz.	66	8	10 $\frac{2}{3}$	
Answers.												

C A S E XXV.

When the price is at any of the rates in the second Practice Table of aliquot parts : Multiply the given quantity by the numerator, and divide that product by the denominator ; if the price be pence, the quotient will be the answer in shillings ; if shillings, the answer will be pounds.

EXAMPLES.

1 ft. What will 379 yards, at $4\frac{1}{2}d.$ per yard come to?

$$\begin{array}{r} 379 \\ 3 \\ \hline 8 \overline{) 1137} \\ 2 \overline{) 0} 14 \overline{) 2} \quad 1 \frac{1}{2} \\ \hline \text{Ans. } £ 7 \quad 2 \quad 1 \frac{1}{2} \end{array}$$

2d. What will 149 yards, at 6s. per yard come to?

$$\begin{array}{r} 149 \\ 3 \\ \hline 1 \overline{) 044} 7 \\ \hline \end{array}$$

Ans. £ 44 14

Questions.

	Yds.	s.	d.	£.	s.	d.
Questions.	3d. 127 at	0	$7\frac{1}{2}$ per yd.	3	19	$4\frac{1}{2}$
	4th. 159 —	0	8 —	5	6	0
	5th. 173 —	0	9 —	6	9	9
	6th. 241 —	0	10 —	10	0	10
	7th. 249 —	7	6 —	93	7	6
	8th. 357 —	12	6 —	223	2	6
	9th. 345 —	13	4 —	230	0	0
	10th. 323 —	14	0 —	226	2	0
	11th. 287 —	15	0 —	215	5	0
	12th. 253 —	16	8 —	210	16	8
Answers.						

C A S E XXVI.

When the price is any even number of shillings, if it be required to know what quantity of any thing may be bought for so much money; Annex a cypher to the money, and divide it by half of the price, and the quotient will be the quantity to be purchased.

E X A M P L E S.

1st. How many yards of cloth, at 18s. per yard, may I have for £345?

Half the price = 9)3450 = money with a cypher annexed.

383 $\frac{1}{3}$ yards, Ans.

	s.	£.	Yds.
2d. How many yds, at 2 peryd. for 427?	—	427	—
3d. —	4	312	—
4th. —	6	917	—
5th. —	8	195	—
6th. —	10	247	—
7th. —	12	439	—
Answers.			
			4270
			1560
			3056 $\frac{2}{3}$
			487 $\frac{1}{2}$
			494
			731 $\frac{2}{3}$

C A S E XXVII.

To find the discount of any invoice, or bill of parcels, at any rate per cent. Multiply the pounds in the invoice by the amount of the discount of 1 pound, at the rate per cent. and take parts for the shillings and pence; then add them together, and the sum will be the discount required.

N. B. The discount for 1 pound at any rate per cent. is in the 3d. Practice Table.

E X A M P L E S.

1st. What is the discount of an invoice, amounting to £65 18s. 4d. at $7\frac{1}{2}$ per cent?

OPERATION

O P E R A T I O N.

	£.	s.	d.
The discount of 65 <i>l.</i> at 5 <i>l.</i> per cent. is 65 <i>s.</i> or	3	5	0
The discount of 65 <i>l.</i> at 2½ <i>l.</i> per cent. is	1	12	6
The discount of 10 <i>s.</i> being half a pound, at 7½ <i>l.</i> per cent. is	0	0	9
The discount of 3 <i>s.</i> 4 <i>d.</i> being ⅙ of a pound at 7½ <i>l.</i> per cent. is	0	0	3

The sum is, £4 18 6 *Ans.*

2d. What is the discount of a bill of parcels, amounting to 8*l.* 17*s.* 8*d.* at 2½*l.* per cent?

	£.	s.	d.
The discount of 8 <i>l.</i> is	0	4	0
The discount of 10 <i>s.</i> is	0	0	3
The discount of 6 <i>s.</i> 8 <i>d.</i> is	0	0	2
The discount of 1 <i>s.</i> is	0	0	0½

Ans. £0 4 5½

N. B. When the rate per cent. is any even part of £100, it may be performed by dividing the amount by *that even part*.

3d. What is the discount of an invoice amounting to 57*l.* 13*s.* 9*d.* at 12½*l.* per cent.?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ | 12\frac{1}{2} | \frac{1}{4} | 57 \quad 13 \quad 9 \end{array}$$

£7 4 2½ *Ans.*

C A S E XXVIII.

To find the value of goods sold by particular quantities, viz. I. By the score. II. Round timber. III. By 5 score to the hundred. IV. By 112 to the hundred. V. By 6 score to the hundred. VI. By the great gros. VII. By the 1000.

I. To find the value of goods sold by the score.

The price of one is given, to find the price of one score.

If the given price be shillings and pence, or only pence, divide the given price, in pence, by 12. The quotient will be the answer in pounds, and the remainder will be so many times 1*s.* 8*d.*

E X A M P L E S.

1*ft.* At 9*d.* each; What is that per score?

12)9*d.* (.75 = £0 15 0 *Ans.*

Or by inverting the question.

1 score = 20 = 1*s.* 8*d.*

9

15*s.* 0

2d. At 4*s.* 9*d.* each; What is that per score?

4*s.* 9*d.*

12

12)57*d.*

£4 15 *Ans.*

It may be remarked, that when the price is shillings and pence, the answer will be just so many pounds as there are shillings, and so many times 1s. 8d. as there are pence. If farthings are given, for $\frac{1}{4}d.$ reckon 5d. for $\frac{1}{2}d.$ 10d. and for $\frac{3}{4}d.$ 1s. 3d.

T A B L E of Aliquot Parts. 20 the Integer.

2 is $\frac{1}{10}$	6 is $\frac{3}{10}$	12 is $\frac{6}{10}$	16 is $\frac{8}{10}$
4 — $\frac{2}{5}$	8 — $\frac{4}{5}$	14 — $\frac{7}{10}$	18 — $\frac{9}{10}$
5 — $\frac{1}{2}$	10 — $\frac{1}{1}$	15 — $\frac{3}{4}$	

3d. What cost 7 ; at 2s. 9d. per score ?

$$\begin{array}{r|l}
 5 & \frac{1}{4} \\
 2 & \frac{1}{10} \\
 \hline
 & 2 \quad 9 \\
 & 0 \quad 8\frac{1}{4} \\
 & 0 \quad 3\frac{1}{4} \\
 \hline
 7 & = 0 \quad 11\frac{1}{2}
 \end{array}$$

4th. What cost 17 ; at 19s. 10d. per score ?

$$\begin{array}{r|l}
 10 & \\
 5 & \frac{1}{2} \\
 2 & \frac{1}{4} \\
 & \frac{1}{10} \\
 \hline
 & 19 \quad 10 \\
 & 9 \quad 11 \\
 & 4 \quad 11\frac{1}{2} \\
 & 1 \quad 11\frac{3}{4} \\
 \hline
 17 & = 16 \quad 10\frac{1}{4}
 \end{array}$$

II. Round Timber.

Forty feet make a load or ton of round timber.

If the given price of a foot be shillings,

R U L E.

Multiply the given price by 2, and the product will be the answer in pounds.

5th. What cost a ton, at 3s. per foot ?

$$3s. \times 2 = 6l. \text{ Anf.}$$

6th. What cost a ton at 9s.

$$\text{per foot ? } 9s. \times 2 = 18l. \text{ Anf.}$$

If the given price of 1 foot be pence only, or shillings and pence, divide the given price, in pence, by 6. The quotient will be the answer in pounds, and the remainder will be so many times 3s. 4d.

7th. What cost 40 feet, at 17d. per foot ?

$$6)17$$

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$£2 \quad 16 \quad 8 \text{ Anf.}$$

8th. At 1s. 9d. per foot, What cost a ton ?

$$6)21$$

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$£3 \quad 10 \text{ Anf.}$$

If the given price of a foot be farthings only, or pence and farthings, divide the given price, in farthings, by 6 ; then divide that quotient by 4, and this last quotient will be the answer.

9th. At $\frac{3}{4}d.$ per foot, What cost a ton ?

$$6)3$$

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$4)0 \quad 10$$

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$£0 \quad 2 \quad 6 \text{ Anf.}$$

10th. At $13\frac{1}{4}$ per foot, What cost a ton ?

$$13\frac{1}{4}$$

$$\underline{\quad}$$

$$6)53$$

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$4)8 \quad 1 \quad 6 \quad 8$$

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$£2 \quad 4 \quad 2 \text{ Anf.}$$

Or,

Or, suppose every shilling in the price to be 2*l.* every penny to be 3*s.* 4*d.* and every farthing to be 10*d.*

11th. What cost 40 feet at $\frac{3}{4}$ *d.* per foot?

$$\frac{3}{4}d. \times 10 = \text{£}0 \ 2 \ 6 \text{ Ans.}$$

12th. What cost 40 at 15 $\frac{1}{2}$ *d.* per foot?

s. d.

$$1 \ 0 \times 2 = \text{£}2 \ 0 \ 0$$

$$3 \ 4 \times 3 = \ 0 \ 10 \ 0$$

$$0 \ \frac{1}{2} \times 10 = \ 0 \ 1 \ 8$$

$$\text{£}2 \ 11 \ 8 \text{ Ans.}$$

III. To find the value of goods sold by 5 score to the hundred.

1st. If the given price be pounds and shillings, or shillings only.

R U L E.

Multiply the given price, in shillings, by 5, and the quotient will be the answer in pounds.

13th. At 19*s.* per yard, What cost 100 yards?

19*s.*

5

£95 *Ans.*

14th. At 4*l.* 13*s.* per cwt. What cost 100 cwt. or 5 tons?

4 13

20

93

5

£465 *Ans.*

2d. If the given price of 1 be pence only, or shillings and pence.

R U L E.

Multiply the given price, in pence, by 5; then divide that product by 12. The quotient will be pounds, and the remainder so many times 1*s.* 8*d.*

15th. If 1 yard cost 9*d.* What cost 100 yards?

9

5

12)45

£3 15 *Ans.*

16th. What cost 100 bushels, at 35*s.* 4*d.* per bushel?

s. d.

35 4

12

424

5

12)2120

£176 13 4 *Ans.*

Or,

35*s.* 4*d.*

| 4*d.* | $\frac{1}{3}$ | 5

175

1 13 4

£176 13 4

Here 5 is divided by $\frac{1}{3}$.
3*d.* If

3. If the given price of 1 be shillings and pence ; Multiply the price by 5, and the product, under the place of shillings, will be the answer in pounds, and the product under the place of pence, will be so many times 1s. 8d.

17th. At 2s. 5d. per bushel ;
What cost 100 bushels ?

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 2 \quad 5 \\ \quad 5 \\ \hline 12 \quad 1 \end{array}$$

Ans. £12 1 8

18th. At 25s. 3d. per ton ;
What cost 100 tons ?

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 25 \quad 3 \\ \quad 5 \\ \hline 126 \quad 3 \end{array}$$

Ans. £126 5

4. To find the price of one, at so much per hundred of 5 score.

GENERAL RULE.

Multiply the given price by 12 ; divide the product by 5, and the quotient will be the answer in pence.

But if the price be pounds only ;

RULE.

Divide the given price by 5, and the quotient will be the answer in shillings.

19th. If 100 yds. cost 65*l.*
What cost 1 yd. ?

$$\begin{array}{r} 5 \overline{)65} \\ \hline 13 \text{ s. } \text{Ans.} \end{array}$$

21st. If 100 yards cost 11*l.*
7s. 9d. What cost 1 yard ?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 11 \quad 7 \quad 9 \\ \hline 12 \end{array}$$

20th. If 100 yds. cost 2*l.* 18s. 4d.
What is that per yard ?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 2 \quad 18 \quad 4 \\ \hline 12 \end{array}$$

$$5 \overline{)35} \quad 0 \quad 0$$

7d. Answer.

$$5 \overline{)136} \quad 13 \quad 0$$

$$12 \overline{)27} \quad 6 \quad 7$$

2s. 3½d. Ans.

In dividing 27 by 12 (in the 21st question) the quotient is 2s. and the remainder 3d. the 6 is $\frac{6}{20}$ of a penny = one farthing, and the 7 is of no account.

TABLE of Aliquot Parts. 100 the Integer.

5 is $\frac{1}{20}$	25 is $\frac{1}{4}$	50 is $\frac{1}{2}$	75 is $\frac{3}{4}$
10 — $\frac{1}{10}$	30 — $\frac{3}{10}$	60 — $\frac{3}{5}$	80 — $\frac{4}{5}$
20 — $\frac{1}{5}$	40 — $\frac{2}{5}$	70 — $\frac{7}{10}$	90 — $\frac{9}{10}$

22d. At

22d. At 3*l.* 7*s.* 6*d.* per 100 ; What will 23 cost ?

$$\begin{array}{r|l|l}
 20 & \frac{1}{5} & \begin{array}{r} \text{£. s. d.} \\ 3. 7 6 \end{array} \\
 2 & \frac{1}{10} & \begin{array}{r} 0 13 6 \\ 0 1 4 \end{array} \\
 1 & \frac{1}{2} & \begin{array}{r} 0 0 8 \end{array} \\
 \hline
 23 & & \begin{array}{r} \text{£} 0 15 6 \text{ Ans.} \end{array}
 \end{array}
 \quad \text{Add.}$$

23d. At 2*l.* 1*s.* 10*d.* per 100 ; What cost 18 ?

$$\begin{array}{r|l|l}
 20 & \frac{1}{5} & \begin{array}{r} \text{£. s. d.} \\ 2 1 10 \end{array} \\
 2 & \frac{1}{10} & \begin{array}{r} 0 8 4\frac{2}{5} \\ 0 0 10 \end{array} \\
 \hline
 18 & = & \begin{array}{r} \text{£} 0 7 6\frac{2}{5} \text{ Ans.} \end{array}
 \end{array}
 \quad \text{Sub.}$$

24th. At 5*l.* 9*s.* 6*d.* per 100 ; What cost 35 ?

$$\begin{array}{r|l|l}
 & & \begin{array}{r} \text{£. s. d.} \\ 5 9 6 \end{array} \\
 & & 3 \\
 \hline
 10 & 16 8 6 \\
 \hline
 5 & \frac{1}{5} & \begin{array}{r} 1 12 10 \\ 5 5\frac{2}{3} \end{array} \\
 \hline
 \text{£} 1 18 3\frac{2}{3} \text{ Ans.}
 \end{array}
 \quad \text{Add.}$$

IV. To find the value of goods, sold by 112*lb* the Cwt.

The price of 1*lb* is given to find the value of 1 cwt.

R U L E.

For a farthing, account 2*s.* 4*d.* per cwt. For a half penny, 4*s.* 8*d.* For three farthings, 7*s.* And for every penny, 9*s.* 4*d.* per cwt.

25th. What cost 1cwt. at 3*½d.* per *lb* ?

$$\begin{array}{rcl}
 \text{At 1d. per } \text{lb} & \text{s. d.} & \\
 1\text{cwt. costs} & - & 9 \ 4 \\
 & & 3 \\
 \hline
 & &
 \end{array}$$

$$\begin{array}{rcl}
 \text{At 3d.} & - & \text{£} 1 \ 8 \ 0 \\
 \text{At } \frac{1}{2}\text{d.} & - & 0 \ 4 \ 8 \\
 \hline
 & & \text{£} 1 \ 12 \ 8 \text{ Ans.}
 \end{array}
 \quad \text{Add.}$$

26th. At 8*¾d.* per *lb*, What cost 1cwt. ?

$$\begin{array}{rcl}
 \text{At 1d. per } \text{lb} & \text{s. d.} & \\
 1\text{cwt. costs} & - & 9 \ 4 \\
 & & 8 \\
 \hline
 & &
 \end{array}$$

$$\begin{array}{rcl}
 \text{At 8d.} & - & \text{£} 3 \ 14 \ 8 \\
 \text{At } \frac{3}{4}\text{d.} & - & 0 \ 7 \ 0 \\
 \hline
 & & \text{£} 4 \ 1 \ 8 \text{ Ans.}
 \end{array}
 \quad \text{Add.}$$

V. To find the value of goods sold by 6 score to the hundred.

The price of 1 is given, to find the price of 1 hundred.

R U L E.

Suppose every penny in the price to be so many pounds, and for the farthings, such a part of a pound, as they are of a penny ; then, half of that sum will be the answer.

A a

27th. At

27th. At $4\frac{1}{2}d.$ per yard, What cost 120 yards?

$$\begin{array}{r} \text{£. s.} \\ 2) 4 \ 10 \\ \hline \text{£} 2 \ 5 \text{ Ans.} \end{array}$$

28th. At $16s. 9\frac{1}{4}d.$ per yard; What cost 120 yards?

$$\begin{array}{r} \text{s. d.} \\ 16 \ 9\frac{1}{4} \\ \hline 12 \\ \hline 201 \\ \hline 2) 201 \text{ l. } 5 \text{ s.} \\ \hline \text{£} 100 \ 12 \ 6 \text{ Ans.} \end{array}$$

To find the price of *one*, at so much per hundred of 6 score.

R U L E.

Multiply the price by 2, then call the pounds so many pence, and the shillings, such a part of a penny, as they are of a pound, and you will have the answer.

29th. If 120 yds. cost $3\text{ l. } 12\text{ s.}$ What cost 1 yard?

$$\begin{array}{r} \text{£. s.} \\ 3 \ 12 \\ 2 \\ \hline 7 \ 4 \\ \hline \text{Ans. } 7\frac{1}{2}d. \end{array}$$

30th. If 120 yds. cost $5\text{ l. } 18\text{ s.}$ 6d. What cost 1 yard?

$$\begin{array}{r} \text{£. s. d.} \\ 5 \ 18 \ 6 \\ 2 \\ \hline 11 \ 17 \ 0 \end{array}$$

Ans. $11\frac{3}{4}d. + \frac{2}{5}$ of a farthing.

TABLE of Aliquot Parts. 120 the Integer.

Also,

6 is $\frac{1}{20}$	24 is $\frac{1}{5}$	36 is $\frac{3}{10}$	72 is $\frac{3}{5}$	96 is $\frac{4}{5}$
10 — $\frac{1}{12}$	30 — $\frac{1}{4}$	45 — $\frac{3}{8}$	75 — $\frac{5}{8}$	100 — $\frac{5}{6}$
12 — $\frac{1}{10}$	40 — $\frac{1}{3}$	48 — $\frac{2}{5}$	80 — $\frac{2}{3}$	105 — $\frac{7}{8}$
15 — $\frac{1}{8}$	60 — $\frac{1}{2}$	50 — $\frac{1}{2}$	84 — $\frac{7}{10}$	108 — $\frac{9}{10}$
20 — $\frac{1}{6}$		70 — $\frac{7}{12}$	90 — $\frac{3}{4}$	110 — $\frac{11}{12}$

31st. At $3\text{ l. } 17\text{ s. } 6\text{ d.}$ per hundred; What cost 14?

$$\begin{array}{r} \text{£. s. d.} \\ \left| \begin{array}{c} 12 \\ 2 \end{array} \right| \left| \begin{array}{c} \frac{1}{10} \\ \frac{1}{6} \end{array} \right| \left| \begin{array}{c} 3 \ 17 \ 6 \\ 0 \ 7 \ 9 \\ 0 \ 1 \ 3\frac{1}{2} \end{array} \right\} \text{Add.} \\ \hline 14 = \text{£} 0 \ 9 \ 0\frac{1}{2} \text{ Ans.} \end{array}$$

32d. At

32d. At 2*l.* 13*s.* 6½*d.* per hundred; What cost 49?

		£.	s.	d.
40	$\frac{1}{3}$	2	13	6½
8	$\frac{1}{5}$	0	17	10 0½
1	$\frac{1}{8}$	0	3	6 ¾
		0	0	5 1

49 = £1 1 10 1 *Anf.*

33d. At 1*l.* 19*s.* 3*d.* per hundred; What cost 75?

	£.	s.	d.
	1	19	3
			5
8)9	16	3	
	£1	4	6½ <i>Anf.</i>

VI. To find the value of goods sold by the great grofs.

NOTE. 12 make 1 dozen, 12 dozen 1 small grofs, 12 small grofs 1 great grofs.

The price of 1 dozen being given, in pence, to find the price of a great grofs.

R U L E.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c. For proof, do the contrary.

N. B. If the price of 1 be given, the price of 1 small grofs is found after the same manner.

34th. What cost 1 great grofs, at 18*d.* per dozen?

$$\begin{array}{r} 3 \\ 5 \overline{)54} \\ \hline \end{array}$$

£10 16 *Anf.*

35th. At 4*s.* 3*d.* per dozen; What cost 1 great grofs.

4 <i>s.</i> 3 <i>d.</i>	Or,	Or,
12	s. d.	£. s.
—	4 3	144 = 7 4
51 <i>d.</i>	12	4
3		
—	2 11 0	34. ¼ 28 16
5)153	12	1 16 } <i>Add.</i>
<i>Anf.</i> £30 12	£30 12 0	£30 12

TABLE of Aliquot Parts. 144 the Integer.

12 is $\frac{1}{12}$	36 is $\frac{1}{4}$	Alfo,	32 is $\frac{2}{9}$	84 is $\frac{7}{12}$	128 is $\frac{8}{9}$
16 — $\frac{1}{9}$	48 — $\frac{1}{3}$		60 — $\frac{1}{2}$	96 — $\frac{2}{3}$	132 — $\frac{11}{12}$
18 — $\frac{2}{9}$	72 — $\frac{1}{2}$		64 — $\frac{4}{9}$	108 — $\frac{3}{4}$	
24 — $\frac{1}{6}$			80 — $\frac{5}{9}$	120 — $\frac{5}{6}$	

36th. At

86th. At 2*l.* 12*s.* 9*d.* per great gross ; What cost 45 dozen ?

Doz.		£.	s.	d.
36	$\frac{1}{4}$	2	12	9
9	$\frac{1}{4}$	0	13	$2\frac{1}{4}$
		0	3	$3\frac{1}{2}$

} *Add.*

$45 = \text{£}0 \ 16 \ 5\frac{3}{4} \text{ Ans.}$

37th. What cost 117 dozen, at 9*l.* 13*s.* 7*d.* per great gross ?

£.	s.	d.
9	13	7
		3

Doz.		£.	s.	d.
108	$\frac{3}{4}$	29	0	9
9	$\frac{1}{12}$	7	5	$2\frac{1}{4}$
			12	1

} *Add.*

$117 = \text{£}7 \ 17 \ 3\frac{1}{4} \text{ Ans.}$

38th. At 3*l.* 16*s.* 8*d.* per great gross ; What cost 7 great gross, and 96 dozen ?

£.	s.	d.
3	16	8
		2

Doz.		£.	s.	d.
37	$\frac{1}{4}$	7	13	4
96				$1\frac{1}{4}$

} *Add.*

$Top \ line \times 7 = 26 \ 16 \ 8$

$\text{£}29 \ 7 \ 9\frac{1}{4} \text{ Ans.}$

VII. To find the value of goods sold by the thousand.

The price of 1 is given, to find the price of 1000.

R U L E.

Multiply the given price, in pence, by 50, then divide the product by 12, and the quotient will be the answer in pounds, &c.

39th. At 6*d.* each ; Or, as 1000*s.* are 50*l.* What cost 1000 ?

6
50
12)300

$\text{£}25 \text{ Ans.}$

take parts, for the pence out of 50.

6 <i>d.</i>	$\frac{1}{2}$	50

$\text{Ans. } 25$

40th. What cost 1000 at 2*l.* 4*s.* each.

2 <i>d.</i>		£.	s.	d.
$\frac{1}{6}$	$\frac{1}{6}$	50		
$\frac{1}{4}$	$\frac{1}{8}$	8	6	8
			1	0

} +

$\text{£}9 \ 7 \ 6 \text{ Ans.}$

To find the price of one, at so much per thousand.

R U L E.

Multiply the price by 12 ; divide the product by 50 ; then take the pounds for so many pence, and the shillings for such a part of a penny as they are of a pound, which will be the answer.

41st. At

41st. At 5*l.* 4*s.* 2*d.* per 1000; What cost 1?

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 5 \quad 4 \quad 2 \\
 \quad \quad \quad 12 \\
 \hline
 50 \left\{ \begin{array}{l} 5) 62 \quad 10 \quad 0 \\ \hline 10) 12 \quad 10 \end{array} \right. \\
 \hline
 \text{£} 1 \quad 5 \\
 \hline
 \text{Ans. } 1 \frac{1}{4} \text{d.}
 \end{array}$$

42d. At 35*l.* 3*s.* 4*d.* per 1000; What cost 1?

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 354 \quad 3 \quad 4 \\
 \quad \quad \quad 12 \\
 \hline
 50 \left\{ \begin{array}{l} 10) 4250 \quad 0 \quad 0 \\ \hline 5) 425 \\ \hline 85 \end{array} \right. \\
 \hline
 \text{Ans. } 7 \text{s. } 1 \text{d.}
 \end{array}$$

Or,

$$\begin{array}{c|c|c}
 100 & \frac{1}{10} & \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 354 \quad 3 \quad 4 \\ \hline 35 \quad 8 \quad 4 \\ \hline 3 \quad 10 \quad 10 \end{array} \\
 10 & \frac{1}{10} & \\
 1 & \frac{1}{10} & \\
 \hline
 \text{Ans. } 0 \quad 7 \quad 1
 \end{array}$$

T A B L E of Aliquot Parts. 1000 the Integer.

$$\begin{array}{l}
 \text{Also,} \\
 \begin{array}{l}
 50 \text{ is } \frac{1}{20} \quad \left| \quad 200 \text{ is } \frac{1}{5} \right. \\
 100 \text{ — } \frac{1}{10} \quad \left| \quad 250 \text{ — } \frac{1}{4} \right. \\
 125 \text{ — } \frac{1}{8} \quad \left| \quad 500 \text{ — } \frac{1}{2} \right.
 \end{array}
 \end{array}$$

43d. At 1*l.* 17*s.* 9*d.* per 1000; What cost 115?

$$\begin{array}{c|c|c}
 100 & \frac{1}{10} & \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 1 \quad 17 \quad 9 \\ \hline 0 \quad 3 \quad 9\frac{1}{2} \\ 0 \quad 0 \quad 4\frac{1}{2} \\ 0 \quad 0 \quad 2\frac{1}{4} \end{array} \\
 10 & \frac{1}{10} & \\
 5 & \frac{1}{2} & \\
 \hline
 115 = \text{£} 0 \quad 4 \quad 4 \text{ Answer.}
 \end{array}$$

44th. At

44th. At 2*l.* 1*s.* 8*d.* per 1000; What cost 875?

$$\begin{array}{r}
 \text{£. s. d.} \\
 2 \quad 1 \quad 8 \\
 \hline
 7 \\
 8 \overline{) 14 \quad 11 \quad 8} \\
 \hline
 \text{£ } 1 \quad 16 \quad 5\frac{1}{2} \text{ Ans.}
 \end{array}$$

45th. What cost 33, at 24*s.* 8*d.* per 1000?

$$\begin{array}{r|l|l}
 \text{£. s. d.} & & \\
 \hline
 50 & \frac{1}{20} & 1 \quad 4 \quad 8 \\
 \hline
 25 & \frac{1}{2} & 0 \quad 1 \quad 2\frac{1}{4} \\
 \hline
 5 & \frac{1}{5} & 0 \quad 0 \quad 7\frac{1}{4} \left. \vphantom{\frac{1}{5}} \right\} \text{Add.} \\
 \hline
 30 & = & 0 \quad 0 \quad 8\frac{1}{2} \left. \vphantom{=}\right\} \text{Add.} \\
 3 & \frac{1}{10} & 0 \quad 0 \quad \frac{3}{4} \\
 \hline
 33 & = & 0 \quad 0 \quad 9\frac{1}{4} \text{ Answer.}
 \end{array}$$

PRACTICE BY DECIMALS.

I. Since 2*s.* is $\frac{1}{10}$ of 1*l.* the decimal of 2*s.* is .1: Wherefore any quantity being given at 2*s.* per *lb.*, yard, &c. the price is found in pounds and decimal parts of a pound, by separating the unit figure of the given quantity from the rest, for a decimal.

Let it be required to find the value of 356 yards at 2*s.* per yard?

By pointing off the unit figure 6 for a decimal, I find the } amount to be £35.6, which is known to be equal to 35*l.* 12*s.* } £35.6

II. Consequently, if the price be a multiple of 2*s.* (viz. any even number of shillings) the amount at 2*s.* being first found in pounds and decimal parts, as above, and that amount multiplied by the number which shews how often 2*s.* is contained in the given price, the product will be the amount required in pounds and decimal parts of a pound.

What cost 427 gallons of wine, at 8*s.* per gallon?

£42.7 amount at 2*s.* per gallon.

4

Ans. £170.8 or 170*l.* 16*s.*

The examples in Case 5th may be worked in this manner. Likewise, if the price be pounds and even shillings.

754 yards at 1*l.* 8*s.*

75.4 amount at 2*s.*
14 × 2 = 28*s.*

30 16
754

Or,
754
75.4 × 4 = 301.6 } Add.
£1055.6

Ans. £1055.6 = 1055*l.* 12*s.*

III. If

III. *If the price be an aliquot part of 2s.—Find the amount at 2s. and divide it by the denominator of the part, and the quotient will be the answer.*

At 8d. per lb What cost 976 lb ?

$$\begin{array}{r} | \text{8d.} | \frac{1}{3} | \hline 97,6 \end{array}$$

$$\pounds 32,533 = \pounds 32 \text{ } 10 \text{ } 8 \text{ Ans.}$$

IV. *If the price be an aliquant (that is, uneven) part ; Divide it into aliquot parts.*

7235 yards, at 7d.

$$\begin{array}{r} | \text{4d.} | \frac{1}{6} | \hline 723,5 \\ | \text{3d.} | \frac{1}{8} | \hline 120,583 \\ \hline 90,437 \end{array}$$

$$211,02 = \pounds 211 \text{ } 0 \text{ } 4\frac{3}{4} \text{ Ans.}$$

V. *If the price be pounds and shillings, or pounds, shillings and pence ; Reduce the shillings, &c. to the decimal of a pound, and multiply the quantity thereby, or the price by the quantity.*

At 15l. 12s. 6d. per Cwt. ; What cost 75 Cwt. ?

$$\pounds 15 \text{ } 12 \text{ } 6 = \pounds 15,625$$

75

78,125

109375

1171,875

$$\pounds 1171 \text{ } 17 \text{ } 6 \text{ Ans.}$$

VI. *If the quantity likewise be of divers denominations ; Reduce the less denominations to the decimal of that, whereof the price is given.*

9lb 10oz. of silk, at $\pounds 4 \text{ } 5 \text{ } 9 = \pounds 4,287$

9lb 10oz. = 9,625

21435

8574

25722

38583

41,262875

$$\pounds 41 \text{ } 5 \text{ } 3 \text{ Answer.}$$

Cases 12th, and 13th, may be wrought in this manner.

Or,

Or, You may take parts for the lower denominations.

$$\begin{array}{r}
 \left| \begin{array}{l} 8\text{oz.} \\ 202. \end{array} \right| \left| \begin{array}{l} \frac{1}{2} \\ \frac{1}{4} \end{array} \right| \begin{array}{l} 4,287 \\ 9 \end{array} \\
 \hline
 38,583 \\
 2,1435 \\
 .535875 \\
 \hline
 41,262375 \\
 \hline
 £4: 5: 2\frac{1}{4}
 \end{array}$$

VII. When the price is any *odd* number of shillings : If it be required to know what quantity of any thing may be bought for any sum of money, in pounds : Annex *two* cyphers to the money, and divide it by half the price.

Note. As half a shilling (or 6 pence) is ,5, therefore, to halve any odd number of shillings, is only to annex ,5 to half of the greatest even number in the price.

1st. How many yds. at 7s. per yd. may I have for 435l. ?

Half = 3.5)43500(1242 $\frac{3}{5}$ yds. *Anf.*

$$\begin{array}{r}
 35 \\
 \hline
 85 \\
 70 \\
 \hline
 150 \\
 140 \\
 \hline
 100 \\
 70 \\
 \hline
 30
 \end{array}$$

2d. How many pounds of tea, at 5s. per lb for 37l. ?

2,5)3700(148 lb *Anf.*

$$\begin{array}{r}
 25 \\
 \hline
 120 \\
 100 \\
 \hline
 200 \\
 200 \\
 \hline
 \hline
 \end{array}$$

3d. How many yards, at 9s. per yard, may I have for 540l. ?

Anf. 1200 yards.

B I L L of P A R C E L S.

Newburyport, June 1st, 1787.

Mr. Timothy Huckster

Bought of Samuel Merchant.

- 25 $\frac{1}{2}$ lb Bohea tea, at 3s. 6d. per lb.
- 48lb Cheese at 9d. per lb.
- 15 Pair of worsted hose, at 5s. 8d. per pair.
- 4 $\frac{1}{2}$ Dozen women's gloves, at 3s. 6d. per dozen.
- 19 Dozen knives and forks, at 5s. 9d. per dozen.
- 9 Grindstones at 15s. 9d. per stone.
- $\frac{1}{2}$ Cwt. Brown sugar, at 51s. per cwt.
- 31lb Loaf sugar, at 1s. 0 $\frac{1}{2}$ d. per lb.

£34 3 3 $\frac{1}{2}$

Received payment in full.

Samuel Merchant.

TARE

T A R E A N D T R E T.

Tare and Tret are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of $\frac{4}{100}$ in every 100 lb for waste, dust, &c.

Cloff is an allowance of $\frac{2}{3}$ upon every 3 Cwt.

Gross weight is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. which contains them.

Suttle is, when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

C A S E I.*

When the tare is at so much per box, barrel or bag, &c.—Multiply the number of boxes, barrels, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight required.

E X A M P L E S.

1st. In 6 hogsheds of sugar, each weighing 9 Cwt. 2 qrs. 10 lb gross, tare 25 lb per hoghead; How much neat?

Cwt. qr. lb	Cwt. qr. lb	
$25 \times 6 = 1$	9	gross wt. of one hhd.
1	2	
10	10	
	6	

57	2	4	gross.
1	1	10	tare.

Anf. 56 0 22 neat.

2d. In 5 bags of cotton, marked with the gross weight as follows, tare 23 lb per bag; What neat weight?

	Cwt. qr. lb
A =	7 1 19
B =	3 3 27
C =	5 1 13
D =	6 0 15
E =	8 1 0

Cwt. qr. lb
Anf. 30 0 14 neat.

3d. What is the neat weight of 15 hogsheds of tobacco, each 7 Cwt. 1 qr. 13 lb. Tare 100 lb per hoghead?

Anf. 97 Cwt. 0 qr. 11 lb.

C A S E

* This, as well as every other case in this rule, is only an application of the rules of Proportion and Practice.

C A S E II.

When the tare is at so much per cwt.—Divide the gross weight by the aliquot parts of a cwt. subtract the quotient from the gross, and the remainder will be the neat weight.

E X A M P L E S.

1st. In 129cwt. 3qrs. 16lb gross, tare 14lb per cwt. What neat weight?

		Cwt.	qr.	lb	
14lb	$\frac{1}{8}$	129	3	16	gross.
		16	0	26 $\frac{1}{2}$	tare.

Anf. 113 2 17 $\frac{1}{2}$ neat.

2d. In 97cwt. 1qr. 7lb gross, tare, 20lb per cwt. What neat weight?

		Cwt.	qr.	lb	
16	$\frac{1}{7}$	97	1	7	gross.
		13	3	17	} Add.
4	$\frac{1}{4}$	3	1	25	

Subtract 17 1 14 tare.

Anf. 79 3 21 neat.

4th. What is the value of the neat weight of 7hhds. of tobacco, at 5l. 7s. 6d. per cwt. each weighing 8cwt. 3qrs. 10lb gross, tare 21lb per cwt.?

Anf. £270 4 4 $\frac{1}{2}$ reckoning the odd ozs.

C A S E III.

When tret is allowed with tare.—Divide the futtle weight by 26, and the quotient will be the tret, which subtract from the futtle, and the remainder will be the neat.

E X A M P L E S.

1st. In 247cwt. 2qrs. 15lb gross, tare 28 per cwt. and tret 4lb per 104lb, What neat weight?

		Cwt.	qr.	lb	
28	$\frac{1}{4}$	247	2	15	gross.
		61	3	17	12 tare, subtract.

		Cwt.	qr.	lb	
4	$\frac{1}{26}$	185	2	25	4 futtle.
		7	0	16	0 tret, subtract.

Anf. 178 2 9 4 neat.

2d. What is the neat weight of 4hhds. of tobacco, weighing as follow: The 1st. 5cwt. 1qr. 12lb gross, tare 65lb per hoghead; the 2d. 3cwt. 0qr. 19lb gross, tare 75lb; the 3d. 6cwt. 3qrs. gross, tare,

tare 49^{lb}; and the 4th. 4^{cwt}. 2^{qrs}. 9^{lb} gros, tare 35^{lb} and allowing tret to each as usual? *Ans.* 17^{cwt}. 0^{qrs}. 19^{lb} +

CASE IV.

When tare, tret and cloff are allowed.—Deduct the tare and tret as before, and divide the futtle by 168, and the quotient will be the cloff, which subtract from the futtle, and the remainder will be the neat.

EXAMPLES.

1st. What is the neat weight of 1^{hhd}. of tobacco, weighing 16^{cwt}. 2^{qrs}. 20^{lb} gros, tare 14^{lb} per cwt. tret 4^{lb} per 104, and cloff 2^{lb} per 3^{cwt}.?

	<i>Cwt.</i>	<i>qrs.</i>	<i>lb</i>	<i>oz.</i>	
14 ^{lb} is $\frac{1}{8}$)	16	2	20	0	<i>gros.</i>
	2	0	9	8	<i>tare, subtract.</i>

4 ^{lb} is $\frac{1}{26}$)	14	2	10	8	
	0	2	6	13	<i>tret, subtract.</i>

2 ^{lb} is $\frac{1}{168}$)	14	0	3	11	<i>futtle.</i>
	0	0	9	5	<i>cloff, subtract.</i>

Ans. 13 3 22 6 neat.

2d. If 9^{hhd}s. of tobacco, contain 85^{cwt}. 0^{qrs}. 2^{lb}, tare 30^{lb} per ^{hhd}. tret and cloff as usual, What will the neat weight come to at 6 $\frac{1}{2}$ ^{d.} per ^{lb}, after deducting for duties and other charges, 5^{l.} 11^{s.} 8^{d.}? *Ans.* 187^{l.} 18^{s.} 5^{d.}

INVOLUTION, OR TO RAISE POWERS.

A power is the product arising from multiplying any given number into itself continually a certain number of times, thus:

$3 \times 3 = 9$ is the 2d. power, or square of 3. $= 3^2$

$3 \times 3 \times 3 = 27$ is the 3d. power, or cube of 3. $= 3^3$

$3 \times 3 \times 3 \times 3 = 81$ is the 4th. power, or the biquadrate of 3, &c. $= 3^4$

The number denoting the power is called the *index*, or the *exponent* of that power. Thus, the fourth power of 3 is 81, or 3^4 ; the second power of 5 is 25, or 5^2 , &c.

$2 \times 2 = 4$, the square of 2; $4 \times 4 = 16 = 4^{\text{th}}$ power of 2; $16 \times 16 = 256 = 8^{\text{th}}$ power of 2, &c.

RULE.

Multiply the given number, root, or first power continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

Note. Whence, because fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved by raising each of their terms to the power required, and if a mixed number be proposed, either reduce it to an improper

fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

E X A M P L E S.

1st. What is the 5th power of 9?

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 = 2d. \text{ power.} \end{array}$$

$$\begin{array}{r} 9 \\ \hline 729 = 3d. \text{ power.} \end{array}$$

$$\begin{array}{r} 9 \\ \hline 6561 = 4th. \text{ power.} \end{array}$$

$$59049 = 5th. \text{ power, or answer} = 9^5.$$

2d. What is the 5th. power of $\frac{3}{5}$? *Anf.* $\frac{243}{3125}$.

3d. What is the 4th. power of .045? *Anf.* .00004100625.

Here we see, that in raising a fraction to a higher power, we decrease its value.

EVOLUTION, OR THE EXTRACTION OF ROOTS.

The Root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d. 3d. 4th. root, &c. accordingly as it is, when raised to the 2d. 3d. 4th. &c. power, equal to that power. Thus, 4 is the square root of 16, because $4 \times 4 = 16$, and 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$; and so on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which precise roots can never be determined. But, by the help of decimals, we can approximate towards the root, to any assigned degree of exactness.

The roots, which approximate, are called *furd roots*, and those which are perfectly accurate, are called *rational roots*.

Roots are sometimes denoted by writing the character $\sqrt{}$ before the power, with the index of the root over it; thus the 3d. root of 36 is expressed $\sqrt[3]{36}$, and the 2d. root of 36 is $\sqrt{36}$, the index 2 being omitted when the square root is designed.

If the power be expressed by several numbers, with the sign + or — between them, a line is drawn from the top of the sign over all the parts of it, Thus the 3d. root of $42 + 22$ is $\sqrt[3]{42+22}$, and the 2d. root of $59 - 17$ is $\sqrt{59-17}$, &c.

Sometimes roots are designed like powers, with fractional indices. Thus, the square root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and 4th. root of $37 - 20$ is $37-20^{\frac{1}{4}}$, &c.

A T A B L E o f P O W E R S.

Roots, - - -	or 1 st . Pow. 1	2	3	4	5	6	7	8	9
Squares, - -	or 2 ^d . Pow. 1	4	9	16	25	36	49	64	81
Cubes, - - -	or 3 ^d . Pow. 1	8	27	64	125	216	343	512	729
Biquadrates, -	or 4 th . Pow. 1	16	81	256	625	1296	2401	4096	6561
Surfolids, - -	or 5 th . Pow. 1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes, -	or 6 th . Pow. 1	64	729	4096	15625	46656	117649	262144	531441
Second Surfolids,	or 7 th . Pow. 1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Sqd.	or 8 th . Pow. 1	256	6561	65536	390625	1679616	5764801	16777216	43946721
Cubes Cubed, -	or 9 th . Pow. 1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surfolids Squared,	or 10 th . Pow. 1	1024	59049	1248576	9765625	60466176	282475249	1073741824	4386784401
Third Surfolids,	or 11 th . Pow. 1	2048	177147	4194304	48828125	362797056	1977326743	8389934592	31381059609
Square Cubes Sqd.	or 12 th . Pow. 1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
Fourth Surfolids,	or 13 th . Pow. 1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	254186588329
2d. Surfolids Sqd.	or 14 th . Pow. 1	16384	4782969	268435456	6103515625	78964164096	678223072849	4398046511104	22876792454961
Surfolids Cubed.	or 15 th . Pow. 1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

The EXTRACTION of the SQUARE ROOT.

R U L E.

*1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand, period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor: Multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend: To the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it) and from these, find the next figure in the root as last directed, and continue the operation, in the same manner, till you have brought down all the periods.

Note 1. If when the given power is pointed off as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

Note 2. If there be decimals in the given number, it must be pointed both ways from the place of units: If, when there are integers, the first period in the decimals be deficient, it may be completed

* In order to shew the reason of the rule, it will be proper to premise the following Lemma. The product of any two numbers can have, at most, but so many places of figures as are in both the factors, and at least but one less.

Demonstration. Take two numbers consisting of any number of places; but let them be the least possible of those places, viz. Unity with cyphers, as 100 and 10: Then their product will be 1 with so many cyphers annexed as are in both the numbers, viz. 1000; but 1000 has one place less than 100 and 10 together have: And since 100 and 10 were taken the least possible, the product of any other two numbers, of the same number of places, will be greater than 1000; consequently, the product of any two numbers can have, at least, but one place less than both the factors.

Again, take two numbers, of any number of places, which shall be the greatest possible of those places, as 99 and 9. Now, 99×9 is less than 99×10 ; but $99 \times 10 (= 990)$ contains only so many places of figures as are in 99 and 9; therefore, 99×9 , or the product of any other two numbers, consisting of the same number of places, cannot have more places of figures, than are in both its factors.

Corollary 1. A square number cannot have more places of figures than double the places of the root, and at least but one less.

Corollary 2. A cube number cannot have more places of figures than triple the places of the root, and at least but two less.

completed by annexing so many cyphers as the power requires : And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each ; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing cyphers.

E X A M P L E S.

1st. Required the square root of 30138696025 ?

$$\begin{array}{r} \dot{3}0\dot{1}3\dot{8}6\dot{9}6\dot{0}2\dot{5} (173605 \text{ the root.} \\ \hline \end{array}$$

$$\begin{array}{r} 1st. \text{ Divisor} = 27 \overline{)201} \\ \underline{189} \end{array}$$

$$\begin{array}{r} 2d. \text{ Divisor} = 343 \overline{)1238} \\ \underline{1029} \end{array}$$

$$\begin{array}{r} 3d. \text{ Divisor} = 3466 \overline{)20969} \\ \underline{20796} \end{array}$$

$$\begin{array}{r} 4th. \text{ Divisor} = 347205 \overline{)1736025} \\ \underline{1736025} \end{array}$$

2d. Required the square root of 575,5 ?

$$\begin{array}{r} \dot{5}75,5\dot{0} (23,98 + \text{ the root.} \\ \hline \end{array}$$

$$\begin{array}{r} 43 \overline{)175} \\ \underline{129} \end{array}$$

$$\begin{array}{r} 469 \overline{)4650} \\ \underline{4221} \end{array}$$

$$\begin{array}{r} 4788 \overline{)42900} \\ \underline{38304} \end{array}$$

4596 Remainder.

3d. What is the square root of 10342656 ? *Ans.* 3216.

4th. What is the square root of 964,5192360241 ? *Ans.* 31,05671.

5th. What is the square root of 234,09 ? *Ans.* 15,3.

6th. What is the square root of ,0000316969 ? *Ans.* ,00563.

7th. What is the square root of ,045369 ? *Ans.* ,213.

RULES for the SQUARE ROOT of VULGAR FRACTIONS and MIXED NUMBERS.

After reducing the fraction to its lowest terms, for this and all other roots ; then,

1st. Extract

1st. Extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator, which is the best method, provided the denominator be a complete power. But if it be not,

2d. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required.* Or,

3d. Reduce the vulgar fraction to a decimal, and extract its root.

4th. Mixed numbers may either be reduced to improper fractions, and extracted by the first or second rule, or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

EXAMPLES.

1st. What is the square root of $\frac{144}{15129}$?

By Rule 1.

$$\frac{144}{15129} = \frac{16}{1681}$$

16 (4 root of the numerator.
16

1681 (41 root of the denominator.
16

81)81 Therefore, $\frac{4}{41}$ = the root of the given fraction.
81

By Rule 2.

$16 \times 1681 = 26896$, and $\sqrt{26896} = 164$. Then,

$$\frac{164}{1681} = \frac{16}{164} = \frac{4}{41} = .09756+$$

By Rule 3.

$1681)16(.0095181439+$. And $\sqrt{.0095181439} = .09756+$.

2d. What is the square root of $\frac{2703}{8208}$?

Ans. $\frac{7}{12}$.

3d. What is the square root of $4\frac{3}{4}$?

Ans. $6\frac{1}{2}$.

Note. In extracting the square or cube root of any furd number, there is always a remainder or fraction left, when the root is found. To find the value of which, the common method is, to annex pairs of cyphers to the resolvend, for the square, and ternaries of cyphers to that of the cube, which makes it tedious to discover the value of the remainder, especially in the cube, whereas this trouble might be saved if the true denominator could be discovered.

Now,

* That is, suppose $a = 7$, and $b = 2$. the rule may be thus expressed: $\sqrt{\frac{a}{b}} =$

$\frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}$. Or, numerically thus: $\sqrt{\frac{7}{2}} = \frac{\sqrt{7 \times 2}}{2} = \frac{7}{\sqrt{7 \times 2}} = 1.87+$, and this rule will serve whether the root be finite or infinite.

Now, all numbers whatever, which are to be extracted by the square or cube, contain in themselves their own fractions, and their own denominators; but neither of them can be known till an operation has taken place, when they are separated, and the fractions plainly appear. For, as in division the divisor is always the denominator to its own fraction, so likewise it is in the square and cube, each of their divisors being the denominators to their own particular fractions or numerators.

In the square, the quotient is always doubled for a new divisor; therefore, when the work is completed, the root doubled is the true divisor or denominator to its own fraction; as, if the root be 12, the denominator will be 24, to be placed under the remainder, which vulgar fraction, or its equivalent decimal, must be annexed to the quotient, or root, to complete it.*

If to the remainder, either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

APPLICATION AND USE OF THE SQUARE ROOT.

PROB. I. *To find a mean proportional between two numbers.*

RULE. Multiply the given numbers together, and extract the square root of the product; which root will be the mean proportional sought.

EXAMPLE.

What is the mean proportional between 24 and 96?

$$\sqrt{96 \times 24} = 48 \text{ Answer.}$$

PROB. II. *To find the side of a square equal in area to any given superficies whatever.*

RULE. Find the area, and the square root is the side of the square sought.

EXAMPLES.

1st. If the area of a circle be 184,125, What is the side of a square equal in area thereto?

$$\sqrt{184,125} = 13,569 + \text{Answer.}$$

2d. If the area of a triangle be 160, What is the side of a square equal in area thereto?

$$\sqrt{160} = 12,649 + \text{Answer.}$$

PROB. III. A certain general has an army of 5625 men; pray How many must he place in rank and file, to form them into a square?

$$\sqrt{5625} = 75 \text{ Answer.} +$$

PROB. IV.

* Although these denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use, and are much more expeditious than the operation with cyphers.

+ If you would have the number of men be double, triple, or quadruple, &c. as many in rank as in file, extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c. and the product will be the number in rank.

PROB. IV. Let 10952 men be so formed, as that the number in rank may be double the file.

$$\sqrt{\frac{10952}{2}} = 74 \text{ in file, and } 74 \times 2 = 148 \text{ in rank.}$$

PROB. V. If it be required to place 2016 men so as that there may be 56 in rank, and 36 in file, and to stand 4 feet distance in rank, and as much in file, How much ground do they stand on?

To answer this, or any of the kind, use the following proportion: As unity † to the distance :: so is the number in rank less by one † to a fourth number; next, do the same by the file, and multiply the two numbers together, found by the above proportion, and the product will be the answer.*

As $1 : 4 :: 56 - 1 : 220$. And, as $1 : 4 :: 36 - 1 : 140$. Then, $220 \times 140 = 30800$ square feet, *the Answer*.

PROB. VI. Suppose I would set out an orchard of 600 trees, so that the length shall be to the breadth as 3 to 2, and the distance of each tree, one from the other, 7 yards; How many trees must it be in length, and how many in breadth? and, How many square yards of ground do they stand on?

To resolve any question of this nature, say, as the ratio in length † is to the ratio in breadth :: so is the number of trees † to a fourth number, whose square root is the number in breadth. And as the ratio in breadth † is to the ratio in length :: so is the number of trees † to a fourth, whose root is the number in length.

As $3 : 2 :: 600 : 400$. And $\sqrt{400} = 20 = \text{number in breadth}$.

As $2 : 3 :: 600 : 900$. And $\sqrt{900} = 30 = \text{number in length}$.

As $1 : 7 :: 30 - 1 : 203$. And, as $1 : 7 :: 20 - 1 : 133$. And $203 \times 133 = 26999$ square yards, *the Answer*.

PROB. VII. Admit a leaden pipe $\frac{3}{4}$ inch diameter will fill a cistern in 3 hours; I demand the diameter of another pipe which will fill the same cistern in 1 hour.

RULE.† As the given time is to the square of the given diameter, so is the required time † to the square of the required diameter.

$\frac{3}{4} = .75$; and, $.75 \times .75 = .5625$. Then, as $3h. : .5625 :: 1h. : 1.6875$ *inversely*, and $\sqrt{1.6875} = 1.3$ *inch nearly, Ans.*

PROB. VIII. If a pipe, whose diameter is 1.5 inch, fill a cistern in 5 hours, In what time will a pipe, whose diameter is 3.5 inches fill the same?

1.5

* The above rule will be found useful in planting trees, having the distance of ground between each given.

† The areas of circles are to one another as the squares of their diameters.

The area of a circle is the same as the superficial content.

The circumference and periphery are the same thing.

The diameter of a circle is a line drawn through the centre from one side of the circumference to the other, and is the longest line that can be drawn across a circle.

The centre of a circle is the point round which the circle is described.

Note. For the measuring of a circle and its parts, see Mensuration of Superficies, from Art. 12th to Art. 26th and the Note.

$1,5 \times 1,5 = 2,25$; and $3,5 \times 3,5 = 12,25$. Then, as $2,25 : 5 :: 22,25 : 9183$ hour, *inversely*, $= 55$ min. 5 sec. *Answer*.

PROB. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water, In what time will 3 pipes, each 4 inches bore, be in discharging double the quantity?

$6 \times 6 = 36$. $4 \times 4 = 16$, and $16 \times 3 = 48$. Then, as $36 : 4h. :: 48 : 3h.$ *inversely*, and as $1w. : 3h. :: 2w. : 6h.$ *Answer*.

PROB. X. Given the diameter of a circle to make another circle, which shall be 2, 3, 4, &c. times greater or less than the given circle.

RULE. Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the root of the product will be the required diameter. But if the required circle be less, divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle whose diameter is 4 inches; I demand the diameter of a circle 3 times as large?

$4 \times 4 = 16$; and $16 \times 3 = 48$; and $\sqrt{48} = 6,928 +$ inches, *Ans*.

PROB. XI. To find the diameter of a circle, equal in area, to an ellipsis, (or oval) whose transverse and conjugate diameters are given.*

RULE. Multiply the two diameters of the ellipsis together, and the square root of that product will be the diameter of a circle, equal to the ellipsis.

Let the transverse diameter of an ellipsis be 48, and the conjugate, 36; What is the diameter of an equal circle?

$48 \times 36 = 1728$, and $\sqrt{1728} = 41,569 +$ the *Ans*.

Note. The square of the hypotenuse, or the longest side of a right angled triangle, (by 47th B. 1. *Euc.*) is equal to the sum of the squares of the other two sides; and consequently the difference of the squares of the hypotenuse and either of the other sides is the square of the remaining side.

PROB. XII. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad. The height of the wall is required?

$36 \times 36 = 1296$; and $24 \times 24 = 576$. Then, $1296 - 576 = 720$, and $\sqrt{720} = 26,83 +$ yards, the *Answer*.

PROB. XIII. The height of a tree, growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet. What is the breadth of the stream, provided the land on each side of the water be level?

* The transverse and conjugate are the longest and shortest diameters of an ellipsis; they pass through the centre, and cross each other at right angles.

$256 \times 256 = 65536$; and $75 \times 75 = 5625$: Then $65536 - 5625 = 59911$ and $\sqrt[3]{59911} = 24476 +$ and $24476 - \frac{44}{2} = 22276$ feet, *Anf.*

PROB. XIV. Suppose a ladder 60 feet long be so planted as to reach a window 37 feet from the ground, on one side of the street, and without moving it at the foot, will reach a window 23 feet high on the other side ; I demand the breadth of the street ?

$60 \times 60 = 3600$. $37 \times 37 = 1369$. $23 \times 23 = 529$: Then, $3600 - 1369 = 2231$, and $\sqrt[3]{2231} = 47.23 +$, and $3600 - 529 = 3071$, and $\sqrt[3]{3071} = 55.41 +$, then, $47.23 + 55.41 = 102.64$ feet, the answer.

PROB. XV. Two ships sail from the same port ; one goes due north 45 leagues, and the other due west 76 leagues : How far are they asunder ?*

$45 \times 45 = 2025$. $76 \times 76 = 5776$. Then, $5776 + 2025 = 7801$ and $\sqrt[3]{7801} = 88.32$ leagues, the answer.

EXTRACTION of the CUBE ROOT.

A Cube is any number multiplied by its square. To extract the cube root, is to find a number which, being multiplied into its square, shall produce the given number.

FIRST METHOD.

R U L E.

1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend*.

4. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor*.

5. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. Multiply the triple square by the last quotient figure, and write the product under the dividend ; multiply the square of the last

* The square root may in the same manner be applied to navigation ; and, when deprived of other means of solving problems of that nature, the following proportion will serve to find the course.

As the sum of the hypotenuse (or distance) and half the greater leg (whether difference of latitude or departure) is to the less leg ; so is 86, to the angle opposite the less leg.

† The reason of pointing the given number, as directed in the rule, is obvious from *Coroll. 2*, to the *Lemma* made use of in demonstrating the square root.

last quotient figure by the triple-quotient, and place this product under the last ; under all, set the cube of the last quotient figure and call their sum the *subtrahend*.

7. Subtract the subtrahend from the dividend, and to the remainder, bring down the next period for a new dividend with which proceed as before, and so on till the whole be finished.

Note. The same rule must be observed for continuing the operation, and pointing for decimals, as in the square-root.

EXAMPLES.

1st. Required the cube root of 436036824287 ?

436036824287	(7583 root.	$7 \times 7 \times 300 =$	14700	= 1st. Trip. sq.
343		$7 \times 30 =$	210	= 1st. do. quo.
<hr/>				
1st. Divf. = 14910	93036 = 1st. Dividend.		14910	= 1st. Divisor.
	73500	$14700 \times 5 =$	73500	
	5250	$5 \times 5 \times 210 =$	5250	
	125	$5 \times 5 \times 5 =$	125	
<hr/>				
	78875	= 1st. Subtrahend.	78875	= 1st. Subtra.
<hr/>				
2d. Div. = 1689750	14161824 = 2d. Divid.	$75 \times 75 \times 300 =$	1687500	= 2d. Tri. sq.
		$75 \times 30 =$	2250	= 2d. do. quo.
	13500000			
	144000			
	512			
<hr/>				
	13644512	= 2d. Subtra.	1689750	= 2d. Divif.
<hr/>				
3d. Div. = 172391940	517312287 = 3d. Divid.	$1687500 \times 8 =$	13500000	
		$2250 \times 8 \times 8 =$	144000	
		$8 \times 8 \times 8 =$	512	
<hr/>				
	517107600		13644512	= 2d. Subtra.
	204660	$758 \times 758 \times 300 =$	172369200	= 3d. Trip. sq.
	27	$758 \times 30 =$	22740	= 3d. do. quo.
<hr/>				
	517313287	= 3d. Subtrah.	172391940	= 3d. Divifor.
<hr/>				
.....		$172369200 \times 3 =$	517107600	
		$22740 \times 3 \times 3 =$	204660	
		$3 \times 3 \times 3 =$	27	
<hr/>				
	517312287	= 3d. Subtra.		

2d. What is the cube root of 34965783 ?

Ans. 327.

3d. What is the cube root of 84,604519 ?

Ans. 4,39.

4th. What is the cube root of ,008649 ?

Ans. ,2052+.

5th. What is the cube root of $\frac{125}{343}$?

Ans. $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finished.

In the extraction of the cube root, the quotient is said to be squared and tripled for a new divisor ; but is not really so, till the triple number of the quotient be added to it, therefore when the operation is finished, it is but squaring the quotient, or root, then

then multiplying it by 3, and to that number adding the triple number of the root, when it will become the divisor, or true denominator to its own fraction, which fraction must be annexed to the quotient, to complete the root.

Suppose the root to be 12, when squared it will be 144, and multiplied by 3, it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.*

SECOND METHOD.

R U L E.

1. Having pointed the given number into periods of three figures each, find the greatest cube in the left hand period, subtracting it therefrom and placing its root in the quotient; to the remainder bring down the next period and call it the *dividend*.

2. Under this dividend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former; and under the said triple square, write the triple root, removed one place to the right hand; and call the sum of these the *divisor*.

3. Seek how often the divisor may be had in the dividend, exclusive of the place of units, and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient-figure, setting down the unit's place of this line, under the place of tens in the divisor; under this line, write the product of the triple root by the square of the last quotient-figure, so as to be removed one place beyond the right hand figure of the former; and, under this line, removed one place forward to the right hand, write down the cube of the last quotient-figure, and call their sum the *Subtrahend*.

5. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on until the whole be finished.

EXAMPLE.

* It may not be amiss to remark here, that the denominators, both of the square and cube, shew how many numbers they are denominators to, that is, what numbers are contained between any square or cube number and the next succeeding square or cube number, exclusive of both numbers, for a complete number, of either, leaves no fraction, when the root is extracted, and consequently has no use for a denominator, but all the numbers contained between them have occasion for it:—Suppose the square root to be 12, then its square is 144, and the denominator 24, which will be a denominator to all the succeeding numbers, until we come to the next square number, *viz.* 169, whose root is 13, with which it has nothing to do, for between the square numbers 144 and 169 are contained 24 numbers, excluding both the square numbers. It is the same in the cube; for, suppose the root to be 6, the cube number is 216, and its denominator 126 will be a denominator to all the succeeding numbers, until we come to the next cube number, *viz.* 343, whose root is 7, with which it has nothing to do, as ceasing then to be a denominator; for between the cube 343 and 216 are 126 numbers, excluding both cubes. And so it is with all other denominators, either in the square or cube.

EXAMPLE.

Required the Cube Root of 16194277?

16194277 (253 = Root.

8

8194 = First dividend.

12 = Triple square of 2.

6 = Triple of 2.

126 = First divisor.

60 = Triple square of 2 multiplied by 5.

150 = Triple of 2 multiplied by the square of 5.

125 = Cube of 5.

7625 = First Subtrahend.

569277 = Second dividend.

1875 = Triple square of 25.

75 = Triple of 25.

18825 = Second divisor.

5625 = Triple square of 25 multiplied by 3.

675 = Triple of 25 multiplied by the square of 3.

27 = Cube of 3.

569277 = Second Subtrahend.

FIRST METHOD by APPROXIMATION.

RULE.

1. Find, by trial, a cube near to the given number, and call it the *supposed* cube.

2. Then, as twice the *supposed* cube, added to the given number, is to twice the given number, added to the *supposed* cube, so is the root of the *supposed* cube, to the true root, or an approximation to it.

3. By taking the cube of the root, thus found, for the *supposed* cube, and repeating the operation, the root will be had to a greater degree of exactness.

EXAMPLES.

It is required to find the cube root of 54854153?

Let

EXTRACTION OF THE CUBE ROOT.

Let $64000000 =$ supposed cube, whose root is 400;

Then, 64000000 54854153

2

2

 128000000

 109708306

 54854153

 64000000

As $182854153 : 173708306 :: 400$

 400

$182854153)69483322400(379 = \text{root nearly.}$

Again, let $54439939 =$ supposed cube, whose root is 379.

Then, 54439939 54854153

2

2

 108879878

 109708306

 54854153

 54439939

As $163734031 : 164148245 :: 379$

 379

 1477334205

 1149037715

 492444735

$163734031)62212184855(379,958793+ =$ *refined.* root cor-

SECOND METHOD by APPROXIMATION.

RULE.

1. Divide the resolvend by three times the assumed root, and reserve the quotient.
2. Subtract one twelfth part of the square of the assumed root from the quotient.
3. Extract the square root of the remainder.
4. To this root add one half of the assumed root, and the sum will be the true root, or an approximation to it; take this approximation as the assumed root, and, by repeating the process, a root farther approximated will be found, which operation may be farther repeated, as often as necessary, and the root discovered to any assigned exactness.

Note. In order to find the value of the first assumed root, in this or any other power, divide the resolvend into periods by beginning at the place of units, and including in each period, so many figures as there are units in the exponent of the root; viz. 3 figures in the cube root; 4 for the biquadrate, and so on; then, by a table of powers, or otherwise, find a figure, which (being involved to the power whose exponent is the same with that

of

of the required root) is the nearest to the value of the first period of the resolvend at the left hand, and to that figure annex so many cyphers as there are periods remaining in the integral part of the resolvend; this figure, with the cyphers annexed, will be the assumed root, and equal to r in the theorem; and it is of no importance whether the figure thus chosen be, when involved, greater or less than the left hand period, as the theorem is the same in both cases.

1st. What is the cube root of 436036824287 ?

$7000 =$ assumed root.

3

$21,000)436036824287(20763658,2994$
Subtract $7000 \times 7000 \div 12 = 4083333,3333$

$\sqrt{16680324,9661} = 4084,15$

Add $\frac{1}{2}$ the assumed root $= 3500$

And it gives the approximated root $= 7584,15$

For the second operation, use the approximated root as the assumed one, and proceed as above.

THIRD METHOD by APPROXIMATION.

1. Assume the root in the usual way, then multiply the square of the assumed root by 3, and divide the resolvend by this product; to this quotient add $\frac{2}{3}$ of the assumed root, and the sum will be the true root, or an approximation to it.

2. For each succeeding operation let the last approximated root be the assumed root, and, proceeding in this manner, the root may be extracted to any assigned exactness.

1st. What is the cube root of 7?

Let the assumed root be 2. Then, $2 \times 2 \times 3 = 12$ the divisor.

$12)7,0(,583$ to this add $\frac{2}{3}$ of 2 $= 1,333$, &c. that is, $,583 + 1,333 = 1,916$ approximated root.

Now assume 1,916 for the root. Then, by the second process, the root is $\frac{7}{3 \times 1,916} 2 + \frac{2}{3} \times 1,916 = 1,9126$, &c.

2d. What is the cube root of 9? Let 2 be the assumed root as before. Then, $\frac{9}{12} + \frac{2}{3} \times 2 = 2,08$ the approximated root. Now assume 2,08. Then, $\frac{9}{3 \times 2,08} 2 + \frac{2}{3} \times 2,08 = 2,08008$, &c.

3d. What is the cube root of 282?—Let 6 be the assumed root. Then, $6 \times 6 \times 3 = 108)282(2,611$, &c. and $2,611 + \frac{2}{3}$ of 6 $= 6,611$ approximated root. Now assume 6,611, and it will be $6,611 \times 6,611 \times 3 = 131,116)282(2,1507$, &c. and $2,1507 + \frac{2}{3}$ of 6,611 $= 6,558$ a farther approximated root.

D d

4th. What

210 APPLICATION AND USE OF THE CUBE ROOT.

4th. What is the cube root of 1728?—Here the assumed root is 10. Then, $10 \times 10 \times 3 = 300$ 1728(5,76, and $5,76 + \frac{2}{3}$ of 10 = 12,426.——Now assume 12,426, then $12,426 \times 12,426 \times 3 = 463,216428$ 1728(3,732, and $3,732 + \frac{2}{3}$ of 12,426 = 12,014 a farther approximated root, and so on.

APPLICATION and USE of the CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

RULE.—1. Divide the *greater* by the *less*, and extract the cube root of the quotient.

2. Multiply the root, so found, by the *least* of the given numbers, and the product will be the *least*.

3. Multiply this product by the same root, and it will give the *greatest*.

E X A M P L E S.

1st. What are the two mean proportionals between 6 and 750?

$750 \div 6 = 125$, and $\sqrt[3]{125} = 5$. Then, $5 \times 6 = 30 = \text{least}$, and $30 \times 5 = 150 = \text{greatest}$. *Ans.* 30 and 150.

Proof. As $6 : 30 :: 150 : 750$.

2d. What are the two mean proportionals between 56 and 12096? *Ans.* 336 and 2016.

Note. The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

3d. If a bullet 6 inches diameter weigh 32lb, What will a bullet of the same metal weigh, whose diameter is 3 inches?

$6 \times 6 \times 6 = 216$. $3 \times 3 \times 3 = 27$. As $216 : 32\text{lb} :: 27 : 4\text{lb}$ *Ans.*

4th. If a globe of silver of 3 inches diameter, be worth £45, What is the value of another globe, of a foot diameter?

$3 \times 3 \times 3 = 27$. $12 \times 12 \times 12 = 1728$. As $27 : 45 :: 1728 : £2880$ *Ans.*

The side of a cube being given, to find the side of that cube which shall be double, triple, &c. in quantity to the given cube.

RULE.—Cube your given side, and multiply it by the given proportion between the given and required cube, and the Cube root of the product will be the side sought.

5th. If a cube of silver, whose side is 4 inches, be worth £50, I demand the side of a cube of the like silver, whose value shall be 4 times as much?

$4 \times 4 \times 4 = 64$, and $64 \times 4 = 256$. $\sqrt[3]{256} = 6,349 + \text{inches}$, *Ans.*

6th. There is a cubical vessel, whose side is 2 feet, I demand the side of a vessel, which shall contain three times as much?

$2 \times 2 \times 2 = 8$, and $8 \times 3 = 24$. $\sqrt[3]{24} = 2,884 = 2\text{ft. } 10\frac{3}{4}\text{inch}$. *Ans.*

7th. The diameter of a bushel measure being $18\frac{1}{2}$ inches, and the height 8 inches, I demand the side of a cubic box, which shall contain that quantity? *Ans.* 12,908 inches.

8th. Suppose

EXTRACTION OF THE BIQUADRATE ROOT. 211

8th. Suppose a ship of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold ; What are the dimensions of a ship of 200 tons, of the same mould and shape ?

$$89 \times 89 \times 89 = 704969 = \text{cubed keel.}$$

As 500 : 200 :: 704969 : 281987,6 cube of the required keel.

$$\sqrt[3]{281987,6} = 65,57 \text{ feet the required keel.}$$

As 89 : 65,57 :: 36 : 26,522 = 26½ feet, beam, nearly.

As 89 : 65,57 :: 16 : 11,7 feet, depth of the hold.

9th. From the proof of any cable to find the strength of any other.

RULE.—The strength of cables, and consequently the weights of their anchors, are as the cubes of their peripheries.

If a cable, 12 inches about, require an anchor of 18 Cwt. Of what weight must an anchor be, for a 15 inch cable ?

Cwt.

Cwt.

$$\text{As } 12 \times 12 \times 12 : 18 :: 15 \times 15 \times 15 : 35,15625 \text{ Ans.}$$

10th. If a 15 inch cable require an anchor 35,15625 Cwt. : What must the circumference of a cable be, for an anchor of 18 Cwt. ?

$$\text{As } 35,15625 : 15 \times 15 \times 15 :: 18 : 1728, \text{ and } \sqrt[3]{1728} = 12 \text{ Ans.}$$

EXTRACTION of the BIQUADRATE ROOT.

R U L E.

Extract the square root of the resolvend, and then, the square root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736 ?

$$\begin{array}{r} \cdot \cdot \cdot \\ 20736(144 \\ 1 \end{array}$$

$$\begin{array}{r} 24)107 \\ 96 \end{array}$$

$$\begin{array}{r} 284)1136 \\ 1136 \end{array}$$

$$\begin{array}{r} \cdot \cdot \\ 144(12 \text{ root required.} \\ 1 \end{array}$$

$$\begin{array}{r} 22)44 \\ 44 \end{array}$$

$$\cdot \cdot$$

Two Methods of extracting the Biquadrate Root by Approximation, according to the two General Theorems for extracting the roots of all powers, in pages 214 and 216.

R U L E I.

1. Divide the resolvend by six times the square of the assumed root, and from the quotient subtract $\frac{1}{18}$ part of the square of the assumed root.

2. Extract the square root of the remainder.

3. Add $\frac{2}{3}$ of the assumed root to the square root, and the sum will be the true root, or an approximation to it.

4. For

212 EXTRACTION OF THE SURSOLID ROOT.

4. For every succeeding operation) either in this or the following method) proceed in the same manner, as in the first, each time using the last approximated root for the assumed root,

The biquadrate root of 20736 is required.

Here 10 is the assumed root.

$$10 \times 10 \times 6 = 600 \quad 20736(34,56$$

$$\text{Subtract. } 10 \times 10 \div 18 = 5,5555$$

$$\sqrt{29,0044} = 5,38$$

$$\text{Add } \frac{2}{3} \text{ of } 10 = 6,66$$

Approximated root 12,04, To be made the assumed root for the next operation.

R U L E II.

Divide the resolvend by *four* times the cube of the assumed root : To the quotient add *three fourths* of the assumed root, and the sum will be the true root, or an approximation to it.

Let the biquadrate of 20736 be required, as before ?

The assumed root is 10.

$$10 \times 10 \times 10 \times 4 = 4000 \quad 20736(5,184$$

$$\text{Add } \frac{3}{4} \text{ of } 10 = 7,5$$

Approximated root 12,684, To be made the assumed root for the next operation.

EXTRACTION of the SURSOLID ROOT by APPROXIMATION.

A particular R U L E.*

1. Divide the resolvend by *five* times the assumed root, and to the quotient add *one twentieth* part of the *fourth* power of the same root.

2. From the square root of this sum subtract *one fourth* part of the square of the assumed root.

3. To the square root of the remainder add *one half* of the assumed root, and the sum is the root required, or an approximation to it.

Note. This Rule will give the root true to *five* places, at the least, (and generally to eight or nine places) at the first process.

Required

$$* r \pm e = \sqrt{\sqrt{\frac{G}{6r} + \frac{r^4}{20} - \frac{rr}{4} + \frac{r}{2}}}$$

Required the sursolid root of 281950621875^P

$200 = \text{assumed root,}$

5

$1000 \overline{) 281950621875} \text{ quotient.}$
 $\text{Add } 200 \times 200 \times 200 \times 200 \div 20 = 80000000$

$\sqrt[5]{361950621875} = 19025 \text{ nearly.}$
 Subtract $200 \times 200 \div 4 = 10000$

$\sqrt[5]{9025} = 95$
 Add half the assumed root $= 100$

Required root 195

A General RULE for EXTRACTING the ROOTS of all POWERS.

R U L E.

*1. Prepare the given number, for extraction, by pointing off from the unit's place, as the required root directs.

2. Find the first figure of the root by trial, or by inspection into the table of powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the *dividend*.

4. Involve the root to the next inferior power to that which is given, and multiply it by the number, denoting the given power, for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the *given number*, as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and, in like manner, proceed till the whole be finished.

EXAMPLES.

* The extracting of roots of very high powers will, by this rule, be a tedious operation:—The following method, when practicable, will be much more convenient.

When the index of the power, whose root is to be extracted, is a composite number, take any two or more *indices*, whose product is equal to the given *index*, and extract out of the given number a root answering to another of the indices, and so on to the last.

Thus, the fourth root = square root of the square root;—the sixth root = square root of the cube root;—the eighth root = square root of the fourth root;—the ninth root = the cube root of the cube root;—the tenth root = square root of the fifth root;—the twelfth root = cube root of the fourth, &c.

214 EXTRACTION OF ROOTS BY APPROXIMATION.

E X A M P L E S.

1st. What is the cube root of 20346417 ?

$$\begin{array}{rcl}
 20346417 & \overset{273}{=} & 2 \times 2 \times 2 = 8 \text{ root of the 1st. period, or 1st. Subtrahend.} \\
 2^3 = 8 & \overset{1st. Subtrah.}{=} & 2 \times 2 = 4 (= \text{next inferior power,}) \text{ and,} \\
 & & 4 \times 3 \text{ (the index of the given pow.)} = 12 \text{ 1st. divisor.} \\
 2^2 \times 3 = 12 & \overset{123}{=} & \text{Dividend.} \\
 & & 27 \times 27 \times 27 = 19683 = 2d. \text{ Subtrahend.} \\
 27^3 = 19683 & \overset{2d. Subtr.}{=} & 27 \times 27 = 729 \text{ (next inferior power) and,} \\
 & & 729 \times 3 (= \text{index of the given power}) = 2187 = 2d. \text{ divisor.} \\
 27^2 \times 3 = 2187 & \overset{6634}{=} & 2d. \text{ Div.} \\
 & & 273 \times 273 \times 273 = 20346417 = 3d. \text{ Subtrahend.} \\
 273^3 = 20346417 & \overset{3d. Subt.}{=} & \text{.....}
 \end{array}$$

2d. What is the biquadrate root of 34827998976 ? *Ans.* 431,94.

3d. Extract the fursolid, or fifth root of 281950621875 ?

Ans. 195.

4th. Extract the square cubed, or sixth root of 1178420166015625 ?

Ans. 325

A General Rule for EXTRACTING ROOTS by APPROXIMATION.

R U L E.*

1. Subtract *one* from the exponent of the root required, and multiply half of the remainder by the same exponent, and farther

* The general theorem for the extraction of all roots, by approximation, from whence the rule was taken, and the Theorems deducible from it, as high as the twelfth power. Let G = resolvent whose root is to be extracted. $r \pm e$ = root required ; r being assumed, as near the true root, and m = exponent of the power—then the equation will stand thus,

$$r \pm e = \sqrt[m \times \frac{m-1}{2}]{G} - \frac{m \times m-3+2}{m \times m-1} \frac{rr}{3} : + \frac{m-2}{m-1} r. \text{ Hence,}$$

Theorem for the cube root. $r \pm e = \sqrt[3]{\frac{G}{3r}} - \frac{rr}{12} + \frac{r}{2}$

For the Biquadrate - - $\sqrt[4]{\frac{G}{6rr}} - \frac{rr}{18} + \frac{2r}{3}$

For the Surfsolid - - - $\sqrt[5]{\frac{G}{10r^3}} - \frac{3rr}{80} + \frac{3r}{4}$

For the squared cube root $\sqrt[6]{\frac{G}{15r^4}} - \frac{2rr}{75} + \frac{4r}{5}$ (See next page.)

§ Or, by the following Theorem in which the fraction is obtained in numbers to the lowest terms in all the odd powers ; and in the even powers only by halving the numerator and denominator found by this equation.

$$r \pm e = \sqrt[m-1]{\frac{G}{\frac{m-1}{2} m^{m-2}}} - \frac{m-2r^2}{m^{m-1} 2} + \frac{m-2}{m-1} r.$$

EXTRACTION OF ROOTS BY APPROXIMATION. 215

ther multiply this product by that power of the assumed root, whose exponent is *two* less than the exponent of the root required, and with this last product divide the resolvend and reserve the quotient.

2. From this quotient is to be subtracted the following number, viz. the square of the assumed root multiplied by a fraction whose numerator is found by subtracting *three* from the exponent of the root, multiplying the remainder by that exponent, and adding *two* to the product; and the denominator is found by subtracting *one* from that exponent, and multiplying the cube of the remainder by the same exponent.

3. After this subtraction is made, extract the square root of the remainder.

4. Multiply the assumed root by a fraction whose numerator is the remainder after *two* is subtracted from the exponent, and the denominator is the remainder after *one* is subtracted from the exponent: Add this product to that square root and the sum is the root required, or an approximation to it.

EXAMPLE.

What is the square cubed root of 1178420166015625?

Let the assumed root = 300.

Exponent of the required root is 6. Then, $\frac{6-1}{2} \times 6 = 15$.

$300^{15} = 8100000000$ and this multiplied by 15 = 121500000000.

$1178420166015625 \div 121500000000 = 9698,9314$, from this

Subtract $\frac{6-3 \times 6 + 2}{6-1 \times 6-1 \times 6-1 \times 6} \times 90000 = 2400$

And $\sqrt{7258,9314} = 85,43$

Add $\frac{6-2}{6-1} \times 300 =$

$\frac{240}{325,43}$ Approximated root =

For the 2d. operation, let 325,43 = assumed root.

Another

For the second sursolid

$$\sqrt{\frac{G}{21r^5} - \frac{5rr}{25^2} + \frac{5r}{6}}$$

For the squared Biquadrate

$$\sqrt{\frac{G}{28r^6} - \frac{3rr}{146} + \frac{6r}{7}}$$

For the cubed cube

$$\sqrt{\frac{G}{36r^7} - \frac{7rr}{576} + \frac{7r}{8}}$$

For the squared sursolid

$$\sqrt{\frac{G}{45r^8} - \frac{4rr}{405} + \frac{8r}{9}}$$

For the third sursolid

$$\sqrt{\frac{G}{55r^9} - \frac{9rr}{1100} + \frac{9r}{10}}$$

For the squared square cube

$$\sqrt{\frac{G}{66r^{10}} - \frac{5rr}{726} + \frac{10r}{11}}$$

216 EXTRACTION OF ROOTS BY APPROXIMATION.

Another METHOD by APPROXIMATION.*

R U L E.

1. Having assumed the root in the usual way, involve it to that power denoted by the exponent less 1.
2. Multiply this power by the exponent.
3. Divide the resolvend by this product, and reserve the quotient.
4. Divide the exponent of the given power, less 1, by the exponent, and multiply the assumed root by the quotient.
5. Add this product to the reserved quotient, and the sum will be the true root, or an approximation.
6. For every succeeding operation, let the root, last found, be the assumed root.

E X A M P L E.

What is the square cubed root of 1178420166015625?

The exponent is 6. Let the assumed root be 300.

Then, $300^5 \times 6 = 1458000000000$.

1458000000000)1178420166015625(80,824.

$$\text{Add } \frac{5}{6} \times 300 = 250$$

330,824 = approximated root.

For the next operation, let 330,824 be the assumed root.

of

* A rational formula for extracting the root of any pure power by approximation. Let the resolvend be called G , and let $r \pm e$ be the required root, r being assumed in the usual way.

Let $G^{\frac{1}{m}}$ be required; then $r \pm e = \frac{G}{m-1} + \frac{m-1}{m}r$, the general Theorem.

Hence, For the cube root $r \pm e = \frac{G}{3r^2} + \frac{2}{3}r$.

For the biquadrate $- - - \frac{G}{4r^3} + \frac{3}{4}r$.

For the sursolid $- - - \frac{G}{5r^4} + \frac{4}{5}r$.

For cube cubed $- - - \frac{G}{6r^5} + \frac{5}{6}r$.

For the seventh root $- - - \frac{G}{7r^6} + \frac{6}{7}r$.

For the eighth $- - - \frac{G}{8r^7} + \frac{7}{8}r$.

For the ninth $- - - \frac{G}{9r^8} + \frac{8}{9}r$.

For the tenth $- - - \frac{G}{10r^9} + \frac{9}{10}r$.

For the eleventh $- - - \frac{G}{11r^{10}} + \frac{10}{11}r$.

For the twelfth $- - - \frac{G}{12r^{11}} + \frac{11}{12}r$.

OF PROPORTION in GENERAL.

Numbers are compared together to discover the relations they have to each other.

There must be two numbers to form a comparison : The number, which is compared, being written first, is called the *antecedent* ; and that, to which it is compared, the *consequent*.

Numbers are compared with each other two different ways : The one comparison considers the *difference* of the two numbers, and is called arithmetical relation, the difference being sometimes named the arithmetical ratio ; and the other considers their *quotient*, which is termed geometrical relation, and the quotient, the geometrical ratio. Thus, of the numbers 12 and 4, the difference, or arithmetical ratio, is $12 - 4 = 8$; and the geometrical ratio is

$$\frac{12}{4} = 3.*$$

If two, or more, couplets of numbers have equal ratios, or differences, the equality is termed proportion ; and their terms similarly posited, that is, either all the greater, or all the less taken as antecedents, and the rest as consequents, are called proportionals. So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals ; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.†

Proportion is distinguished into continual and discontinual. If, of several couplets of proportionals, written down in a series, the difference or ratio of each consequent, and the antecedent of the next following couplet, be the same as the common difference or ratio of the couplets, the proportion is said to be continual, and the numbers themselves, a series of continual proportionals, or an arithmetical or geometrical proportion. So 2, 4, 6, 8, form an arithmetical proportion ; for $4 - 2 = 6 - 4 = 8 - 6 = 2$; and 2, 4, 8, 16, a geometrical proportion ; for $\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$.

But

* Ratios are, here, always considered as the result of the greater term of comparison diminished, or divided, by the less ; not regarding which of them be the antecedent.

† To denote numbers as being geometrically proportional, the couplets are separated by a double colon, and a colon is written between the terms of each couplet ; we may, also, denote arithmetical proportionals by separating the couplets by a double colon, and writing a colon turned horizontally between the terms of each couplet. So the above arithmetics may be written thus, $2 :: 4 :: 6 :: 8$, and $4 :: 2 :: 8 :: 6$; where the first antecedent is less or greater than its consequent by just so much as the second antecedent is less or greater than its consequent : And the geometricals thus, $2 : 4 :: 8 : 16$, and $4 : 2 :: 16 : 8$; where the first antecedent is contained in, or contains its consequent, just so often, as the second is contained in, or contains its consequent.

Four numbers are said to be *reciprocally* or *inversely* proportional, when the fourth is less than the second, by as many times, as the third is greater than the first, or when the first is to the third, as the fourth to the second, and vice versa. Thus 2, 9, 6 and 3, are reciprocal proportionals.

Note. It is common to read the geometricals $2 : 4 :: 8 : 16$, thus, 2 is to 4 as 8 to 16, or, As 2 to 4 so is 8 to 16.

Harmonical

218 ARITHMETICAL PROPORTION.

But if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinual. So 4, 2, 8, 6, are in discontinued arithmetical proportion; for $4-2=8-6=2$ =common difference of the couplets, $8-2=6$ =difference of the consequent of one couplet and the antecedent of the next; also, 4, 2, 16, 8, are in discontinued geometrical proportion; for $\frac{4}{2}=\frac{16}{8}=2$ =common ratio of the couplets, and $\frac{16}{2}=8$ =ratio of the consequent of one couplet and the antecedent of the next.

ARITHMETICAL PROPORTION.

THEOREM I.

If any four quantities a, b, c, d , (2, 4, 6, 8) be in Arithmetical Proportion,

Harmonical Proportion is that, which is between those numbers which assign the lengths of musical intervals, or the lengths of strings sounding musical notes; and of three numbers it is, when the first is to the third, as the difference between the first and second is to the difference between the second and third, as the numbers 3, 4, 6. Thus, if the lengths of strings be as these numbers, they will sound an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between four numbers, when the first is to the fourth, as the difference between the first and second is to the difference between the third and fourth, as in the numbers 5, 6, 8, 10: For strings of such lengths will sound an octave 5 to 10; a sixth greater 6 to 10; a third greater 8 to 10; a third less 5 to 6; a sixth less 5 to 8; and a fourth 6 to 8.

A series of numbers in harmonical proportion is, reciprocally, as another series in arithmetical proportion.

As $\left\{ \begin{array}{l} \text{Harmonical } 10 \dots 12 \dots 15 \dots 20 \dots 30 \dots 60 \\ \text{Arithmetical } 6 \dots 5 \dots 4 \dots 3 \dots 2 \dots 1 \end{array} \right\}$ for here $10:12::5:6$; and $12:15::4:5$, and so of all the rest. Whence those series have an obvious relation to, and dependence on, each other.

1. Let a, b, c be the three numbers in musical proportion. Then, because we have $a:c::a-b:b-c$; therefore, $ab-ac=ac-bc$; whence, if any two of the three be given, the other may be found by the following Theorems.

$$\text{I. } \frac{ab}{2a-b} = c. \quad \text{II. } \frac{2ac}{2a-b} = b. \quad \text{III. } \frac{cb}{2c-b} = a.$$

For example. Suppose you would find a musical mean proportional between the monochord 50 = a , and the octave 25 = c ; then, by Theor. II. $\frac{2ac}{a+c} = b = \frac{2500}{75} = 33,3$, which is the length of that chord, called a *fifth*.

2. If there be four numbers in musical proportion, as a, b, c, d ; then, since it is $a:d::a-b:b-c$, we have $ac-ad=ad-db$. From which equation we have the following Theorems.

$$\text{I. } \frac{db}{2d-c} = a. \quad \text{II. } \frac{a}{d} \times \frac{2d-c}{2} = b. \quad \text{III. } \frac{2ad-db}{a} = c. \quad \text{IV. } \frac{ac}{2a-b} = d.$$

Hence, when any three of those numbers are given, the fourth may be found.

Thus, let 10, 8, 6 be given to find a fourth harmonical proportion.

$$\frac{a \times c}{2a-b} = \frac{10 \times 6}{20-8} = \frac{60}{12} = 5, \text{ the octave.}$$

This harmonical theory may be carried much farther. See *Martin's Newtonian Philosophy*, Vol. II. page 123.

Proportion,* the sum of the two means is equal to the sum of the two extremes.†

And if any three quantities, a, b, c , (2, 4, 6) be in Arithmetical Proportion, the double of the mean is equal to the sum of the extremes.

THEOREM 2.

In any continued Arithmetical Proportion (1, 3, 5, 7, 9, 11) the sum of the two extremes, and that of every other two terms, equally distant from them, are equal. Thus, $1+11=3+9=5+7$.‡

When the number of terms is odd, as in the proportion 3. 8. 13. 18. 23, then, the sum of the two extremes being double to the mean or middle term, the sum of any other two terms, equally remote from the extremes, must likewise be double to the mean.

THEOREM 3.

In any continued Arithmetical Proportion, $a, a+b, a+2b, a+3b, a+4b$, &c. (4, 4+2, 4+4, 4+6, 4+8, &c.) the last or greatest term is equal to the first or least more the common difference of the terms drawn into the number of all the terms after the first, or into the whole number of the terms, less one.§

THEOREM 4.

The sum of any rank, or series, of quantities in continued Arithmetical Proportion (1. 3. 5. 7. 9. 11) is equal to the sum of the two extremes multiplied into half the number of terms.||

ARITHMETICAL

* Although, in the comparison of quantities according to their differences, the term *proportion* is used; yet the word, *progression*, is frequently substituted in its room, and is indeed more proper; the former from being, in the common acceptation of it, synonymous with ratio, which is only used in the other kind of comparison.

† For, since $b-a$ ($4-2$) $=d-c$ ($8-6$), therefore, $b+c$ ($4+6$) $=a+d$ ($2+8$.)

‡ Since, by the nature of progressionals, the second term exceeds the first by just so much as its corresponding term, the last but one, wants of the last, it is evident that when these corresponding terms are added, the excess of the one will make good the defect of the other, and so their sum be exactly the same with that of the two extremes, and in the same manner it will appear that the sum of any two other corresponding terms must be equal to that of the two extremes.

§ For since each term, after the first, exceeds that preceding it by the common difference, it is plain that the last must exceed the first by so many times the common difference as there are terms after the first; and therefore must be equal to the first, and the common difference repeated that number of times.

|| For, because (by the second Theorem) the sum of the two extremes, and that of every other two terms, equally remote from them, are equal, the whole series, consisting of half so many such equal sums as there are terms, will therefore be equal to the sum of the two extremes, repeated half as many times as there are terms.

The same thing also holds, when the number of terms is odd, as in the series 4, 8, 12, 16, 20; for then, the mean, or middle term, being equal to half the sum of any two terms, equally distant from it on contrary sides, it is obvious that the value of the whole series is the same as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the sum of the two extremes) multiplied by the whole number of terms; or to the sum of the extremes multiplied by half the number of terms.

ARITHMETICAL PROGRESSION.

Any rank of numbers, more than two, increasing by a common excess, or decreasing by a common difference, is said to be in Arithmetical Progression.

If the succeeding terms of a progression exceed each other, it is called an ascending series or progression; if the contrary, a descending series.

So { 0. 2. 4. 6. 8. 10, &c. is an ascending arithmetical series.
 1. 2. 4. 8. 16. 32, &c. is an ascending geometrical series.
 And { 10. 8. 6. 4. 2. 0, &c. is a descending arithmetical series.
 32. 16. 8. 4. 2. 1, &c. is a descending geometrical series.

The numbers which form the series, are called the *terms* of the progression.

Note.—The first and last terms of a progression are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be easily found.

1. The *first* term.
2. The *last* term.
3. The *number* of terms.
4. The *common* difference.
5. The *sum* of all the terms.

PROBLEM I.

The *first* term, the *last* term, and the *number* of terms being given, to find the *common* difference.

RULE.*

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

EXAMPLES.

The sum of any number of terms (x) of the arithmetical series of odd numbers 1, 3, 5, 7, 9, &c. is equal to the square (x^2) of that number.

For, $1+1$ or the sum of 1 term $= 1^2$ or 1

$1+3$ or the sum of 2 terms $= 2^2$ or 4

$1+3+5$ or the sum of 3 terms $= 3^2$ or 9

$1+3+5+7$ or the sum of 4 terms $= 4^2$ or 16

$1+3+5+7+9$ or the sum of 5 terms $= 5^2$ or 25, &c.

Whence, it is plain, that, let x be any number whatever, the sum of x terms will be x^2 .

EXAMPLE.

* The first term, the ratio, and number of terms given, to find the sum of the series.

A gentleman travelled 29 days, the first day he went but 1 mile, and increased every day's travel 2 miles; How far did he travel? $29 \times 29 = 841$ miles, *Ans.*

* The difference of the first and last term evidently shews the increase of the first term by all the subsequent additions, till it becomes equal to the last; and as the number of those additions was one less than the number of terms, and the increase, by every addition, equal, it is plain that the total increase, divided by the number of additions, must give the difference of every one separately; whence the rule is manifest.

E X A M P L E S.

1st. The extremes are 3 and 39, and the number of terms is 19; What is the common difference?

$$\begin{array}{r} 39 \\ - 3 \\ \hline 36 \end{array} \left. \vphantom{\begin{array}{r} 39 \\ - 3 \\ \hline 36 \end{array}} \right\} \text{Extremes.}$$

Divide by the number of terms less 1 = 19 - 1 = 18 $\frac{36}{18} = 2$ Ans.

Or, $\frac{39-3}{19-1} = 2.$

2d. A man had 10 sons, whose several ages differed alike; the youngest was 3 years old, and the eldest 48; What was the common difference of their ages?

$$\frac{48-3}{10-1} = 5 \text{ Ans.}$$

3d. A man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 37 miles; Required the daily increase?

$$\frac{37-5}{9-1} = 4 \text{ Ans.}$$

P R O B L E M II.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

RULE.*—Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

E X A M P L E S.

1st. The extremes of an arithmetical series are 3 and 39, and the number of terms 19; Required the sum of the series?

39

* Suppose another series of the same kind with the given one be placed under it in an inverse order; then will the sum of every two corresponding terms be the same as that of the first and last; consequently, any one of those sums, multiplied by the number of terms, must give the whole sum of the two series.

Let 1. 2. 3. 4. 5. 6. 7. 8, be the given series.

And 8. 7. 6. 5. 4. 3. 2. 1, the same inverted.

Then, $9+9+9+9+9+9+9+9=9 \times 8=72$, and

$$1+2+3+4+5+6+7+8=\frac{72}{2}=36.$$

$$\begin{array}{r} 39 \\ + 3 \end{array} \} \text{Extremes.}$$

$$\text{Sum} = 42$$

$$\text{Number of terms} = \times 19$$

$$\begin{array}{r} 378 \\ 42 \end{array}$$

$$2)798$$

$$\text{Or, } \frac{39+3 \times 19}{2} = 399. \quad 399 \text{ Ans.}$$

2d. It is required to find how many strokes the hammer of a clock would strike in a week, or 168 hours, provided it increased 1 at each hour?

$$\frac{168+1 \times 168}{2} = 14196 \text{ Ans.}$$

3d. Suppose a number of stones were laid a yard distant from each other for the space of a mile, and the first a yard from a basket; What length of ground will that man travel over, who gathers them up singly, returning with them one by one to the basket?

$$\frac{3520+2 \times 1760}{2} = 3099360 \text{ yards} = 1761 \text{ miles, Ans.}$$

N. B. In this question, there being 1760 yards in a mile, and the man returning with each stone to the basket, his travel will be doubled; therefore the first term will be 2, and the last 1760×2 , and the number of terms 1760.

4th. A man bought 25 yards of linen in Arithmetical Progression; for the 4th yard he gave 12 shillings, and for the last yard 3*l.* 15*s.* What did the whole amount to, and what did it average per yard?

$$\frac{75-12}{22-1} = 3 \text{ the common difference by which the first term is found to be 3.}$$

$$\text{Then } \frac{75+3 \times 25}{2} = 48*l.* 15*s.* and the average price is 1*l.* 19*s.* per yd.$$

5th. Required the sum of the first 1000 numbers in their natural order?

$$\frac{1000+1 \times 1000}{2} = 500500 \text{ Ans.}$$

PROBLEM III.

Given the extremes and the common difference, to find the number of terms.

RULE.

ARITHMETICAL PROGRESSION. 223

RULE.*—Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the number of terms required.

EXAMPLES.

1st. The extremes are 3 and 39, and the common difference 2; What is the number of terms?

$$\begin{array}{r} 39 \\ - 3 \\ \hline \end{array} \} \text{Extremes.}$$

$$\text{Common difference} = 2 \overline{)36}$$

$$\begin{array}{r} \text{Quotient} = 18 \\ \text{Add } 1 \\ \hline \end{array}$$

$$19 \text{ Ans.}$$

$$\text{Or, } \frac{39-3}{2} + 1 = 19.$$

2d. A man, going a journey, travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles; How many days did he travel, and how far?

$$\frac{51-7}{4} + 1 = 12 \text{ days, and } \frac{51+7 \times 12}{2} = 348 \text{ miles, Ans.}$$

PROBLEM IV.

The extremes and common difference given, to find the sum of all the series.

RULE.—Multiply the sum of the extremes by their difference increased by the common difference, and the product divided by twice the common difference, will give the sum.

EXAMPLES.

1st. If the extremes are 3 and 39, and the common difference 2; What is the sum of the series?

$$39+3=42 \text{ sum of the extremes.}$$

$$39-3=36=\text{difference of extremes.}$$

$$36+2=38=\text{difference of extremes increased by the common difference.}$$

42

* By the first Problem, the difference of the extremes, divided by the number of terms less 1, gave the common difference; consequently, the same divided by the common difference, must give the number of terms less 1; hence, this quotient, augmented by 1, must be the answer to the question.

$$\begin{array}{r} 42 \\ \times 38 \\ \hline 336 \\ 126 \\ \hline \end{array}$$

Twice the common diff. = $4)1596$

399 Answer.

$$\text{Or, } \frac{39+3 \times 39-3+2}{2 \times 2} = 399.$$

2d. A owes B a certain sum, to be discharged in a year, by paying 6d. the first week, 18d. the second, and thus to increase every weekly payment by a shilling, till the last payment be 2l. 11s. 6d. ; What is the debt ?

$$\frac{51.5+.5 \times 51.5-.5+1}{1 \times 2} = £67 \ 12s. \text{ Ans.}$$

PROBLEM V.

The extremes and the sum of the series given, to find the common difference.

RULE. Divide the product of the sum and difference of the extremes, by the difference of twice the sum of the series, and the sum of the extremes, and the quotient will be the common difference.

EXAMPLES.

1st. Let the extremes be 3 and 39, and the sum 399 ; What is the common difference ?

$$\text{Sum of the extremes} = 39 + 3 = 42$$

$$\text{Diff. of the extremes} = 39 - 3 = \times 36$$

$$\begin{array}{r} 252 \\ 126 \\ \hline \end{array}$$

$$399 \times 2 - 42 = 756)1512(2 \text{ Ans.}$$

$$\text{Or, } \frac{39+3 \times 39-3}{399 \times 2 - 39+3} = 2.$$

2d. A owes B 67l. 12s. to be discharged in a year, by weekly payments ; the first payment to be 6d. and the last, 2l. 11s. 6d. ; What is the common difference of the payments, and what will each payment be ?

$$\frac{51.5+.5 \times 51.5-.5}{1352 \times 2 - 51.5+.5} = 1s. \text{ and } 6d. + 1s. = 1s. 6d. = 2d. \text{ payment, } 1s.$$

$$6d. + 1s. = 2s. 6d. = 3d. \text{ payment, } \&c.$$

PROBLEM VI.

P R O B L E M VI.

The extremes and sum of the series given, to find the number of terms.

RULE.—Twice the sum of the series, divided by the sum of the extremes, will give the number of terms.

E X A M P L E S.

1st. Let the extremes be 3 and 39, and the sum of the series 399 ; What is the number of terms ?

$$\begin{array}{r} \text{Sum of the series} = 399 \\ \times 2 \\ \hline \end{array}$$

$$\text{Sum of extremes} = 39 + 3 = 42 \quad 798 \quad (19 \text{ Ans.})$$

42

$$\begin{array}{r} 378 \\ 378 \\ \hline \end{array}$$

$$\text{Or, } \frac{399 \times 2}{39 + 3} = 19.$$

2d. A owes B 67*l.* 12*s.* to be paid weekly in Arithmetical Progression, the first payment to be 6*d.* and the last 51*s.* 6*d.* ; How many payments will there be ? and, How long will he be in discharging the debt ?

$$\frac{1352 \times 2}{51.5 + 5} = 52 \text{ payments, and as many weeks, Ans.}$$

P R O B L E M VII.

The first term, the common difference, and sum of the series given, to find the number of terms.

RULE.—To the square of the difference of twice the first term and the common difference, add the rectangle (or product) of the sum and the common difference multiplied by 8, and extract the square root of the sum, from which root take twice the first term less the common difference ; divide the remainder by twice the common difference, and the quotient will be the number of terms.

E X A M P L E.

If the first term be 3, the common difference 2, and the sum of the series 399 ; Required the number of terms ?

$$3 \times 2 = 6 = \text{Twice the sum of the first term.}$$

$$6 - 2 = 4 = \text{Diff. of twice the first term and the common diff.}$$

$$4 \times 4 = 16 = \text{Square of the said difference.}$$

$$399 \times 2 \times 8 = 6384 = \text{Rect. of the sum and com. diff. mult. by 8.}$$

$$6384 + 16 = 6400 = \text{Sum of the said eightfold rectangle and the square of the aforefaid difference.}$$

$$\sqrt{6400} = 80 = \text{Square root of the last mentioned sum.}$$

F f

80—

226 ARITHMETICAL PROGRESSION.

$80 - 4 = 76 =$ Difference of the said root and twice the first term less the common difference.

$\frac{76}{4} = 19$ The number of terms.

$$\text{Or, } \sqrt{\frac{3 \times 2 - 2}{2 \times 2} + 399 \times 2 \times 8 - 3 \times 2 - 2} = 19.$$

PROBLEM VIII.

The first term, the common difference, and the sum of the series given, to find the last term.

RULE.—To the square of the difference of twice the first term and the common difference, add the rectangle of the sum and the common difference, and extract the square root of their sum, from which root take the common difference; and the remainder, divided by 2, will be the last term.

EXAMPLE.

If the first term be 3, the common difference 2, and the sum of the series 399; What is the last term?

$3 \times 2 = 6$. $6 - 2 = 4$. $4 \times 4 = 16$. $399 \times 2 \times 8 = 6384$. $6384 + 16 = 6400$. $\sqrt{6400} = 80$. $80 - 2 = 78$. And $78 \div 2 = 39$ the Answer.

$$\text{Or, } \sqrt{\frac{3 \times 2 - 2}{2} + 399 \times 2 \times 8 - 2} = 39.$$

PROBLEM IX.

The first term, the common difference, and the number of terms given, to find the last term.

RULE.—The number of terms less 1, multiplied by the common difference, and the first term added to the product, will give the last term.

EXAMPLES.

1st. If the first term be 3, the common difference 2, and the number of terms 19; What is the last term?

Number of terms $= 19$

$- 1$

Number of terms less 1 $= 18$

Common difference $= \times 2$

First term $= + 3$

39 the Ans.

$$\text{Or, } 19 - 1 \times 2 + 3 = 39.$$

2d. A owes B a certain sum, to be paid in Arithmetical Progression; the first payment is 6d, the number of payments 52, and the common

common difference of the payments is 12d. ; What is the last payment ?
 $52 - 1 \times 12 + 6 = 618d. = 2l. 11s. 6d. \text{ Ans.}$

PROBLEM X.

The first term, common difference, and number of terms given, to find the sum of the series.

RULE.—To the first term add the product of the number of terms less 1 by half the common difference, and their sum, multiplied by the number of terms, will give the sum of the progression.

EXAMPLES.

1st. If the first term be 3, the common difference 2, and number of terms 19 ; What is the sum of the series ?

Add the product of the number of terms less 1 by $\frac{1}{2}$ com. diff. $= 19 - 1 \times 1 = 18$	First term $= 3$
	Their sum 21
Multiply by the number of terms $= 19$	<u>189</u>
	21
	<u>399</u>

Or, $19 \times 3 + 19 - 1 \times 1 = 399$ Ans. $= 399$

2d. Sixteen persons gave charity to a poor man ; the first gave 5d. and the second 9d. and so on in Arithmetical Progression ; I demand what sum the last person gave, and how much the poor man received in all ?

Answer $16 - 1 \times 4 + 5 = 65d. = 5s. 5d. \text{ the last gave.}$

And $16 \times 5 + 16 - 1 \times 4 = 560d. = 2l. 6s. 8d. \text{ the whole sum.}$

PROBLEM XI.

Given the first term, the number of terms, and the sum of the series, to find the common difference.

RULE.—From the sum subtract the rectangle of the first term and number of terms ; twice the remainder, divided by the product of the number of terms and number of terms less 1, will give the common difference.

EXAMPLE.

If the first term be 3, the number of terms 19, and the sum 399 ; What is the common difference ?

Subtract the product of the first term and number of terms $= 3 \times 19 = 57$	Sum of the series $= 399$
	Remainder $= 342$
	Multiplied by 2
Divide by the product of the number of terms } and number of terms less 1	$= 19 \times 18 = 342 \overline{)684} (2 \text{ Ans.}$
	684
	<u>684</u>
	...

Or, $\frac{2 \times 399 - 3 \times 19}{19 - 1 \times 19} = 2$

PROBLEM XII.

Given the first term, number of terms, and the sum of the series, to find the last term,

RULE

228 ARITHMETICAL PROGRESSION.

RULE.—Divide twice the sum by the number of terms; from the quotient take the first term, and the remainder will be the last.

EXAMPLES.

1st. If the first term be 3, the number of terms 19, and the sum 399; What is the last term?

$$\text{Sum of the terms} = 399$$

$$\text{Multiply by } 2$$

$$\text{Divide by the number of terms} = 19 \overline{)798}$$

$$\text{Quotient} = 42$$

$$\text{Subtract the first term} = 3$$

$$\text{Or, } \frac{399 \times 2}{19} - 3 = 39.$$

$$\text{Answer} = 39$$

2d. A merchant being indebted to 12 creditors £2460, ordered his clerk to pay the first £40, and the rest increasing in Arithmetical Progression; I demand the difference of the payments, and the last payment?

$$\text{Ans. } \frac{2 \times 2460 - 40 \times 12}{12 - 1 \times 12} = 30\text{ l.} = \text{diff. } \frac{2460 \times 2}{12} - 40 = 370\text{ l. last paym.}$$

PROBLEM XIII.

The common difference, the last term, and sum of the progression given, to find the first term.

RULE.—From the square of twice the last term plus the common difference, take 8 times the rectangle of the sum and common difference, and extract the square root of the remainder, which (root) either add to, or subtract from the common difference, (as the case may require) and half the sum or difference will be the first term.

EXAMPLES.

1st. If the common difference be 2, the last term 39, and the sum of the terms 399; Required the first term?

$$\begin{array}{r} \text{Last term } 39 \\ \text{Multiplied by } 2 \end{array}$$

$$\text{Product} = 78$$

$$\text{Add the common difference} = 2$$

$$80$$

$$\text{Multiplied by } 80$$

$$\text{From the square of twice the last term plus the common difference} = 6400$$

$$\text{Take 8 times the rectangle of the sum and common diff.} = 399 \times 2 \times 8 = 6384$$

$$\text{Remainder} = 16$$

$$\text{Square root of } 16 = 4$$

$$\text{Sum of the common difference and the square root of } 16 = 2 + 4 = 6$$

$$\text{And half the sum} = \frac{6}{2} = 3 \text{ Ans.}$$

Or,

$$\text{Or, } \frac{2 + \sqrt{39 \times 2 + 2^2} - 399 \times 2 \times 8}{2} = 3.$$

2d. A merchant being indebted to several persons 1080*l.* he ordered his clerk to pay the greatest creditor 142*l.* the greatest but one 132*l.* and so on, to decrease in Arithmetical Progression; What did the least creditor receive?

$$\text{Ans. } \frac{10 - \sqrt{142 \times 2 + 10^2} - 1080 \times 10 \times 8}{2} = 2.$$

PROBLEM XIV.

Given the common difference, the last term, and sum of the series, to find the number of terms.

RULE.—From the square of twice the last term plus the common difference take 8 times the rectangle of the sum and common difference, and extract the square root of the remainder, which (root) either subtract from, or add to, twice the last term plus the common difference (as the case may require) and the remainder or sum, divided by twice the common difference, will give the number of terms.

EXAMPLES.

1st. If the common difference be 2, the last term 39, and the sum of the terms 399; I demand the number of terms.

$$\begin{array}{r} \text{Last term } 39 \\ \text{Multiply by } 2 \\ \hline 78 \\ \text{Add the common difference } = 2 \\ \hline 80 \\ \hline 80 \end{array}$$

$$\begin{array}{r} \text{Square of twice the last term plus the com. diff.} = 6400 \\ \text{Sub. 8 times the rect. of the sum \& com. diff.} = 399 \times 2 \times 8 = 6384 \\ \hline 16 \end{array}$$

$$\begin{array}{r} \text{Square root of } 16 = 4 \\ \text{Sum of twice the last term plus the com. diff.} = 39 \times 2 + 2 = 80 \\ \text{Sum of twice the last term and com. diff. minus } \left. \begin{array}{l} \text{the square root of } 16 = 80 - 4 \end{array} \right\} = 76 \end{array}$$

$$\text{Which remainder divided by twice the com. diff.} = \frac{76}{4} = 19 \text{ Ans.}$$

$$\text{Or, } \frac{39 \times 2 + 2 - \sqrt{39 \times 2 + 2^2} - 399 \times 2 \times 8}{2 \times 2} = 19$$

2d. A merchant being indebted to several persons 1080*l.* he ordered his clerk to pay the greatest creditor 142*l.*; the greatest but one 132*l.* and so on, to decrease in Arithmetical Progression. How many creditors had he?

$$\text{Ans. } \frac{142 \times 2 + 10 + \sqrt{142 \times 2 + 10^2} - 1080 \times 10 \times 8}{10 \times 2} = 15 \text{ Creditors.}$$

PROBLEM XV.

Given the last term, the number of terms, and the sum of the terms, to find the first term.

RULE.—Divide twice the sum by the number of terms; from the quotient subtract the last term, and the remainder will be the first.

EXAMPLES.

1. If the last term be 39, the number of terms 19, and the sum of the series 399; What is the first term?

$$\text{Sum of the series} = 399$$

$$\text{Multiply by } 2$$

$$\text{Divide by the number of terms} = 19 \overline{)798}$$

$$\text{Quotient} = 42$$

$$\text{From the quotient take the last term} = 39$$

$$\text{Remainder} = 3 \text{ Ans.}$$

$$\text{Or, } \frac{399 \times 2}{19} - 39 = 3.$$

2. A man had 10 sons, whose several ages differed alike; the eldest was 48 years old, and the sum of all their ages was 255; What was the age of the youngest?

$$\frac{255 \times 2}{10} - 48 = 3 \text{ years, Ans.}$$

PROBLEM XVI.

Given the last term, the number of terms, and the sum of the series, to find the common difference.

RULE.—Double the rectangle of the number of terms and the last term minus the sum of the series; divide the product by the rectangle of the number of terms and the number of terms minus 1, and the quotient will be the common difference.

EXAMPLES.

1. If the last term be 39, the number of terms 19, and the sum of the series 399; What is the common difference?

$$\text{Number of terms} = 19$$

$$\text{Multiply by the last term} = 39$$

$$171$$

$$57$$

$$\left. \begin{array}{l} \text{Rectangle of the number of} \\ \text{terms, and the last term} \end{array} \right\} = 19 \times 39 = 741$$

$$\text{Subtract the sum of the series} = 399$$

$$\text{Remainder} = 342$$

$$\text{Multiply by } 2$$

$$\left. \begin{array}{l} \text{Divide by the rectangle of} \\ \text{the number of terms, and} \\ \text{number of terms minus 1} \end{array} \right\} = 19 \times 18 = 342 \overline{)684} \text{ Ans.}$$

$$684$$

$$\text{Or,}$$

$$\text{Or, } \frac{2 \times 19 \times 39 - 399}{19 - 1 \times 19} = 2.$$

2. Sixteen persons gave charity to a poor man in such proportion as to form an arithmetical series : The last gave 5s. 5d. and the whole sum amounted to 2l. 6s. 8d. What did each give less than the other from the last down to the first ?

$$\frac{2 \times 16 \times 65 - 560}{16 - 1 \times 16} = 4d. \text{ Ans.}$$

PROBLEM XVII.

The common difference, number of terms, and the last term given, to find the first term.

RULE.—From the last term subtract the product of the terms less 1 by the common difference, and the remainder will be the first term.

EXAMPLES.

1. If the common difference be 2, the number of terms 19, and the last term 39 ; What is the first ?

$$\left. \begin{array}{l} \text{Subtract the number of terms less 1} \\ \text{multiplied by the common difference} \end{array} \right\} \begin{array}{l} \text{Last term} = 39 \\ = 19 - 1 \times 2 = 36 \end{array}$$

Remains 3 Ans.

$$\text{Or, } 39 - 19 - 1 \times 2 = 3.$$

2. A man travelled 6 days, each day going 4 miles farther than on the preceding day, till the last day's journey was 40 miles ; How far did he ride the first day ?

$$40 - 6 - 1 \times 4 = 20 \text{ miles, Ans.}$$

PROBLEM XVIII.

The common difference, the number of terms, and last term given, to find the sum of the series.

RULE.—From the last term take the number of terms minus 1, multiplied by half the common difference, and the remainder multiplied by the number of terms will give the sum.

EXAMPLES.

1. If the common difference be 2, number of terms 19, and the last term 39 ; What is the sum of the series ?

$$\left. \begin{array}{l} \text{Subtract the number of terms less 1} \\ \text{multiplied by } \frac{1}{2} \text{ the common difference} \end{array} \right\} \begin{array}{l} \text{Last term} = 39 \\ = 19 - 1 \times 1 = 18 \end{array}$$

Remainder = 21

Multiply by the number of terms = 19

189

21

Answer, 399 Or,

$$\text{Or, } 19 \times 39 - 19 - 1 \times 1 = 399.$$

2. A man performed a journey in 6 days, and, each day, travelled 4 miles farther than on the preceding day, till his last day's travel was 40 miles : How far did he travel in the whole ?

$$\text{Ans. } 6 \times 40 - 6 - 1 \times \frac{4}{2} = 180 \text{ miles.}$$

PROBLEM XIX.

The sum of the terms, the number of terms, and the common difference given, to find the first term.

RULE.—Divide the sum by the number of terms ; from the quotient take half the product of the number of terms, minus unity, by the common difference, and the remainder will be the first term.

EXAMPLES.

1. If the sum of the series be 399, the number of terms 19, and the common difference 2 ; What is the first term ?

$$\text{Number of terms} = 19 \quad 399 = \text{sum.}$$

$$\begin{array}{l} \text{Subtract } \frac{1}{2} \text{ the product of the number of } \left. \begin{array}{l} \text{terms, less 1, by the common difference,} \end{array} \right\} \begin{array}{l} \text{Quotient} = 21 \\ = \frac{19 - 1 \times 2}{2} = 18 \end{array} \\ \text{Ans. } 3 \end{array}$$

$$\text{Or, } \frac{399}{19} - \frac{2 \times 19 - 1}{2} = 3.$$

2. A man travelled 180 miles in 6 days ; he increased his journey, each day, by 4 miles : How far did he travel the first day ?

$$\frac{180}{6} - \frac{4 \times 6 - 1}{2} = 20 \text{ miles, Ans.}$$

PROBLEM XX.

The sum of the terms, number of terms, and the common difference given, to find the last term.

RULE.—Divide the sum of the series by the number of terms ; to the quotient add half the product of the number of terms, minus unity by the common difference, and the sum will be the last term.

EXAMPLES.

1. If the sum of the series be 399, the number of terms 19, and the common difference 2 ; What is the last term ?

$$\text{Divide by the number of terms} = 19 \quad 399 = \text{sum}$$

$$\begin{array}{l} \text{Add } \frac{1}{2} \text{ the product of the number of } \left. \begin{array}{l} \text{terms, less 1, by the common difference} \end{array} \right\} \begin{array}{l} \text{Quotient} = 21 \\ = \frac{19 - 1 \times 2}{2} = 18 \end{array} \\ \text{Ans.} = 39 \end{array}$$

$$\text{Or, } \frac{399}{19} + \frac{2 \times 19 - 1}{2} = 39.$$

9. A person bought a farm for 510*l.* to be paid monthly in Arithmetical Progression, and to be completed in a year, each payment

The following Table contains a summary of the whole doctrine of Arithmetical Progression.

CASES OF ARITHMETICAL PROGRESSION.			
Case	Given	Required	Solution.
1.	aln	d	$\frac{l-a}{n-1}$
		s	$\frac{a+l \times n}{2}$
2.	ald	n	$\frac{l-a}{d} + 1$
		s	$\frac{l+a \times l-a+d}{2d}$
3.	als	d	$\frac{l+a \times l-a}{2s-l+a}$
		n	$\frac{2s}{a+l}$
4.	ads	n	$\frac{\sqrt{2a-d}^2 + 8ds - 2a-d}{2d}$
		l	$\frac{\sqrt{2a-d}^2 + 8ds - d}{2}$
5.	adn	l	$\overline{n-1} \times d + a$
		s	$n \times a + \overline{n-1} \times \frac{d}{2}$

payment to exceed that preceding by 5*l*. What were the first and last payments?

Ans. $\frac{510}{12} - \frac{5 \times 12 - 1}{2} = 15*l*$. the first payment, and

$\frac{510}{12} + \frac{5 \times 12 - 1}{2} = 70*l*$. the last payment.

GEOMETRICAL

Case	Given	Required	Solution.
6.	ans	d	$\frac{2 \times s - an}{n-1 \times n}$
		l	$\frac{2s}{n} - a$
7.	lds	a	$\frac{d \pm \sqrt{2l+d ^2 - 8ds}}{2}$
		n	$\frac{2l+d \pm \sqrt{2l+d ^2 - 8ds}}{2d}$
8.	lns	a	$\frac{2s}{n} - l$
		d	$\frac{2 \times nl - s}{n-1 \times n}$
9.	lnd	a	$l - n-1 \times d$
		s	$n \times l - n-1 \times \frac{d}{2}$
10.	dns	a	$\frac{s}{n} - \frac{d \times n-1}{2}$
		l	$\frac{s}{n} + \frac{d \times n-1}{2}$
Here $\begin{cases} a = \text{first term.} \\ l = \text{last term.} \\ n = \text{number of terms.} \\ d = \text{common difference.} \\ s = \text{sum of all the terms.} \end{cases}$			

GEOMETRICAL PROPORTION.

THEOREM 1.

If four quantities, a, b, c, d . (2. 6. 4. 12.) be in Geometrical Proportion, the product of the two means, bc (6×4) will be equal to that of the two extremes, ad (2×12) whether they are continued, or discontinued,* and, if three quantities, a, b, c . (2. 4. 8.) the square of the mean is equal to the product of the two extremes.

THEOREM 2.

If four quantities, a, b, c, d . (2. 6. 4. 12.) are such, that the product of two of them, ad (2×12) is equal to the product of the other two, bc (6×4), then are those quantities proportional.†

THEOREM 3.

If four quantities, a, b, c, d . (2. 6. 4. 12.) are proportional, the rectangle of the means, divided by either extreme, will give the other extreme.‡

THEOREM 4.

The products of the corresponding terms of two Geometrical Proportions are also proportional.

That is, if $a : b :: c : d$ (2 : 6 :: 4 : 12,) and $e : f :: g : h$ (2 : 4 :: 5 : 10,) then will $ae : bf :: cg : dh$ ($2 \times 2 : 6 \times 4 :: 4 \times 5 : 12 \times 10$.)§

THEOREM

* For since the ratio of a to b (2 to 6,) or the part, which a is of b (2 is of 6) is expressed by $\frac{a}{b}$ ($\frac{2}{6}$) and the ratio of c to d (4 to 12,) in like manner, by $\frac{c}{d}$ ($\frac{4}{12}$); and since, by supposition, the two ratios are equal, let them both be multiplied by bd , (6×12) and the products $\frac{a}{b} \times bd$ ($\frac{2}{6} \times 6 \times 12$) and $\frac{c}{d} \times bd$ ($\frac{4}{12} \times 6 \times 12$) will likewise be equal; that is, $\frac{abd}{b} = \frac{cbd}{d}$ or $ad = cb$ ($\frac{2 \times 6 \times 12}{6} = \frac{4 \times 6 \times 12}{12}$, or, $2 \times 12 = 6 \times 4$.)

† For since, by supposition, the products ad (2×12) and bc (6×4) are equal, let both be divided by bd (6×12) and the quotients $\frac{ad(a)}{bd(b)}$ and $\frac{bc(c)}{bd(d)}$ ($\frac{2 \times 12}{6 \times 12}$ and $\frac{6 \times 4}{6 \times 12}$) will also be equal; and therefore $a : b :: c : d$.

‡ For, by the second Theorem, $ad = bc$ ($2 \times 12 = 6 \times 4$) whence dividing both sides of the equation by a (2) we have $d = \frac{bc}{a}$ ($12 = \frac{6 \times 4}{2}$) Hence, if the two means and one extreme be given, the other extreme may be found.

§ For $\frac{a}{b} = \frac{c}{d}$ ($\frac{2}{6} = \frac{4}{12}$), and $\frac{e}{f} = \frac{g}{h}$ ($\frac{2}{4} = \frac{5}{10}$) by supposition; whence, $\frac{a}{b} \times \frac{e}{f} = \frac{c}{d} \times \frac{g}{h}$ ($\frac{2}{6} \times \frac{2}{4} = \frac{4}{12} \times \frac{5}{10}$) by equal multiplication; and consequently $\frac{ae}{bf} = \frac{cg}{dh}$ ($\frac{2 \times 2}{6 \times 4} = \frac{4 \times 5}{12 \times 10}$) that is, $ae : bf :: cg : dh$ ($2 \times 2 : 6 \times 4 :: 4 \times 5 : 12 \times 10$.) Hence it follows, that if any quantities be proportional, their squares, cubes, &c. will likewise be proportional.

THEOREM 5.

If four quantities, a, b, c, d . (2. 6. 4. 12.) are directly proportional.

Then,	1. Directly,	$a \dagger b :: c \dagger d$	(2 \dagger 6 :: 4 \dagger 12)
	2. Inversely,	$b \dagger a :: d \dagger c$	(6 \dagger 2 :: 12 \dagger 4)
	3. Alternately,	$a \dagger c :: b \dagger d$	(2 \dagger 4 :: 6 \dagger 12)
	4. Compoundedly,	$a \dagger a+b :: c \dagger c+d$	(2 \dagger 8 :: 4 \dagger 16)
	5. Dividedly,	$a \dagger b-a :: c \dagger d-c$	(2 \dagger 4 :: 4 \dagger 8)
	6. Mixtly,	$b+a \dagger b-a :: d+c \dagger d-c$	(8 \dagger 4 :: 16 \dagger 8)
	7. By Multiplication,	$ra \dagger rb :: c \dagger d$	(2r \dagger 6r :: 4 \dagger 12)
	8. By Division,	$\frac{a}{r} \dagger \frac{b}{r} :: c \dagger d$	($\frac{2}{r} \dagger \frac{6}{r} :: \frac{4}{r} \dagger \frac{12}{r}$)

Because the product of the means, in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 2.

THEOREM 6.

If three numbers, a, b, c . (2. 4. 8.) be in continued proportion, the square of the first will be to that of the second, as the first number to the third; that is, $a^2 : b^2 :: a : c$ (2 \times 2 : 4 \times 4 :: 2 : 8.)*

THEOREM 7.

In any continued Geometrical Proportion (1. 3. 9. 27. 81. &c.) the product of the two extremes, and that of every other two terms, equally distant from them, are equal.†

THEOREM 8.

The sum of any number of quantities, in continued Geometrical Proportion, is equal to the difference of the rectangle of the second and last terms and the square of the first, divided by the difference of the first and second terms.‡

GEOMETRICAL

* For since $a \dagger b :: b \dagger c$ (2 \dagger 4 :: 4 \dagger 8) thence will $ac=bb$ (2 \times 8=4 \times 4) by Theorem 1; and therefore $aac=abb$ (2 \times 2 \times 8=2 \times 4 \times 4) by equal multiplication; consequently, $a^2 \dagger b^2 :: a \dagger c$ (2 \times 2 \dagger 4 \times 4 :: 2 \dagger 8) by Theorem 2.

In like manner it may be proved that, of four quantities continually proportional, the cube of the first is to that of the second, as the first quantity to the fourth.

† For, the ratio of the first term to the second being the same as that of the last but one to the last, these four terms are in proportion; and therefore, by Theorem 1, the rectangle of the extremes is equal to that of their two adjacent terms; and after the same manner, it will appear that the rectangle of the third and last but two is equal to that of their two adjacent terms, the second and last but one, and so. of the rest; whence the truth of the proposition is manifest.

‡ For, let the first term of the proportion be denoted by a , the common ratio by r , the number of terms by n , and the sum of the whole series by s ; then, it is plain that the second term will be expressed by $a \times r$, or, ar ; the third by $ar \times r$, or ar^2 ; the fourth by $ar^2 \times r$, or, ar^3 ; and the n th, or last, term by ar^{n-1} ; and therefore the proportion will stand thus, $a \dagger ar \dagger ar^2 \dagger ar^3 \dots \dagger ar^{n-1} = s$; which equation multiplied by r , gives $ar \dagger ar^2 \dagger ar^3 \dagger ar^4 \dots \dagger ar^n \dagger ar^{n+1} = sr$; from the

GEOMETRICAL PROGRESSION.

A Geometrical Progression is, when a *rank*, or *series*, of numbers increases, or decreases, by the continual multiplication, or division, of some equal number.

PROBLEM I.

Given one of the extremes, the ratio, and the number of the terms of a geometrical series, to find the other extreme.

RULE.—Multiply, or divide, (as the case may require) the given extreme by such power of the ratio, whose exponent* is equal to the number of terms less 1, and the product, or quotient, will be the other extreme.

EXAMPLES.

the first equation being subtracted, there will remain $-a + ar^n = rs - s$: Whence,

$$s = \left\{ \frac{(ar^n - a) \times (ar^{n-1} - a)}{r - 1} \right\} = \frac{ar \times ar^{n-1} - aa}{ar - a} : \text{(Or, take any series of numbers}$$

whatever, as 2. 6. 18. 54. 162. 486. and their sum will be $2+6+18+54+162+486=728$: This equation multiplied by the ratio, will stand thus, $6+18+54+162+486+1458=2184$; now it is plain that the sum of the second series will be so many times the first, as is expressed by the ratio; subtract the first series from the second, and it will give $1458-2=2184-728$, which is evidently so many times the sum of the first series, as is expressed by the ratio less 1; whence $\frac{1458-2}{3-1} = \frac{2916-4}{6-2}$, as was to be demonstrated.)

* As the *last term*, or any term near the last, is very tedious to be found by continual multiplication, it will often be necessary, in order to ascertain them, to have a series of numbers in Arithmetical Proportion, called *indices*, or *exponents*, beginning either with a cypher, or an unit, whose common difference is one.

When the *first term* of the series and the *ratio* are equal, the *indices* must begin with an unit, and, in this case, the product of any two terms is equal to that term, signified by the *sum* of their *indices*.

Thus, $\left\{ \begin{array}{l} 1. 2. 3. 4. 5. 6, \&c. \text{ indices, or arithmetical series.} \\ 2. 4. 8. 16. 32. 64, \&c. \text{ geometrical series (leading terms.)} \end{array} \right.$

Now, $6+6=12$ the index of the twelfth term, and

$$64 \times 64 = 4096 \text{ the twelfth term.}$$

But, when the *first term* of the series and the *ratio* are different, the *indices* must begin with a cypher, and the sum of the *indices*, made choice of, must be one less than the number of terms, given in the question; because 1 in the *indices* stands over the second term, and 2, in the *indices*, over the third term, &c. And, in this case, the product of any two terms, divided by the *first*, is equal to that term beyond the first, signified by the *sum* of their *indices*,

Thus, $\left\{ \begin{array}{l} 0. 1. 2. 3. 4. 5. 6, \&c. \text{ indices.} \\ 1. 3. 9. 27. 81. 243. 729, \&c. \text{ geometrical series.} \end{array} \right.$

Here, $6+5=11$ the index of the 12th term.

$729 \times 243 = 177147$ the 12th term, because the first term of the series and the ratio are different, by which mean a cypher stands over the first term.

Thus, by the help of these *indices*, and a few of the first terms in any geometrical series, any term, whose distance from the first term is assigned, though it were ever so remote, may be obtained without producing all the terms.

Note. If the *ratio* of any geometrical series be double, the difference of the greatest and least terms is equal to the sum of all the terms, except the greatest; if the ratio be triple, the difference is double the sum of all but the greatest; if the ratio be quadruple, the difference is triple the sum of all but the greatest, &c.

In any geometrical series decreasing, and continued *ad infinitum*, half the greatest term is equal to the sum of all the remaining terms, *ad infinitum*.

E X A M P L E S.

1. If the first term be 4, the ratio 4, and the number of terms 9; What is the last term?

$$1. \quad 2. \quad 3. \quad 4. + 4 = 8$$

4. 16. 64. 256. $\times 256 = 65536 =$ power of the ratio, whose exponent is less by 1, than the number of terms.

$$65536 = 8\text{th power of the ratio.}$$

Multiply by 4 = first term.

$$262144 = \text{last term.}$$

$$\text{Or, } 4 \times 4^8 = 262144 = \text{the Answer.}$$

2. If the last term be 262144, the ratio 4, and the number of terms 9, What is the first term?

Last term.

8th power of the ratio $4^8 = 65536$) 262144 (4 = the first term.

$$\text{Or, } \frac{262144}{4^8} = 4 \text{ the first term.}$$

Again, Given the first term, and the ratio, to find any other term assigned

R U L E I.

When the indices begin with an unit.

1. Write down a few of the leading terms of the series, and place their indices over them.

2. Add together such indices, whose sum shall make up the entire index to the term required.

3. Multiply the terms of the geometrical series, belonging to those indices, together, and the product will be the term sought.

E X A M P L E S.

1. If the first term be 2, and the ratio 2, What is the 13th term?

$$1. \quad 2. \quad 3. \quad 4. \quad 5 + 5 + 3 = 13$$

$$2. \quad 4. \quad 8. \quad 16. \quad 32 \times 32 \times 8 = 8192 \text{ Anf.}$$

$$\text{Or, } 2 \times 2^{12} = 8192.$$

2. A merchant wanting to purchase a cargo of horses for the Westindies, a jockey told him he would take all the trouble and expense, upon himself, of collecting and purchasing 30 horses for the voyage, if he would give him what the last horse would come to by doubling the whole number by a half penny, that is, two farthings for the first, four for the second, eight for the third, &c. to which the merchant, thinking he had made a very good bargain, readily agreed: Pray, what did the last horse come to, and what did the horses, one with another, cost the merchant?

$$1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6 + 6 = 12\text{th.}$$

$$12 + 12 + 6 = \text{last term.}$$

$$2. \quad 4. \quad 8. \quad 16. \quad 32. \quad 64 \times 64 = 4096, \text{ and } 4096 \times 4096 \times 64 =$$

$$1073741824 \text{ qrs.} = £1118481. 1s. 4d. \text{ and their average price was } £27282 \text{ 14s. } 0\frac{1}{2}d. \text{ apiece.}$$

R U L E

R U L E II.

When the indices begin with a cypher.

1. Write down a few of the leading terms of the series, as before, and place their indices over them.

2. Add together the most convenient indices to make an index, less by 1 than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power, whose index is one less than the number of terms multiplied, and make the result a divisor, by which divide the dividend, and the quotient will be that term beyond the first, signified by the sum of those indices, or the term sought.

3. If the first term be 5, and the ratio 3; What is the 7th term?

O. 1. 2. $3 + 2 + 1 = 6 =$ index to 6th term beyond the 1st or 7th.

5. $15 \cdot 45 \cdot 135 \times 45 \times 15 = 91125 =$ dividend.

The number of terms, multiplied, is 3 (viz. $135 \times 45 \times 15$), and $3 - 1 = 2$ is the power to which the term 5 is to be raised; but the 2d power of 5 is $5 \times 5 = 25$, and therefore $91125 \div 25 = 3645$ the 7th term required.

P R O B L E M II.

Given the first term, the ratio, and number of terms, to find the sum of the series.

RULE.—Raise the ratio to a power, whose index shall be equal to the number of terms, from which subtract 1; divide the remainder by the ratio, less 1, and the quotient, multiplied by the first term, will give the sum of the series.

E X A M P L E S.

1. If the first term be 5, the ratio 3, and the number of terms 7; What is the sum of the series?

Ratio $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187 =$ 7th power of the ratio,
Subtract 1

Divide by the ratio less 1 $= 3 - 1 = 2 \overline{) 2186}$

Quotient $= 1093$

Multiply by the first term $= 5$

Sum of the series $= 5465$

Or, $\frac{3^7 - 1}{3 - 1} \times 5 = 5465$ Ans.

2. A shopkeeper sold 13 yards of cloth, on the following terms, viz. 2d. for the first yard, 4d. for the second, 8d. for the third, &c. I demand the price of the cloth.

$\frac{2^{13} - 1}{2 - 1} \times 2 = 16382d. = 68l. 5s. 2d.$ Ans.

3. A gentleman, whose daughter was married on a new year's day, gave her a guinea, promising to triple it on the first day of each month in the year; Pray what did her portion amount to?

$$\frac{3^{12}-1}{3-1} \times 1 = 265720 \text{ guineas, } \text{Ans.}$$

4. What debt can be discharged in a year, by paying 1 shilling the first month, 10s. the second, and so on, each month in a tenfold proportion?

$$10^{12}-1 \div 10-1 \times 1 = 111111111111 \text{ s.} = 5555555555 \text{ l. } 11 \text{ s. } \text{Ans.}$$

5. A man threshed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the first day's work, 64 for the second, and so on in an eightfold proportion; I demand what his 9 days' labour amounted to, rating the wheat at 5s. per bushel?*

$$\frac{8^9-1}{8-1} \times 8 = 153391683 \text{ corns. Amount} = 78 \text{ l. } 0 \text{ s. } 5 \frac{1}{2} \text{ d. } \text{Ans.}$$

6. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz. that he should give him a penny for the first door, 2d. for the second, 4d. for the third, and so on, doubling at every door, which were 36 in all: It is a bargain, cried the simpleton, and here is a guinea to bind it; Pray, what would the house have cost him?

$$\frac{2^{36}-1}{2-1} \times 1 = 68719476735 \text{ d.} = \text{£} 286331153 \text{ 1s. } 3 \text{ d. } \text{Ans.}$$

7. A young fellow, well skilled in numbers, agreed with a rich farmer to serve him 10 years, without any other reward, but the produce of one wheat corn for the first year, and that produce to be sowed from year to year, till the end of the time, allowing the increase but in a tenfold proportion; What is the sum of the whole produce, and what will it amount to, at 5s. per bushel?

$$\frac{10^{10}-1}{10-1} \times 10 = 111111111110 \text{ corns. Amount} = \text{£} 5651 \text{ 8s. } \frac{9}{10} \text{ Ans.}$$

8. Suppose one farthing had been put out, at 6 per cent. per annum, Compound Interest,† at the commencement of the Christian Æra; What would it have amounted to in 1784 years; and suppose the amount to be in standard gold, allowing a cubic inch to be worth 53l. 2s. 8d. How large would the mass have been?

$$\text{Ans. } \frac{2^{150}-1}{2-1} \times 1 = \text{£} 148671634656874820943571455150989076706536111 \text{ 3}\frac{1}{2}.$$

$$= 27980859722121230415979571232933594210766 \text{ cubic inches of gold.}$$

As 355 : 113 :: 360 × 69,5 : 7964 earth's diameter, 360 × 69,5 × 7964 × 1327,33 = 264482820122 cubic miles in the globe,

= 67273337308854741368832000 cubic inches in the globe. Then,
27980859722121230415979571232933594210766 ÷ 67273337308854741368832000
= 415930899840288,8, which, however incredible it may appear to some, is more

* Note, 7680 wheat or barley corns are supposed to make a pint.

† Any sum, at 6l. per cent. per annum compound interest, will double in eleven years.

more than four hundred and fifteen millions of millions, nine hundred and thirty thousand, eight hundred and ninety nine million, eight hundred and forty thousand, two hundred and eighty eight times larger than the globe we inhabit.*

PROBLEM III.

The first term, the last term (or the extremes) and the ratio given, to find the sum of the series.

RULE.—Divide the difference of the extremes by the ratio less by 1 : Add the greater extreme to the quotient, and the result will be the sum of all the terms.

Or, Multiply the greatest term by the ratio, from the product subtract the least term ; then divide the remainder by the ratio, less by 1, and the quotient will be the sum of all the terms.

Or, When all the terms are given, then, from the product of the *second* and *last* terms subtract the *square* of the *first* term ; this remainder being divided by the *second* term less the *first*, will give the sum of the series.

EXAMPLES.

1. If the series be 2. 6. 18. 54. 162. 486. 1458. 4374. What is its sum total ?

First method.

$$\begin{array}{r} \text{From the greatest term} = 4374 \\ \text{Subtract the least} = 2 \\ \hline \end{array}$$

Divide by the ratio, less 1 = $3 - 1 = 2$) 4272 diff. of extremes.

$$\begin{array}{r} \text{Quotient} = 2186 \\ \text{Add the greater extreme} = 4374 \\ \hline \end{array}$$

$$6560$$

$$\text{Or, } \frac{4374 - 2}{3 - 1} + 4374 = 6560 \text{ Ans.}$$

Second method.

$$\begin{array}{r} \text{Greatest term} = 4374 \\ \text{Multiply by the ratio} = 3 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Product} = 13122 \\ \text{Subtract the least term} = 2 \\ \hline \end{array}$$

Divide by the ratio, less by $1 \times 3 - 1 = 2$) 13120

$$6560 \text{ Ans.}$$

$$\text{Or, } \frac{4374 \times 3 - 2}{3 - 1} = 6560$$

Third

years and three hundred and twenty five days, or 11,889 years, or 11,89 is near enough, then; if you divide 1784 by 11,89, it will give the number of terms in this case equal to 150 ; the ratio will be 2, and the first term 1.

* To find the solid content of a globe. See Art. 34th of Mensuration of Solids.

Note, that, 523598 is two thirds of, 785398 the area of a circle, whose diameter is 1.

Third method.

Greatest term = 4374

Multiply by the second term = 6

Product = 26244

Subtract the square of the first term = $2 \times 2 = 4$ Divide the remainder by the 2d term less the 1st = $6 - 2 = 4$) 26240

Ans. 6560

$$\text{Or, } \frac{4374 \times 6 - 4}{6 - 2} = 6560.$$

2. A man travelled 6 days, the first day he went 4 miles, and each day doubling his day's travel, his last day's ride was 128 miles; How far did he go in the whole?

$$\frac{128 - 4}{2 - 1} + 128 = 252 \text{ miles, Ans.}$$

3. A gentleman, dying, left 5 sons, to whom he bequeathed his estate as follows, viz. to his youngest son 1000*l.*; to the eldest 5062*l.* 10*s.* and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$; What did the several legacies amount to?

$$\frac{5062,5 - 1000}{1,5 - 1} + 5062,5 = \text{£}13,87 \text{ 10*s.* Ans.}$$

P R O B L E M I V.

Given the extremes and ratio, to find the number of terms.

RULE.—Divide the greatest term by the least; find what power of the ratio is equal to the quotient, then, add 1 to the index of that power, and the sum will be the number of terms.

Or, Subtract the logarithm* of the least term from that of the greatest; divide the remainder by the logarithm of the ratio, and add 1 to the quotient.

E X A M P L E S.

1. If the least term be 2, the greatest term 4374, and the ratio 3; What is the number of terms?

Divide by the least term = 2) 4374 = greatest term.

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = \text{quotient, } 2187 = 7\text{th power, then } 7 + 1 = 8 \text{ answer.}$$

Or,

* Logarithms are artificial numbers, the addition of which answers to multiplication of whole numbers, and subtraction, to division.

GEOMETRICAL PROGRESSION.

243

Or, From the logar. of the greatest term = 3.64088

Subtract the logarithm of the least term = 0.30103

Divide the remainder by } = 47712
the logarithm of the ratio } 3.33985 (7 + 1 = 8, *Ans.*
3.33984

1

2. A gentleman travelled 252 miles ; the first day he rode 4 miles ; the last 128, and each day's journey was double to the preceding one ; How many days was he in performing the journey ?

PROBLEM V.

Given the least term, the ratio, and the sum of the series, to find the last term.

RULE.—Multiply the sum of the series by the ratio, less 1, to that product add the first term, and the result, divided by the ratio, will give the last term.

EXAMPLES.

1. If the first term be 2, the ratio 3, and the sum of the series 6560 ; What is the last term ?

Sum of the series = 6560

Multiply by the ratio less 1 = 2

Product = 13120

Add the least term = 2

Divide their sum by their ratio = 3) 13122

4374 *Ans.*

Or, $\frac{3-1 \times 6560 + 2}{3} = 4374$ *Ans.*

2. A gentleman performed a journey of 252 miles ; the first day he rode 4 miles, and each day after the first, twice so far as the day before ; How far did he ride the last day ?

$\frac{2-1 \times 252 + 4}{2} = 128$ miles, *Ans.*

PROBLEM VI.

Given the least term, the ratio, and the sum of the series, to find the number of terms.

RULE.—To the product of the sum of the series and the ratio minus 1, add the first term ; which sum, divided by the first term, will give that power of the ratio signified by the number of terms.

Or, From the logarithm of the sum of the series plus the first term, multiplied by the ratio minus unity, take the logarithm of the first term ; the remainder, divided by the logarithm of the ratio, will give the number of terms.

EXAMPLE.

GEOMETRICAL PROGRESSION.

EXAMPLE.

If the first term be 2, the ratio 3, and the sum of the series 80; What is the number of terms?

$$\begin{array}{r}
 \text{Sum} = 80 \\
 \text{Multiply by the ratio less } 1 = 3 - 1 = 2 \\
 \hline
 160 \\
 \text{Add the first term} = 2 \\
 \hline
 162 \\
 \text{Divide by the first term} = 2 \quad 162
 \end{array}$$

81 which, found in the Table of Power, is the 4th power of the ratio, therefore, the number of terms is 4.

By Logarithms.

$$\begin{array}{r}
 \text{Sum} = 80 \\
 \text{Add the first term} = 2 \\
 \hline
 82 \\
 \text{Multiply by the ratio less } 1 = 3 - 1 = 2 \\
 \hline
 \text{Logarithm of } 164 = 2.21484 \\
 \text{Subtract the logarithm of the first term} = .30103 \\
 \hline
 \text{Divide by the logarithm of the ratio} = .47712 \quad 1.91381 \quad (4 \text{ Anf.}) \\
 \hline
 1.90848
 \end{array}$$

533

PROBLEM VII.

Given the extremes, and the sum of the series, to find the ratio.

RULE.—From the sum of the series subtract the least term; divide the remainder by the sum of the series minus the greatest term, and the quotient will be the ratio.

EXAMPLES.

1. If the least term be 2, the greatest term 4374, and the sum of the series 6560; What is the ratio?

$$\begin{array}{r}
 \text{Sum of the series} = 6560 \\
 \text{Subtract the least term} = 2 \\
 \hline
 \text{Divide the rem. by the sum of } \left. \begin{array}{l} \text{the series, minus greatest term} \end{array} \right\} = 6560 - 4374 = 2186 \quad 6558 \quad (3 \text{ Anf.}) \\
 \hline
 6558
 \end{array}$$

2, A

2. A debt of 25*l.* was paid in Geometrical Progression, the first payment was 4*l.* and the last 128*l.* In what ratio did the payments exceed each other?

$$\text{Ans. } \frac{252-4}{252-128} = 2, \text{ viz. a double ratio.}$$

PROBLEM VIII.

Given the extremes, and the sum of the series, to find the number of terms.

RULE I.—From the logarithm of the last term subtract the logarithm of the first, and make the remainder a dividend.

2. Subtract the logarithm of the sum minus the last term from the logarithm of the sum minus the first term, and make the remainder a divisor.

3. Divide the dividend by the divisor, and the quotient plus 1, will be equal to the number of terms.

EXAMPLE.

If the least term be 2, the greatest term 4374, and the sum of the series 6560; What is the number of terms?

From the logarithm of the greatest term = 3.64088

Take the logarithm of the least term = 0.30103

Dividend = 3.33985

From the logarithm of the sum minus the first term = 3.81677

Take the logarithm of the sum minus the last term = 3.33965

Divisor = .47712

Then, .47712) 3.33985 (7 + 1 = 8 Ans.

3 33984

1

$$\text{Or, } \frac{L.4374 - L.2}{L.6558 - L.2186} + 1 = 8.$$

PROBLEM IX.

The first term, the number of terms, and the last term given, to find the ratio.

RULE.—Divide the greater extreme by the less, and extract such root of the quotient, whose index is equal to the number of terms, less 1. Or, find the quotient in the Table of Powers, the root of which is the answer.

EXAMPLES.

1. Given the extremes 2 and 4374, and the number of terms 8; What is the ratio?

Divide

Divide by the least term $\div 2$ $4374 =$ greatest term.

$$\sqrt[7]{2187} = 3$$

$$\text{Or, } \frac{4374}{2} \bigg| \frac{1}{8-1} = 3, \text{ Ans.}$$

PROBLEM X.

The extremes and number of terms given, to find the sum of the series.

RULE 1.—Subtract the least term from the greatest, and make the difference a dividend.

2. Divide the greatest term by the least, and extract such root of the quotient, whose index is equal to the number of terms less 1; take 1 from the said root, and make the remainder a divisor. (Or find the quotient in the table of powers, which will shew the root, from which subtract 1.)

3. Divide the dividend by the divisor, and the greatest term, added to the quotient, will give the sum of the series.

EXAMPLE.

Given the extremes 2 and 4374, and the number of terms 8; What is the sum of the series?

From the greatest term $\div 2$ 4374

Take the least \div 2

Make this remainder a dividend 4372

Divide the greatest term by the least $\div 2$ 4374

And extract the 7th root of the quotient, $\sqrt[7]{2187} = 3$: Then,
 $3 - 1 = 2$ 4372

Quotient $= 2186$

Add the greatest term $= 4374$

6560 Ans.

$$\text{Or, } 4374 + \frac{4374 - 2}{\frac{1}{8-1}} = 6560$$

PROBLEM XI.

Given the ratio, the number of terms, and the greatest term, to find the least term.

RULE.—Divide the greatest term by such power of the ratio, whose index is equal to the number of terms, less 1, and the quotient will be the least term.

EXAMPLE.

EXAMPLE.

If the ratio be 2, the number of terms 6, and the greatest term 128; What is the least?

Divide the last term by $2 \times 2 \times 2 \times 2 \times 2$ } = 32) 128 (4 Ans.
= 5th. power of the ratio 128

$$\text{Or, } \frac{128}{2^{6-1}} = 4.$$

PROBLEM XII.

Given the ratio, the number of terms, and the greatest term, to find the sum of the series.

RULE 1.—Divide the greatest term by such power of the ratio, whose index is equal to the number of terms less 1; take the quotient from the last term, and make the remainder a dividend.

2. Divide the dividend by the ratio less 1, and the quotient, added to the greatest term, will give the sum of the series.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the greatest term 3072; What is the sum of the series?

Divide the last term by the } = $4 \times 4 \times 4 \times 4 \times 4 = 1024$) 3072 (3
5th power of the ratio 3072

From the last term = 3072

Take the quotient = 3

Divide by the ratio less 1 = $4 - 1 = 3$) 3069

Quotient = 1023

Add the greatest term = 3072

Ans. = 4095

$$\text{Or, } 3072 + \frac{3072 - \frac{3072}{4^{6-1}}}{4-1} = 4095.$$

PROBLEM XIII.

Given the ratio, the number of terms, and the sum of the series, to find the least term.

RULE.—Divide the ratio, less 1, by such power, less 1, of the ratio, whose index is equal to the number of terms, and the quotient, multiplied by the sum of the series, will give the least term.

EXAMPLE.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the sum of the series 4095 : What is the least term ?

$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$, and $4096 - 1 = 4095$, then, the ratio less 1, divided by 4095, is $\frac{3}{4095}$, and $\frac{3}{4095} \times \frac{4095}{1} = \frac{12285}{4095} = 3$ Answer.

$$\text{Or, } \frac{4^6 - 1}{4^6 - 1} \times 4095 = 3.$$

PROBLEM XIV.

Given the ratio, the number of terms, and the sum of the series, to find the greatest term.

RULE 1.—Subtract that power of the ratio, which is equal to the number of terms less 1, from that power of it, which is equal to the whole number of terms.

2. Divide the remainder by that power of the ratio minus unity which is equal to the number of terms, and the quotient, multiplied by the sum of the series, will give the greatest term.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the sum of the series 4095 : What is the greatest term ?

From $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6 = 4096$

Subtract $4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

Divide by $4^6 - 1 = 4095$ $3072 = \frac{3072}{4095}$ which, multipli-

ed by the sum, is $\frac{3072}{4095} \times \frac{4095}{1} = \frac{12579840}{4095} = 3072$ Ans.

$$\text{Or, } \frac{4^6 - 4^{6-1}}{4^6 - 1} \times 4095 = 3072.$$

The two last problems may be solved by one short operation thus : Divide the sum by the ratio, and the remainder after the operation will be the least term ; then take the quotient from the sum of the series, and the remainder will be the greatest term.

For the least term.

4)4095(1023 quotient.

4

09
8

15

12

3 Ans.

For the greatest term.

From the sum = 4095

Subtract the quotient = 1023

Ans. = 3072

PROBLEM XV.

Given the ratio, the last term, and the sum of the series, to find the first term.

RULE.—From the sum of the series take the last term, and multiply the remainder by the ratio; then take this product from the sum of the series, and the remainder will be the first term.

EXAMPLE.

If the ratio be 4, the last term 3072, and the sum of the series 4095; What is the first term?

From the sum = 4095
Take the last term = 3072

Remainder = 1023
Multiply by the ratio = 4

Subtract 4092 from the sum.

And the remainder 3 is the Answer.

PROBLEM XVI.

Given the ratio, the last term, and the sum of the series, to find the number of terms.

RULE 1.—Multiply the difference between the sum and the last term by the ratio, and note the product.

2. Subtract this product from the sum, and note the remainder.

3. From the logarithm of the last term subtract the logarithm of the remainder.

4. Divide this last remainder by the logarithm of the ratio, and the quotient, plus unity, will give the number of terms.

EXAMPLE.

If the ratio be 3, the last term 54, and the sum of the series 80; What is the number of terms?

From the sum = 80
Take the last term = 54

Remainder = 26
Multiply by the ratio = 3

Product = 78
From the sum = 80
Take the product = 78

Remainder = 2

From the logarithm of 54 = 1.73239

Take the logarithm of the remainder = .30103

Divide by the logar. of the ratio = .47712) 1.43136 (3 + 1 = 4 Ans.

GEOMETRICAL PROGRESSION.

PROBLEM XVII. and XVIII.

Given the number of terms, the last term, and the sum of the series, to find the first term and the ratio.

The solution of these two Problems being very tedious by the Theorems, they may be solved by a very short operation; thus, Divide the sum of the series by the difference between the sum and the last term; the quotient will give the ratio, and the remainder, after the operation, the first term.

EXAMPLE.

The following Table exhibits a summary view of the doctrine of Geometrical Progression.

CASES of GEOMETRICAL PROGRESSION.			
Case	Given	Required	Solution.
1.	arn	l	$\frac{n-1}{ar}$
		s	$\frac{\frac{n}{r-1} - 1}{r-1} \times a$
2.	arl	s	$l + \frac{l-a}{r-1}$
		n	$\frac{L.l - L.a}{L.r} + 1$
3.	ars	l	$\frac{\frac{n-1}{r-1} \times s \times a}{r}$
		n	$\frac{L.r - 1 \times s + a - L.a}{L.r}$
4.	als	r	$\frac{s-a}{s-l}$
		n	$\frac{L.l - L.a}{L.s - a - L.s - l} + 1$
5.	ans	r	$\frac{rs}{a} - r^{n-1} = \frac{s-a}{a}$
		l	$l \times \frac{n-1}{s-l} = a \times \frac{n-1}{s-a}$

GEOMETRICAL PROGRESSION.

251

EXAMPLE.

If the number of terms be 4, the last term 54, and the sum of the series 80; Required the first term and the ratio?

From the sum=80

Take the last term=54

Divide by the difference=26)80(3 the ratio.

The first term= $\frac{78}{2}$ SIMPLE

Case	Given	Required	Solution.
6.	anl	r	$\frac{l}{a} \sqrt[n-1]{\frac{1}{a}}$
		s	$l + \frac{l-a}{r} \sqrt[n-1]{\frac{1}{a}} - 1$
7.	rnl	a	$\frac{l}{r} \sqrt[n-1]{\frac{1}{a}}$
		s	$l + \frac{l}{r} \sqrt[n-1]{\frac{1}{a}} - 1$
8.	rns	a	$\frac{r-1}{n} \times s$
		l	$\frac{r^n - r^{n-1}}{n} \times s$
9.	rls	a	$s - r \times s - l$
		n	$\frac{L.l - L.s - r \times s - l}{L.r} + s$
10.	nls	a	$\frac{a \times s - a}{r-1} \sqrt[n-1]{\frac{1}{a}} = l \sqrt[n-1]{\frac{1}{a}}$
		r	$\frac{n}{l-s} \times \frac{s}{l} = \frac{l}{l-s}$

Here {

a = first or least term.

l = last or greatest term.

s = sum of all the terms.

n = number of terms.

r = ratio.

L = logarithm.

SIMPLE INTEREST.

Interest is a premium allowed by the borrower of any sum of money to the lender, according to a certain rate per cent. agreed on, which by law is stated at 6*l.* that is, 6*l.* for the use of 100*l.* for one year, &c.

Principal is the money lent.

Rate is the sum per cent. agreed on.

Amount is the principal and interest added together.

Interest is of two sorts, *simple* and *compound*.

Simple Interest is that, which is allowed for the principal lent only.

Note. The rules for Simple Interest serve also to calculate commission, brokerage, insurance, purchasing stocks, or any thing else rated at so much per cent.

GENERAL RULE.

1. Multiply the principal by the rate, and divide the product by 100 (or, which is the same, cut off the two right hand figures in the pounds, which must be reduced to the lowest denomination, each time cutting off as at first) and the quotient will be the answer for one year.

2. For more years than one, multiply the interest of one year by the given number of years, and the product will be the answer for that time.

3. If there be parts of a year, as months, or days, work for the months by the aliquot parts of a year, and for the days, by the Rule of Three Direct, or (which is sufficiently exact for common use,) allowing 30 days to the month, take aliquot parts of the same.

Note. When the rate per cent. per annum, is $\left\{ \begin{array}{l} 9 \\ 8 \\ 6 \\ 4 \\ 3 \\ 2 \end{array} \right.$ multiply the principal by $\left\{ \begin{array}{l} \frac{2}{4} \\ \frac{4}{2} \\ \frac{3}{1} \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{6} \end{array} \right.$ of the given number of months, cutting off, as before directed, and you will have the interest for the given time.

EXAMPLES.

1. What is the interest of 573*l.* 13*s.* 9½*d.* for one year, at 6*l.* per cent. per ann.?

£. s. d.
573 13 9½

34|42 2 9

20

8|42

12

5|3

4

Ans. £ 34 8*s.* 5*d.*

What

2. What is the interest of $329\text{ }17\text{ }6\frac{1}{2}\text{d.}$ for 3 years, 7 months, and 12 days, at 5% per cent. per annum?

£.	s.	d.
329	17	$6\frac{1}{2}$
<hr/>		
16	49	$7\text{ }8\frac{1}{2}$
<hr/>		
9	87	
<hr/>		
10	52	
<hr/>		
2	10	

Or thus :

£	s.	d.
329	17	$6\frac{1}{2}$
<hr/>		
16	9	$10\frac{1}{2}$
<hr/>		
49	9	$7\frac{1}{2}$
<hr/>		
8	4	$11\frac{1}{4}$
<hr/>		
1	7	$5\frac{3}{4}$
<hr/>		
9	14	$\frac{1}{2}$
<hr/>		
1	9	
<hr/>		
£59	13	Ans.

Then,

6 months	$\frac{1}{2}$	16	9	$10\frac{1}{2}$	interest of 1 year
<hr/>					3
1 month	$\frac{1}{6}$	49	9	$7\frac{1}{2}$	ditto of 3 years
10 days	$\frac{1}{3}$	8	4	$11\frac{1}{4}$	ditto of 6 months
2 days	$\frac{1}{3}$	1	7	$5\frac{3}{4}$	ditto of 1 month
<hr/>					9
<hr/>					14
<hr/>					9
<hr/>					12
<hr/>					Ans.

59 13

Y. m. d.
ditto of 3 7 12

3. What is the interest of $439\text{ }12\text{ }9\frac{1}{2}\text{d.}$ at 6% per cent. per ann. for 16 months?

£. s. d.
439 12 $9\frac{1}{2}$
 $8 = \frac{1}{2}$ the number of months.

Ans. £35 | 17 2 4
20
s. 3 | 42
12
d. 5 | 08

4. What

4. What is the interest of 59*l.* 15*s.* 9½*d.* for 15 months, at 8*l.* per cent. per ann. ? 10 = $\frac{2}{3}$ the number of months.

$$\begin{array}{r} \text{£} 59 | 17 \quad 17 \quad 8\frac{1}{2} \\ \hline \quad \quad \quad 20 \end{array}$$

$$\begin{array}{r} \text{s. } 3 | 57 \\ \hline \quad \quad 12 \end{array}$$

$$\begin{array}{r} \text{d. } 6 | 92 \\ \hline \quad \quad 4 \end{array}$$

$$\text{qrs. } 3 | 70$$

5. What is the interest of 347*l.* 7*s.* 5½*d.* at 4*l.* per cent. per ann. for 15 months ? 5 = $\frac{1}{3}$ the number of months.

$$\begin{array}{r} \text{£} 17 | 36 \quad 17 \quad 3\frac{1}{2} \\ \hline \quad \quad \quad 20 \end{array}$$

$$\begin{array}{r} \text{s. } 7 | 37 \\ \hline \quad \quad 12 \end{array}$$

$$\begin{array}{r} \text{d. } 4 | 47 \\ \hline \quad \quad 4 \end{array}$$

$$\text{qr. } 1 | 90$$

6. What is the interest of 517*l.* 15*s.* 4*d.* for 1 month, at 6*l.* per cent. per ann. ? Ans. £ 2 11 9¼.

7. Of 457*l.* 12*s.* 8½*d.* for 2 months, at 6*l.* per cent. per ann. ? Ans. £ 4 11 6¼.

8. Of 347*l.* 5*s.* 9*d.* for 8 months, at 6*l.* per cent. per ann. ? Ans. £ 5 4 2.

9. Of 397*l.* 19*s.* for 4 months, at 6*l.* per cent. per ann. ? Ans. £ 7 19 2.

10. 508*l.* 10*s.* 5½*d.* for 5 months, at 6*l.* per cent. per ann. ? Ans. £ 12 14 3.

11. 719*l.* 19*s.* 4*d.* for 6 months, at 6*l.* per cent. per ann. ? Ans. £ 21 11 11¼.

12. 396*l.* 5*s.* 10*d.* for 7 months, at 6*l.* per cent. per ann. ? Ans. £ 13 17 4¾.

13. 517*l.* 11*s.* 11½*d.* for 8 months, at 6*l.* per cent. per ann. ? Ans. £ 20 14 0¾.

14. 245*l.* 5*s.* 8¾*d.* for 9 months, at 6*l.* per cent. per ann. ? Ans. £ 11 0 9.

15. 195*l.* 15*s.* 5½*d.* for 10 months, at 6*l.* per cent. per ann. ? Ans. £ 9 15 9¼.

16. 148*l.* 12*s.* 6½*d.* for 11 months, at 6*l.* per cent. per ann. ?
Ans. £8 3 5¼.
17. 509*l.* 9*s.* 2*d.* for 13 months, at 6*l.* per cent. per ann. ?
Ans. £33 2 3½.
18. 317*l.* 17*s.* 8½*d.* for 14 months, at 6*l.* per cent. per ann. ?
Ans. £22 5 0¼.
19. 443*l.* 10*s.* 3*d.* for 15 months, at 6*l.* per cent. per ann. ?
Ans. £33 5 3.
20. 293*l.* 7*s.* 9*d.* for 16 months, at 6*l.* per cent. per ann. ?
Ans. £23 9 5.
21. 333*l.* 13*s.* 3¼*d.* for 17 months, at 6*l.* per cent. per ann. ?
Ans. £28 7 2¾.
22. 517*l.* 6*s.* 6*d.* for 18 months, at 6*l.* per cent. per ann. ?
Ans. £46 11 2.
23. 347*l.* 11*s.* 7½*d.* for 19 months, at 6*l.* per cent. per ann. ?
Ans. £33 0 4¾.
24. 419*l.* 12*s.* 5*d.* for 20 months, at 6*l.* per cent. per ann. ?
Ans. £41 19 2¾.
25. 537*l.* 13*s.* 5½*d.* for 16 months, at 9*l.* per cent. per ann. ?
Ans. £64 10 4¼.
26. 197*l.* 19*s.* 1½*d.* for 15 months, at 8*l.* per cent. per ann. ?
Ans. £19 15 10¼.
27. 217*l.* 10*s.* 4½*d.* for 18 months, at 4*l.* per cent. per ann. ?
Ans. £13 1.
28. 327*l.* 15*s.* 9*d.* for 16 months, at 3*l.* per cent. per ann. ?
Ans. £13 2 2¼.
29. 487*l.* 16*s.* 4½*d.* for 30 months, at 2*l.* per cent. per ann. ?
Ans. £24 7 9¾.
30. 1*l.* for 1 year, at 6 per cent. per ann. ?
Ans. 1*s.* 2*d.* 1⅙*gr.*
31. 517*l.* 12*s.* 8½*d.* for 5 years, 11 months and 25 days, at 6*l.* per cent. per ann. ?
Ans. £185 18 4.

A TABLE of RATIOS, from one pound, &c. to ten pounds.

Rate per cent.	Ratios.	Rate per cent.	Ratios.	Rate per cent.	Ratios.
1	,01	4	,04	7	,07
1 $\frac{1}{4}$,0125	4 $\frac{1}{4}$,0425	7 $\frac{1}{4}$,0725
1 $\frac{1}{2}$,015	4 $\frac{1}{2}$,045	7 $\frac{1}{2}$,075
1 $\frac{3}{4}$,0175	4 $\frac{3}{4}$,0475	7 $\frac{3}{4}$,0775
2	,02	5	,05	8	,08
2 $\frac{1}{4}$,0225	5 $\frac{1}{4}$,0525	8 $\frac{1}{4}$,0825
2 $\frac{1}{2}$,025	5 $\frac{1}{2}$,055	8 $\frac{1}{2}$,085
2 $\frac{3}{4}$,0275	5 $\frac{3}{4}$,0575	8 $\frac{3}{4}$,0875
3	,03	6	,06	9	,09
3 $\frac{1}{4}$,0325	6 $\frac{1}{4}$,0625	9 $\frac{1}{4}$,0925
3 $\frac{1}{2}$,035	6 $\frac{1}{2}$,065	9 $\frac{1}{2}$,095
3 $\frac{3}{4}$,0375	6 $\frac{3}{4}$,0675	9 $\frac{3}{4}$,0975
				10	1.

Ratio is the Simple Interest of 1*l.* for 1 year, at the rate per cent. agreed on.

A TABLE for the ready finding the decimal parts of a year, equal to any number of days, or quarters of a year.

Days.	Decimal parts.	Days.	Decimal parts.	Days.	Decimal parts.
1	,00274	10	,027397	100	,273973
2	,005479	20	,054794	200	,547945
3	,008219	30	,082192	300	,821918
4	,010959	40	,109589	365	1,000000
5	,013699	50	,136986	$\frac{1}{4}$ of a year = ,25 $\frac{1}{2}$ of a year = ,5 $\frac{3}{4}$ of a year = ,75	
6	,016438	60	,164383		
7	,019178	70	,191781		
8	,021918	80	,219178		
9	,024657	90	,246575		

C A S E I.*

The principal, time, and ratio are given, to find the interest and amount.

RULE.—Multiply the principal, time and ratio continually together, and the last product will be the interest, commission, brokerage, &c. to which add the principal, and the sum will be the amount.

E X A M P L E S.

1. Required the amount of 537*l.* 10*s.* at 6*l.* per cent. per ann. for 5 years? Principal

* The following Theorems will shew all the possible cases of Simple Interest: Where *p* = principal, *t* = time, *r* = ratio, and *a* = amount.

$$I. \text{ } p \cdot t \cdot r = a. \quad II. \frac{a}{t \cdot r} = p. \quad III. \frac{a - p}{t \cdot p} = r. \quad IV. \frac{a - p}{r \cdot p} = t.$$

Principal 537,5
Multiply by the ratio = ,06

Product 32,250
Multiply by the time = 5

Interest = 161,250
Add the principal = 537,5

Amount = £698,75
20

15,00 *Ans.* £698 15s.

Or, $537,5 \times ,06 \times 5 + 537,5 = £698 15s.$

2. What is the simple interest of 917*l.* 16*s.* at 5*l.* per cent. per ann. for 7 years ? *Ans.* £321 4 7.

3. What is the amount of 391*l.* 17*s.* at 4½*l.* per cent. per ann. for 3¼ years ? *Ans.* £449 3 1¼.

4. What is the amount of 235*l.* 3*s.* 9*d.* at 5¼*l.* per cent. per ann. from March 5th, 1784, to November 23d, 1784 ? *Ans.* £244 0 8½.

5. If my correspondent is to have 2½*l.* per cent. ; What will his commission on 785*l.* 15*s.* amount to ? *Ans.* £19 12 10½.

6. What will be the interest and amount of 445*l.* 10*s.* in 3 years and 129 days, at 8½*l.* per cent. per annum ? *Ans.* Interest, £126 19 8½, and the amount = £572 9 8½.

7. If a broker disposes of a cargo for me, to the amount of 637*l.* 10*s.* on commission at 1¼*l.* per cent. and procures me another cargo of the value of 817*l.* 15*s.* on commission at 1¼*l.* per cent. ; What will his commission, on both cargoes, amount to ? *Ans.* £22 5 7.

C A S E II.

The amount, time, and ratio given, to find the principal.

RULE.—Multiply the ratio by the time ; add unity to the product for a divisor, by which sum divide the amount, and the quotient will be the principal.

E X A M P L E S .

1. What principal will amount to 1045*l.* 14*s.* in 7 years, at 6*l.* per cent. per annum ?

Ratio = ,06
Multiply by the time = 7

Product = ,42
Add 1

Divisor = (,42) 1045,7 (736,4084 ÷ = £736 8 2.
K k

Or,

$$\text{Or, } \frac{1045.7}{.06 \times 7 + 1} = £736 \text{ } 8 \text{ } 2 \text{ Ans.}$$

2. What principal will amount to 3810*l.* in 6 years, at $4\frac{1}{2}\%$ per cent. per annum ? *Ans. £3000.*

3. What principal will amount to 666*l.* 9*s.* 0*d.* in $3\frac{1}{2}$ years, at $5\frac{1}{4}\%$ per cent. per annum ? *Ans. £563.*

4. What principal will amount to 335*l.* 7*s.* 3*d.* in 3 years and 97 days, at $9\frac{1}{2}\%$ per cent. per annum ? *Ans. £255 19 0 $\frac{3}{4}$.*

C A S E III.

The amount, principal, and time given, to find the ratio.

RULE.—Subtract the principal from the amount ; divide the remainder by the product of the time and principal, and the quotient will be the ratio.

E X A M P L E S.

1. At what rate per cent. will 543*l.* amount to 705*l.* 18*s.* in 5 years ?

From the amount = 705.9

Take the principal = 543

$$\text{Divide by } 543 \times 5 = 2715 \quad \begin{array}{r} 162.90 \\ 162.90 \end{array}$$

$$\text{Or, } \frac{705.9 - 543}{543 \times 5} = .06 = £6 \text{ Ans.}$$

2. At what rate per cent. will 391*l.* 17*s.* amount to 449*l.* 3*s.* 1*d.* 74*qr.* in $3\frac{1}{4}$ years ? *Ans. £4 $\frac{1}{2}$.*

3. At what rate per cent. will 413*l.* 12*s.* 6*d.* amount to 546*l.* 4*s.* 10*d.* in $4\frac{3}{4}$ years ? *Ans. £6 $\frac{3}{4}$.*

4. At what rate per cent. will 3000*l.* amount to 3810*l.* in 6 years ? *Ans. £4 $\frac{1}{2}$.*

C A S E IV.

The amount, principal, and rate per cent. given, to find the time.

RULE.—Subtract the principal from the amount ; divide the remainder by the product of the ratio and principal ; and the quotient will be the time.

E X A M P L E S.

1. In what time will 543*l.* amount to 705*l.* 18*s.* at 6*l.* per cent. per annum ?

From the amount = 705.9

Take the principal = 543

$$\text{Divide by } 543 \times .06 = 32.58 \quad \begin{array}{r} 162.9 \\ 162.9 \end{array} \quad \text{5 years, Ans.}$$

2. In

2. In what time will 3000*l.* amount to 3810*l.* at $4\frac{1}{2}$ per cent. per annum ? *Ans.* 6 years.

3. In what time will 391*l.* 17*s.* amount to 449*l.* 3*s.* $1\frac{3}{4}$ *d.* at $4\frac{1}{2}$ per cent. per annum ? *Ans.* $3\frac{1}{4}$ years.

To find the Interest of any Sum, at 6 per cent. per ann. for any number of Months.

RULE.—If the months be an even number, multiply the principal by half that number; and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus, the half of 19 is 9,5; and multiply the principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first.

4. What is the interest of 345*l.* 16*s.* 6*d.* for 9 years and 11 months, at 6 per cent. per annum ? *Y. m.*

9 11
12

2) 119 months.

345,825
59,5

59,5 = $\frac{1}{2}$ No. of months.

1729125
3112425
1729125

£205,765875 = £205 15 $3\frac{3}{4}$ *Ans.*
Principal = £345 16 6

Amount = £551 11 $9\frac{3}{4}$

A TABLE of decimal parts for every day in the twelfth part of a year, which consists of $365\frac{1}{4}$ days.

Days	dec. pts.	Days	dec. pts.	Days	dec. pts.	Days	dec. pts.	Days	dec. pts.
1	,033	7	,23	13	,427	19	,624	25	,821
2	,066	8	,263	14	,46	20	,657	26	,854
3	,098	9	,296	15	,493	21	,69	27	,887
4	,131	10	,328	16	,526	22	,723	28	,92
5	,164	11	,361	17	,558	23	,756	29	,953
6	,197	12	,394	18	,591	24	,788	30	,986

To find the Interest of any Sum, either for Months, or Months and Days, at 6 per cent. per annum.

R U L E.

Multiply the principal by the number of months (or months and parts, answering to the given number of days in the table) and

260 SIMPLE INTEREST IN FEDERAL MONEY.

and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the interest for the given time, in shillings and decimal parts of a shilling.

1. What is the interest of 100*l.* for a year?

Principal = 100
Mult. by the months = 12

Anf. $s. 120 | 0 = 6l.$

Note. This Table may also be used for the *parts of a year*, in Compound Interest, after having worked for whole years.

2. What is the interest of 250*l.* 10*s.* for 19 months and 7 days?

Principal = £250.5
Time = 19, 23

7515
5010
22545
2505

Anf. $s. 481, 7115$
 $= £ 24 \ 1 \ 8\frac{1}{2}.$

*Another Method of calculating INTEREST for Months, at 6*l.* per cent. per annum.*

R U L E .

If the principal consist of pounds only, cut off the unit figure, and, as it then stands, it will be the interest for one month in shillings and decimal parts :—If it consist of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a shilling.

E X A M P L E S .

1. What is the interest of 175*l.* for 5 months?

£175 = 17.5 *Shill.* = interest for 1 month.

Mult. by the time = $\begin{array}{r} 17.5 \\ 5 \end{array}$

$2 | 0 | 87.5$

Anf. = £476

2. What is the interest of 255*l.* 16*s.* for 7 months?

Shill.
£255 16 = 25.58 int. for 1 mo.

$\begin{array}{r} 7 \end{array}$

2,0179,06

£819 0½ *Anf.*

SIMPLE INTEREST IN FEDERAL MONEY.

P R O B L E M I.

When the principal is given in lawful money, and the interest is required in federal money, at 6 per cent. per annum.

RULE.—Reduce the shillings, &c. to their equivalent decimal, by inspection, divide the whole by 5, and the quotient is the annual interest : Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off) will be the interest as before.

E X A M P L E S .

E X A M P L E S.

1. Required the annual interest of 517
- l.*
- 3
- s.*
- 7½
- d.*
- at 6 per cent. ?

$$\begin{array}{r}
 3s. = 15 \\
 1\frac{1}{2}d. = 30 \\
 \text{Excess of 12} = 1 \\
 \hline
 ,181
 \end{array}
 \quad
 \begin{array}{r}
 5)517,181 \text{ Dols. c. m.} \\
 103,436 = 103,43,6 \text{ Anf.} \\
 \hline
 \text{Or, } 517,181 \\
 \quad \quad \quad 2 \\
 \hline
 \text{Dols. c. m.} \\
 103,436 = 103,43,6\frac{2}{5}
 \end{array}$$

2. Required the annual interest of 1
- l.*
- in dollars ?

$$\begin{array}{r}
 5)1,00 \\
 20 \text{ cents, Anf.}
 \end{array}$$

P R O B L E M II.

When the principal is given in lawful money, and the interest and amount are required in federal money, at 6 per cent.

RULE.—Reduce the lawful money to federal, then divide the principal by 20 and that quotient by 5; add those quotients together, and they are the interest; or add them to the principal, and their sum is the amount.

E X A M P L E S.

1. Required the amount of 425
- l.*
- 16
- s.*
- 8½
- d.*
- for one year, at 6 per cent. ?

$$\begin{array}{r}
 ,8 \\
 34 \\
 1 \\
 \hline
 ,835
 \end{array}
 \quad
 \begin{array}{r}
 3)425,835 \\
 \hline
 20)1419,450 \\
 5)70,9725 \\
 14,1945 \\
 \hline
 \text{Dols. c. m.} \\
 1504,6170 = 1504,61,7 \text{ Anf.}
 \end{array}$$

2. Required the amount of 112
- l.*
- 4
- s.*
- 6
- d.*
- for 1 year ?

$$\begin{array}{r}
 ,2 \\
 24 \\
 1 \\
 \hline
 ,225
 \end{array}
 \quad
 \begin{array}{r}
 3)112,225 \\
 \hline
 20)374,083 \\
 5)18,7041 \\
 3,7408 \\
 \hline
 \text{Dols. c. m.} \\
 396,52,79 = 396,52,7,9 \text{ Anf.}
 \end{array}$$

P R O B L E M III.

When the principal is lawful money, and the monthly interest is required in federal money.

RULE.—Reduce the shillings, &c. to decimals, by inspection, then separate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the answer in dollars, cents, &c.

EXAMPLE

262 SIMPLE INTEREST IN FEDERAL MONEY.

E X A M P L E.

Required the monthly interest of 425^l. 16s. 8½^d. in federal money?

$$\begin{array}{r}
 8 \\
 34 \\
 1 \\
 \hline
 835
 \end{array}
 \qquad
 \begin{array}{l}
 6)42,5835 \text{ Dols.c. m.} \\
 7,09725 = 7,09,7\frac{1}{4} \text{ Anf.}
 \end{array}$$

P R O B L E M IV.

When the principal is federal money, and the interest is required in the same.

RULE.—Work according to the general rule in simple interest, that is, multiply by the rate of interest, separate the two right hand figures of the dollars in the product, and it will give the interest in dollars, cents, &c.

N. B. The figures, which are more than three places to the right hand of the comma, are of no account.

E X A M P L E S.

1. What is the annual interest of 537^D. 24^c. 6^m. at 6 per cent.?

$$\begin{array}{r}
 \text{D. c. m.} \\
 537,24,6 \\
 6 \\
 \hline
 \text{D. c. m.} \\
 32,23476 = 32,23,4 \text{ Anf.}
 \end{array}$$

2. What is the interest of 1465^D. 46^c. 6^m. for 16 months, at 6 per cent. per annum?

$$\begin{array}{r}
 \text{D. c. m.} \\
 1465,46,6 \\
 8 = \text{half the number of months.} \\
 \hline
 \text{D. c. m.} \\
 117,23728 = 117,23,7 \text{ Anf.}
 \end{array}$$

3. What is the interest of 537^D. 34^c. 7^m. for 19 months, at 6 per cent. per annum?

$$\begin{array}{r}
 \text{D. c. m.} \\
 537,34,7 \\
 9,5 = \text{half the number of months.} \\
 \hline
 2686735 \\
 4836123 \\
 \hline
 \text{D. c. m.} \\
 51,047965 = 51,04,7 \text{ Anf.}
 \end{array}$$

N. B. Because there are 4 decimals in the multiplicand and multiplier, I cut off 4 figures for them, and two more according to the rule.

P R O B L E M V.

When the principal is federal money, and the monthly interest is required in the same, at 6 per cent. per annum.

RULE.—Separate the two right hand figures of the dollars, and you then have the interest of two months ; half of which is the monthly interest in dollars, cents, &c. If there is but one place, or figure, of dollars, a cypher must be prefixed to the left hand.

E X A M P L E S.

1. What is the monthly interest of 9*D.* 59*c.* 7*m.* at 6 per cent. per annum ?

$$\begin{array}{r} 2), 09597 \quad c. m. \\ , 047985 = 4,8 \text{ nearly, } Ans. \end{array}$$

2. What is the monthly interest of 100*D.* 50*c.* 5*m.* ?

$$\begin{array}{r} 2) 1, 00505 \quad c. m. \\ , 50252 = 50, 2\frac{1}{2}, \text{ } Ans. \end{array}$$

Rules for calculating Interest for Days.

R U L E I.

Multiply the given principal by the given number of days, and that product by the rate on the pound : Divide the last product by 365 (the number of days in a year) and it will give the interest.

E X A M P L E.

What is the interest of 360*l.* 10*s.* for 175 days, at 6 per cent. ?

$$\frac{360,5 \times 175 \times ,06}{365} = \text{£} 10,37 = \text{£} 10 \text{ } 7\text{s. } 4\frac{3}{4}\text{d.}$$

R U L E II.

Multiply the given principal by the given number of days, and divide the product by 6083, for 6*l.* per cent. ; (the number of days in which any sum will double, at that rate) the quotient will give the answer.

E X A M P L E.

What is the interest of 327*l.* 10*s.* at 6 per cent. per annum, for 210 days ?

$$\frac{327,5 \times 210}{6083} = \text{£} 11,306 = \text{£} 11 \text{ } 6 \text{ } 1\frac{1}{4} \text{ } Ans.$$

Rule for making a Divisor for any Rate per Cent.

Multiply 365 by 100, and divide the product by the rate.

Thus, for 6 per cent. $\frac{365 \times 100}{6} = 6083$ divisor.

For 5 per cent. $\frac{365 \times 100}{5} = 7300$ divisor, &c.

Perhaps

Perhaps the most convenient way to calculate at 6 per cent. is first to do it for 5 and then add one fifth of the quotient to itself; because, by cutting off the two cyphers in the divisor, you have to divide only by 73.

Hence, when interest is to be calculated on cash accounts, accounts current, or any other accounts, where partial payments are made, or partial debts are contracted; multiply the several balances into the days they are at interest, and the sum of these products, divided as above, will give the interest at 5*l.* or 6*l.* per cent. and for any other rate, make the proper addition or deduction; or find a divisor, as before directed.

E X A M P L E.

On the 1st of January I lent 450*l.* 10*s.* 6*d.* which I received back in the following partial payments, viz. on the 14th of January, 57*l.* 11*s.* 9*d.*; on the 7th of February, 39*l.* 3*s.* 10*d.*; on the 19th of March, 63*l.* 5*s.* 2*d.*; on the 4th of April, 45*l.*; on the 26th of April, 19*l.* 12*s.* 6*d.*; on the 12th of May, 100*l.*; on the 10th of June, 60*l.* 7*s.* 3*d.*; and on the 1st of August, 65*l.* 10*s.*; What interest is due at 6 per cent.?

Dates.			£.	s.	d.	Days.	Products.
January	1	Lent on demand - - - -	450	10	6	13	5856 16 6
	14	Received in part - - - -	57	11	9		
		Balance	392	18	9	24	9430 10 6
February	7	Received in part - - - -	39	3	10		
		Bal.	353	14	11	40	14149 16 8
March	19	Received in part - - - -	63	5	2		
		Bal.	290	9	9	16	4647 16 0
April	4	Received in part - - - -	45	0	0		
		Bal.	245	9	9	22	5400 14 6
Ditto	26	Received in part - - - -	19	12	6		
		Bal.	225	17	3	16	3613 16 0
May	12	Received in part - - - -	100	0	0		
		Bal.	125	17	3	29	3650 0 3
June	10	Received in part - - - -	60	7	3		
		Bal.	65	10	0	52	3406 0 0
August	1	Received in full of the principal	65	10	0		50155 9 14

SIMPLE INTEREST BY DECIMALS.

263

(5)

73,00)501,55 9 11(6 17 $4\frac{3}{4}$ interest at 5 per cent.

438

1 7 $5\frac{3}{4}$

6355

£8 4 $10\frac{1}{2}$ ditto at 6 per cent.

20

)1271,09

73

541

511

3009

12

)361,19

292

6919

4

)276,76

219

5776

I have given Peter Trusty a cash credit for 1000*l.* in consequence of which, on the 12th of May, I paid his bill for 250*l.* ; ditto 27th, paid his draught for 280*l.* ; June 1st, he gave me a bill on the Massachusetts bank at sight for 290*l.* ; July 17th, he paid me per receipt 70*l.* ; August 20th, he drew for 750*l.* at sight ; ditto 31st, he paid me per receipt, 500*l.* ; Sept. 15th, he drew at sight for 135*l.* ; and 3d of October, for 175*l.* ; October 29th, he paid me per receipt 250*l.* ; and November 3d, 125*l.* ; November 12th, he drew at sight for 375*l.* ; and ditto 18th, for 125*l.* ; January 1st, he paid me per receipt 290*l.* ; and 20th ditto, 210*l.* On the 1st of March following he demands a settle-

ment ; What is then due to me, interest at 6 per cent. ?

Dates.			£.	Days.	Products.
May	12	Paid his bill - - - -	250	15	3750
Ditto	27	Paid his draught - - -	280		
		Balance	530	5	2650
June	1	Received in part - - -	290		
		Bal.	240	46	11040
July	17	Received in part - - -	70		
		Bal.	170	34	5780
August	20	Paid - - - - -	750		
		Bal.	920	11	10120
Ditto	31	Received in part - - -	500		
		Bal.	420	15	6300
September	15	Paid - - - - -	135		
		Bal.	555	18	9990
October	3	Paid - - - - -	175		
		Bal.	730	26	18980
Ditto	29	Received in part - - -	250		
		Bal.	480	5	2400 - 71010
November	3	Received in part - - -	125		
		Carried over Bal.	355		

L 1

Dates.

SIMPLE INTEREST BY DECIMALS.

Dates.			£. Days. Products.		
			71010 brought over.		
November	12	Paid - - - - -	Bal. 355	9	3195
			375		

Ditto	18	Paid - - - - -	Bal. 730	6	4380
			125		

January	1	Received in part - -	Bal. 855	44	37620
			290		

Ditto	20	Received in part - -	Bal. 565	19	10735
			210		

March	1		Bal. 355	40	14200

					<u>141140</u>

(5)
Then, 73,00) 141140 (19 6 8 interest at 5 per cent.
3 17 4

£23 4 0 interest at 6 per cent.
355 0 0

£378 4 0 balance in my favour.

When cash credits are given, a balance should be made upon every transaction, which should be multiplied into the days the first leisure minute; then, when the time of settlement comes, you will only have to add up the products, and divide as above, and the account will be finished.

A owes B the following sums, with the interest on them, at 6 per cent. per annum, as follows; viz. 60*l.* for 7 months, 150*l.* for 15 months, 75*l.* 10*s.* for 9 months, 145*l.* 15*s.* for 27 months, and 397*l.* 12*s.* for 45½ months; What is the amount of principal and interest?

£.	Months.	
60	× 7	= 420
150	× 15	= 2250
75.5	× 9	= 679.5
145.75	× 27	= 3935.25
397.6	× 45.5	= 18090.8
		£ 200) 25375.55 (126,877 interest.
		828,85 principal.

£955,727 amount, Answer.

Note. I divide by 200, the number of months, in which any sum will double at 6 per cent. per annum, and it gives the interest.

When partial payments are made upon notes, bonds, &c. at any interval greater than a year, the interest is calculated in a progressive manner, by adding the interest to the principal at the time of the first payment, and from the sum deducting the payment, &c.

ANNUITIES.

ANNUITIES *or* PENSIONS *in* ARREARS,
at SIMPLE INTEREST.

An Annuity is a sum of money, payable every year, half year, or quarter, for a certain number of years, or forever.

When the debtor keeps the annuity in his own hands, beyond the time of payment, it is said to be in *arrears*.

The sum of all the annuities, for the time they have been forborne, together with the interest due upon each, is called the *amount*.

If an annuity is to be bought off, or paid all at once, at the beginning of the first year, the price, which ought to be given for it, is called the *present worth*.

C A S E.

When the annuity, time, and rate of interest are given, to find the amount.

R U L E.*

1. Find the sum of the natural series of numbers 1, 2, 3, 4, &c. to the number of years less one.
2. Multiply this sum by one year's interest of the annuity, and the product will be the whole interest due upon the annuity.
3. To this product add the product of the annuity and time, and the sum will be the amount sought.

E X A M P L E S.

1. What is the amount of an annuity of 90*l.* for 6 years, allowing simple interest, at 6 per cent. per annum?

$$1+2+3+4+5=15=3 \times 5 = \text{sum of the number of years less 1.}$$

$$\begin{array}{r} \text{£. } s. \\ 5 \end{array}$$

$$8 = 1 \text{ year's interest of the annuity.}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 81 \\ \hline \end{array}$$

$$81 = \text{whole interest due.}$$

$$\text{Add } 540 = 90 \times 6 = \text{annuity multiplied by the time.}$$

$$\begin{array}{r} 621 \\ \hline \end{array} = \text{amount required.}$$

2. If

* Whatever the time may be, there is due upon the first year's annuity so many years' interest as the whole number of years less one, and gradually one less upon every succeeding year to the last but one, and none upon the last; therefore, in the whole, there is due so many years' interest of the annuity, as the sum of the series 1, 2, 3, 4, &c. to the number of years less one; consequently, one year's interest, multiplied by this sum, must be the whole interest due; to which, if all the annuities be added, the sum is plainly the amount.

Let r = ratio, a = annuity, t = time, and A = amount: Then will the following theorems give the solution of all the different cases.

2. If a house be let upon a lease for 8 years, at 36*l.* per annum, I demand the amount for that time, at 5 per cent. per annum ?

Ans. £ 338 8*s.*

3. If a salary of 120*l.* payable every half year, remain unpaid 7 years ; What would it amount to in that time, at 6*l.* per cent. per annum ?*

Ans. £ 1003 16*s.*

4. If a salary of 120*l.* payable every quarter, remain unpaid 7 years ; What would it amount to in that time, at 6*l.* per cent. per annum ?

Ans. £ 1010 2*s.*

PRESENT WORTH of ANNUITIES at SIMPLE INTEREST.

C A S E.

The annuity, rate, and time given, to find the present worth.

R U L E.†

Find the present worth of each year, by itself, discounting from the time it falls due, and the sum of all these will be the present worth required.

EXAMPLES.

$$\text{I. } \frac{2a - 2tn}{2} = a. \quad \text{II. } \frac{2a}{t r + 2t - tr} = n. \quad \text{III. } \frac{2a - 2tn}{2} = r.$$

$$\text{IV. } \frac{2a}{rn} + \frac{d^2}{4} \left| \frac{d}{2} = t. \right. \text{ In the last Theorem, } d = \frac{2n - rn}{rn}$$

And in Theorem first, if a sum cannot be found equal to the amount, the problem is impossible in whole years.

* In this Case, when the annuities, &c. are to be paid half yearly, or quarterly, then for half yearly payments, take $\frac{1}{2}$ of the ratio, $\frac{1}{2}$ the annuity, &c. and twice the number of years.

For quarterly payments, take $\frac{1}{4}$ of the ratio, $\frac{1}{4}$ of the annuity, &c. and 4 times the number of years, and work as before.

† If we grant the condition of allowing simple interest to be consistent, the reason of this rule will be evident from the nature of discount ; for all the annuities may be considered separately as so many single and independent debts, due after 1, 2, 3, &c. years ; so that the present worth of each being found, their sum must be the present worth of the whole,

But the purchasing of annuities by simple interest is unjust and absurd, which may easily be made to appear by one instance only. The price of an annuity of 100*l.* to continue 30 years, discounting at 6 per cent. will amount to 2000*l.* nearly, the interest of which for one year only exceeds the annuity : Would it not therefore be highly absurd to give a sum, which would yield me nearly 120*l.* yearly, forever, for an annuity of 100*l.* to continue only 30 years ?

Let p = present worth, and the other letters as before.

$$\text{Then, } \left\{ \begin{array}{l} n \times \frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r} \text{ \&c. to } \frac{1}{1+tr} = p \\ p \div \frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r} \text{ \&c. to } \frac{1}{1+tr} = n \end{array} \right.$$

EXAMPLES.

1. What is the present worth of 400*l.* per annum, to continue 6 years, at 6 per cent. per annum?

106	} : 100 :: 400 :	377,358 ⁴⁹ =	present worth of one year.	
112		357,142 ⁸⁵ =	—	2d year.
118		338,983 ⁰⁵ =	—	3d year.
124		322,580 ⁶⁴ =	—	4th year.
130		307,692 ³ =	—	5th year.
136		294,117 ⁰⁴ =	—	6th year.

1997,87497 = £1997 17*s.* 5³/₄*d.* = pref. worth.

2. What is the yearly rent of a house of 36*l.* to continue 8 years, worth, in ready money, at 5 per cent. per annum?

Ans. £237 3*s.* 9¹/₂*d.*

DISCOUNT

Is an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The *present worth* of any sum, or debt, due some time hence, is such a sum, as if put to interest, would in that time and at the rate per cent. for which the discount is to be made, amount to the sum or debt, then due.

RULE I.*

As the amount of 100*l.* for the given rate and time, is to 100*l.* So is the given sum, or debt, to the present worth.

Subtract

Most authors give the following Theorems, viz.

$$\text{I. } \frac{t^2 r - tr + 2t}{2tr + 2} \times n = p. \quad \text{II. } \frac{tr + 1}{t^2 r - tr + 2t} \times 2p = n.$$

$$\text{III. } \frac{2nt - p \times 2}{2pt - nt^2 - nt} = r. \quad \text{IV. } \frac{2}{r} - \frac{2p}{n} - 1 = x. \quad \text{Then, } \frac{2p}{nr} + \frac{x^2}{4}$$

$$- \frac{x}{2} = t.$$

Note. The same thing is to be observed in this Case as in the Case of Annuities in Arrears, respecting half yearly and quarterly payments.

I have shewn the method of computing Annuities by Simple Interest, more for the gratification of the curious than for real utility, it being not only customary, but also most equitable, to allow compound interest.

It may be observed that the Theorem $\frac{t^2 r - tr + 2t}{2tr + 2}$ gives the answer to the first question = 2028*l.* which therefore appears to be erroneous.

* That an allowance ought to be made for paying money before it becomes due, which is supposed to bear no interest till after it is due, is very reasonable; for if I keep the money in my own hands till the debt shall become due, it is plain I may make an advantage of it by putting it out to interest for that time; but if I pay it

Subtract the present worth from the given sum, and the remainder will be the discount required.

Or,

As the amount of 100*l.* for the given rate and time, is to the interest of 100*l.* for that time : So is the given sum or debt to the discount required.

EXAMPLES.

1. What is the discount of 635*l.* 17*s.* due two years hence, at 5½ per cent. per annum ?

Int. of 100*l.* } = 5 10
per annum } 2 years.

$$\begin{array}{r}
 \text{Add } 100 \\
 \hline
 \text{As } \text{£} 111 : 11 :: 635 \text{ } 17 \\
 \text{20} \qquad \qquad \text{20} \\
 \hline
 2220 \qquad 12717 \\
 \qquad \qquad 11 \\
 \hline
 \qquad \qquad 12717 \\
 \qquad \qquad 12717 \\
 \hline
 \end{array}$$

222|0)13988|7(63*l.* discount.

1332

668

666

27

20

—

222|0)540|0

12

222|0)648|0(2

444

204

Or,

As $\text{£} 111 : 100 :: 635 \text{ } 17 : 572 \text{ } 16 \text{ } 9\frac{1}{4}$
present worth.

And $635 \text{ } 17 - 572 \text{ } 16 \text{ } 9\frac{1}{4} = 63 \text{ } 0 \text{ } 2\frac{1}{4}$
discount.

Ans. £63 Os. 2¼*d.*

2. What

it before it is due, I give that benefit to another; therefore, we have only to inquire what discount ought to be allowed. And here, many suppose that, since by not paying till it becomes due they may employ it at interest, therefore, by paying it before due, they shall lose that interest, and for that reason all such interest ought to be discounted; but the supposition is false; for they cannot be said to lose that interest till the time arrives, when the debt becomes due; whereas we are to consider what would properly be lost, at present, by paying the debt before it becomes

2. What is the present worth of 350*l.* payable in half a year, discounting at 6*l.* per cent. per annum ?

Ans. £339 16*s.* 1½*d.*

3. What is the present worth of 65*l.* due 15 months hence, at 6*l.* per cent. per annum ?

Ans. £60 9*s.* 3½*d.*

4. What is the discount on 97*l.* 10*s.* due January 22*d.*, this being September 7*th.*, reckoning interest at 5*l.* per cent. ?

Ans. £1 15*s.* 11*d.*

5. What ready money will discharge a debt of 475*l.* 10*s.* due 5 months and 20 days hence, at 6 per cent. ?

Ans. £462 7*s.* 11½*d.*

6. Bought a quantity of goods for 250*l.* ready money, and sold them for 300*l.* payable 9 months hence : What was the gain, in ready money, supposing discount to be made at 6*l.* per cent. ?

Ans. £37 1*s.* 7½*d.*

7. What is the present worth of 275*l.* payable as follows ; viz. $\frac{1}{2}$ at 3 months, $\frac{1}{3}$ at 6 months, and the rest at 9 months, supposing the discount to be made at 6*l.* per cent. ?

Ans. £268 6*s.* 5¾*d.*

R U L E II.

As any sum of money, at 6 per cent. per annum, will double, at simple interest, in 200 months ; therefore,

To 200 add the number of months for which the discount is required, by which sum divide the product of the money and time, (in months,) and the quotient will be the discount.

E X A M P L E S.

1. What is the discount of 150*D.* 75*c.* for a year ?

200

becomes due ; this can, in point of equity, be no other than such a sum, which being put out at interest till the debt shall become due, would amount to the interest of the debt for the same time.—It is besides plain that the advantage arising from discharging a debt due some time hence, by a present payment, according to the principles above mentioned, is exactly the same as employing the whole sum at interest till the time, when the debt becomes due, arrives : For, if the discount allowed for present payment be put out at interest for that time, its *amount* will be the same as the interest of the whole debt for the same time ; thus the discount of 106*l.* due one year hence, reckoning interest at 6*l.* per cent. will be 6*l.* and 6*l.* put out to interest at 6*l.* per cent. for one year, will amount to 6*l.* 7*s.* 2¼*d.* which is exactly equal to the interest of 106*l.* for one year at 6*l.* per cent.

The truth of the rule for working is evident from the nature of Simple Interest ; for since the debt may be considered as the *amount* of some principal (called here the present worth) at a certain rate per cent. and for the given time, that amount must be in the same proportion either to its principal or interest, as the amount of any other sum, at the same rate, and for the same time, to its principal or interest.

$$\begin{array}{r} 200 \quad 15075 \\ 12 \quad 12 \\ \hline \end{array}$$

D. c. m.

$$212) 1809,00(8,533 = 8,533 \text{ discount, } \text{Ans.}$$

$$\begin{array}{r} 1695 \\ \hline \end{array}$$

$$\begin{array}{r} 1130 \\ \hline \end{array}$$

$$\begin{array}{r} 1060 \\ \hline \end{array}$$

$$\begin{array}{r} 150,75 \\ 8533 \\ \hline \end{array}$$

$$\begin{array}{r} 700 \\ \hline \end{array}$$

$$\begin{array}{r} 636 \\ \hline \end{array}$$

Present worth 142,217

$$\begin{array}{r} 640 \\ \hline \end{array}$$

$$\begin{array}{r} 636 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \hline \end{array}$$

2. What is the present worth of 426*D.* 55*c.* at 6 per cent. &c. to be paid 8 months hence? *Ans.* 410*D.* 14*c.* 5*m.*

3. What is the discount of 361*l.* 15*s.* 6*d.* to be paid 1 year and 8 months hence? *Ans.* £32 17*s.* 9½*d.*

ABBREVIATIONS in DISCOUNT.

Any principal to be discounted for one year, at any of the following rates, (or for any rate and time, whose product is equal to any of the following rates) being (multiplied by the multiplier, if any, and) divided by the corresponding divisor, the quotient will be the discount.

Rates.

$$\left. \begin{array}{l} \text{At} \left\{ \begin{array}{ll} 1\frac{1}{4} & \div 81 \text{ (or by 9 and 9)} \\ 2 & \div 51 \\ 2\frac{1}{2} & \div 41 \\ 4 & \div 26 \\ 5 & \div 21 \text{ (or by 7 and 3)} \\ 6 & \div 53 \text{ and } \times 3 \\ 7\frac{1}{2} & \div 43 \text{ and } \times 3 \\ 8 & \div 27 \text{ and } \times 2 \text{ (or } \times 2 \text{ and } \div 9 \text{ and } 3) \\ 8\frac{1}{3} & \div 13 \\ 10 & \div 11 \\ 12 & \div 28 \text{ and } \times 3 \text{ (or } \times 3 \text{ and } \div 7 \text{ and } 4) \\ 12\frac{1}{2} & \div 9 \end{array} \right. \right\} \end{array}$$

1. How much must I abate of 5394*l.* 10*s.* due 3 years hence, at 2½ per cent. per annum? £5894 10*s.*

$$\begin{array}{r} 2\frac{2}{3} \\ \times 3 \\ \hline \end{array}$$

8, therefore, $\times 2$, and $\div 27$

$$27 \div 3 = 9) 10789 \quad 0$$

$$3) 1198 \quad 15 \quad 6\frac{1}{2}$$

$$\begin{array}{r} \text{£} 399 \quad 11 \quad 10 \text{ } \text{Ans.} \\ \hline \end{array}$$

2. What

2. What is the discount of 546*l.* 12*s.* 6*d.* for $8\frac{1}{3}$ years, at 1 per cent. per annum, (or for 1 year at $8\frac{1}{3}$ per cent. per annum ?)

$$\begin{array}{r} \text{£. s. d.} \\ 13 \overline{) 546 \ 12 \ 6} \end{array}$$

$$\text{£ } 42 \ 0 \ 11\frac{1}{2} \text{ Ans.}$$

3. What is the discount of 125*l.* at $1\frac{1}{4}$ per cent. per annum, for four years, (or, at $\frac{1}{4}$ per cent. per annum, for $1\frac{1}{4}$ year ?)

$$\begin{array}{r} 1\frac{1}{4} \\ \times 4 \\ \hline \end{array}$$

$$5, \text{ therefore, } \div 21 \overline{) 125}$$

$$\text{£ } 5 \ 19 \ 0\frac{1}{2} \text{ Ans.}$$

DISCOUNT BY DECIMALS.*

The sum to be discounted, the time and the ratio given, to find the present worth.

RULE.—Multiply the ratio by the time, add unity to the product for a divisor ; by which sum divide the sum to be discounted, and the quotient will be the present worth.

Subtract the present worth from the principal or sum to be discounted, and the remainder will be the discount.

Or, as the amount of 1*l.* for the given time, is to 1*l.* so is the interest of the debt for the said time, to the discount required.

Subtract the discount from the principal, and the remainder will be the present worth.

EXAMPLES.

First Method.

1. What is the present worth of 600*l.* due 3 years hence, at 6*l.* per cent. per annum ?

Ratio

* As in Simple Interest, let a = amount of any debt, p = present worth, t = time, and r = ratio ; then will the following Theorems exhibit all the cases in Discount at Simple Interest.

$$\text{I. } \frac{a}{tr+1} = p. \quad \text{II. } ptr+1 = a. \quad \text{III. } \frac{a-p}{tp} = r. \quad \text{IV. } \frac{a-p}{rp} = t.$$

Note. When the ratio is .06 per cent. per annum, and the given time is expressed in months, whether less or more than a year, if the debt be divided by 1 plus half as many hundredths of an unit, as there are months in the given time, the quotient

will be the present worth.—Thus, for 1 month $\frac{a}{1,005}$, 2 months $\frac{a}{1,01}$, 3 months

$\frac{a}{1,015}$, 36 months $\frac{a}{1,18}$, 42 months $\frac{a}{1,21}$, &c. &c.

M m

$$\begin{array}{r}
 \text{Ratio} = .06 \\
 \text{Multiply by the time} = 3 \\
 \hline
 \text{Product} = .18 \\
 \text{Add } 1 \\
 \text{Divisor} = 1.18 \quad 600 \text{ (£.} \\
 \text{Or, } \frac{600}{.06 \times 3 + 1} = \text{£}508 \text{ } 9\text{s. } 5\frac{3}{4}\text{d. } \text{Ans.}
 \end{array}$$

(508,4745 pref. worth.

Present worth = 508,4745 = £508 9s. 5¼d. which subtracted from the principal, will give the discount = £91 10s. 6¼d.

Second Method.

What is the discount of 600*l.* due 3 years hence, at 6*l.* per cent. per annum?

$$\begin{array}{r}
 \text{Ratio} = .06 \\
 \text{Multiply by } 3 \\
 \hline
 .18 \\
 \text{Add } 1 \\
 \hline
 1.18
 \end{array}
 \quad
 \begin{array}{r}
 \text{As } 1.18 : 1 :: 108 \\
 \hline
 1 \\
 1,18)108,00(91,5254
 \end{array}$$

Amount of 1*l.* for the } given time = 1.18

And $600 \times .06 \times 3 = 108$ = interest of the debt for the given time.
 Discount = 91,5254 = £91 10s. 6d. which taken from the principal will leave the present worth = £508 9s. 6d.

2. What is the present worth of 312*l.* 10s. due 2 years hence, at 4½*l.* per cent. per annum? *Ans.* 286*l.* 13s. 11¼d.

3. What is the present worth of 1650*l.* 15s. 6d. at 6¾*l.* per cent. per annum, due 18 months hence? *Ans.* 1499 os. 0¼d.

4. What ready money will discharge a debt of 1354*l.* 8s. due 3 years, 3 months and 12 days hence, at 5⅞*l.* per cent. per annum? *Ans.* £1135 7s. 9d.

B A R T E R

Is the exchanging of one commodity for another, and teaches traders to proportion their quantities without loss.

C A S E I.

When the quantity of one commodity is given, with its value, or that of its integer, that is, of 1*lb.* 1*cwt.* 1*yd.* &c. as also the value of the integer of some other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of selling it.

RULE.—Find the value of the given quantity by the concise method, then find what quantity of the other, at the rate proposed, you may have for the same money: Or, if the quantity be given, find from thence the rate of selling it.

EXAMPLES.

E X A M P L E S.

1. How much tea, at 9s. 6d. per lb, must be given in barter for 156 gallons of wine, at 12s. 3½d. per gallon?

$$\begin{array}{r}
 \begin{array}{c|c|c}
 3d. & \frac{1}{4} & 156 \\
 \hline
 \frac{1}{2} & \frac{1}{8} & 12
 \end{array} \\
 \hline
 1872 \\
 39 \\
 66 \\
 \hline
 19176 \\
 12 \\
 \hline
 23010
 \end{array}$$

9s. 6d. = 114d.

d. lb d.
As 114 : 1 :: 23010

$$\begin{array}{r}
 \text{I} \\
 \hline
 114 \overline{) 23010} (201 \\
 \underline{228} \\
 210 \\
 114 \\
 \hline
 96 \\
 16 \\
 \hline
 576 \\
 96 \\
 \hline
 114 \overline{) 1536} (13 \frac{54}{114} \text{ Ans. } 201 \text{ lb } 13 \frac{54}{114} \text{ oz.} \\
 \underline{114} \\
 396 \\
 342 \\
 \hline
 54
 \end{array}$$

2. How much cloth, at 15s. 8d. per yard, must be given for 5 Cwt. 3 qrs. 19 lb of steel, at 5 guineas per Cwt.

Ans. 52 yds. 3 qrs. 27.

3. Suppose A has 350 yards of linen, at 1s. 4d. per yard, which he would truck with B for sugar, at 25s. 6d. per Cwt. How much sugar will the linen come to?

Ans. 18 Cwt. 1 qr. 5½ lb.

4. A has broadcloths at 44D. per piece, and B has mace, at 1D. 42c. per lb; How many pounds of mace must B give A for 35 pieces of cloth?

Ans. 1084½ lb.

5. A

5. A has $7\frac{1}{2}$ Cwt. of sugar at 12 cents per lb, for which B gave him $12\frac{1}{2}$ Cwt. of flour ; What was the flour rated at per lb ?

Ans. 7c. 2m.

C A S E II.

If the quantities of two commodities be given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.

RULE.—Find the separate values of the two given commodities ; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quantity and rate may be easily found.

E X A M P L E S.

1. Two merchants barter ; A has 30 Cwt. of cheese, at 23s. 6d. per cwt. and B has 9 pieces of broadcloth, at 3l. 15s. per piece ; Which must receive money, and how much ?

Ans. B must pay A £1 10s.

2. A and B would barter ; A has 150 bushels of wheat, at 5s. 9d. per bushel, for which B gives 65 bushels of barley, worth 2s. 10d. per bushel, and the balance in oats at 2s. 1d. per bushel ; What quantity of oats must A receive from B ?

Ans. $325\frac{1}{5}$ bushels.

C A S E III.

Sometimes, in bartering, one commodity is rated above the ready money price ; then, to find the bartering price of the other, say,

As the ready money price of the one, is to its bartering price ; so is *that* of the other, to *its* bartering price : Next, find the quantity required, according to either the bartering or ready money price.

E X A M P L E S.

1. A has ribbands at 2s. per yard ready money ; but in barter he will have 2s. 3d. B has broadcloth at 32s. 6d. per yard ready money :—At what rate must B value his cloth per yard, to be equivalent to A's bartering price, and how many yards of ribband, at 2s. 3d. per yard, must then be given by A for 488 yards of B's broadcloth ?

Ans. B's broadcloth, at £1 16s. $6\frac{1}{2}$ d. per yd. 7930 yds. ribband.

2. A and B barter ; A has 150 gallons of brandy, at 7s. 3d. per gallon, ready money, but in barter he will have 8s. per gallon : B has linen at 3s. 6d. per yard ready money ; How must B sell his linen per yard in proportion to A's bartering price, and how many yards are equal to A's brandy ?

Ans. barter price is 3s. $10\frac{1}{4}$ d. and he must give A 310 yds. 2 qrs. 3n.

3. P

3. P and Q barter. P has Irish linen, at 3s. 7d. per yard; but in barter will have 3s. 10d. Q delivers him broadcloth at 1l. 16s. 6d. per yard, worth only 1l. 13s. per yard: Pray, which has the advantage in Barter, and how much linen does P give Q for 148 yards of broadcloth?

Yd. Yds. s.

As 1 : 36/6 :: 148 : 540²/₃ price of the broadcloth. As 3/10 : 1yd. :: 540²/₃ : 1409⁵/₃ yards of linen. Q has the advantage; for as 3/7 : 3/10 :: 33s. : 35/3²/₃ his proportional price.

4. A has 200 yards of linen, at 1s. 6d. ready money per yard, which he barter with B, at 1s. 9d. per yard, taking buttons at 7¹/₂d. per gross, which are worth but 6d.: How many gross of buttons will pay for the linen; who gets the best bargain, and by how much both in the whole and per cent.?

Yd. d. Yds. d. d. Gros. d. Gros. Yd. d. Yds. £.
As 1 : 21 :: 200 : 4200. As 7¹/₂ : 1 :: 4200 : 560. As 1 : 18 :: 200 : 15
gr. d. gr. £. [value of A's linen.]

As 1 : 6 :: 560 : 14 value of B's goods. So that B gained 1l. of A.

£. £. £. £.

As 14 : 1 :: 100 : 7 2s. 10d. per cent.

5. A has linen cloth, at 30c. per yard, ready money, in barter 36c. B has 3610 yards of ribband, at 22c. per yard ready money, and would have of A 200dolls. in ready money, and the rest in linen cloth; what rate does the ribband bear in barter per yard, and how much linen must A give B?

Ans. The rate of ribband is 26c. 4m. per yard, and B must receive 1980²/₃ yards of linen, and 200dolls. in cash.

LOSS AND GAIN

Is an excellent rule, by which merchants and traders discover their profit, or loss per cent. or by the gross: It also instructs them to raise or fall the price of their goods, so as to gain or lose so much per cent. &c.

Questions in this rule are performed by the Rule of Three.

C A S E I.

To know what is gained or lost per cent.

R U L E.

First see what the gain or loss is, by subtraction; then, as the price it cost is to the gain or loss: So is 100% to the gain or loss per cent.

E X A M P L E S.

1. If I buy serge at 5s. per yard, and sell it again, at 5s. 8d. per yard; What do I gain per cent, or in laying out 100l.?

Sold

	s.	d.		d.	d.	£.
Sold for	5	8	As	60	: 8 :: 100	
Cost	5	0				8

8 gain per yard.

60)800(13
60

N. B. The first questions, in the several Cases, serve to elucidate each other.

200

180

20

20

2. If I buy serge at 5/8 per yard, and sell it again at 5s. per yard ; What do I lose per cent. or in laying out 100% ?

60)400(6
360

	s.	d.		d.	d.	£.
Cost	5	8	As	68	: 8 :: 100	
Sold for	5	0				8

40

12

Loss per yd. 8d.

68)800(11
6860)480(8
480

Ans. £ 13 6s. 8d.

120

68

52

20

68)1040(15
68

360

340

20

12

68)240(3
204

36

4

68)144(2
136

8 Ans. £ 11 15 8d.

3. If

3. If I buy a *Cwt.* of tobacco for *9l. 6s. 8d.* and sell it again at *1s. 10d.* per *lb.*, do I gain or lose, and what per cent. ?

<i>lb.</i>	<i>£. s. d.</i>
Sold for 10	5 4
Cost 9	6 8

£. s. o 18 8 gained in the gross.
 | 2d. | $\frac{1}{12}$ | 11 4 value at 2s. per *lb.*

 o 18 8 value at 2d. per *lb.*

10 5 4 value at *1s. 10d.* per *lb.*

<i>£. s. d.</i>	<i>s. d.</i>	<i>£.</i>	<i>£.</i>
As 9 6 8	18 8	::	100 10

Ans. £10 per cent.

4. A draper bought 60 yards of cloth at *28s.* per yard, and 38 yards of ditto at *14s.* per yard, and sold them, one with another, at *26s.* per yard : Did he gain or lose, and what per cent. ?

<i>yd. s.</i>	<i>yd. £.</i>	<i>yd. s.</i>	<i>yd. £. s.</i>
As 1 : 28 :: 60	18 4	As 1 : 26 :: 98	127 8
<i>yd. s.</i>	<i>yd. £. s.</i>		<i>£. s.</i>
As 1 : 14 :: 38	26 12	Sold for	127 8
		Prime cost	110 12

£110 12 Gain in the gross 16 16

£. s. *£. s.* *£. £. s. d.*
 As 110 12 : 16 16 :: 100 : 15 3 $9\frac{1}{2}$ per cent.

5. Bought sugar at $6\frac{1}{2}d.$ per *lb.*, and sold it at *2l. 3s. 9d.* per *Cwt.* What was the gain or loss per cent. ?

<i>lb. d.</i>	<i>lb. £. s. d.</i>
As 1 : $6\frac{1}{2}$:: 112	3 0 8
Prime cost <i>£3 0 8</i> per <i>Cwt.</i>	<i>£. s. d.</i>
Sold at 2 3 9 per <i>Cwt.</i>	As 3 0 8 : 16 11 :: 100 : 27 17 $8\frac{1}{2}$
	loss per cent. <i>Ans.</i>

Lost *£0 16 11* in the whole.

6. At $2\frac{1}{2}d.$ in the shilling profit ; How much per cent. ?

s. d. *£. £. s. d.*
 As 1 : $2\frac{1}{2}$:: 100 : 20 16 8 *Ans.*

7. At *4s. 6d.* in the pound profit ; How much per cent. ?

£. s. d. *£. £. s.*
 As 1 : 4 6 :: 100 : 22 10 *Ans.*

8. If I buy candles, at *1s. 6d.* per *lb.*, and sell them again, at *2s.* per *lb.*, and allow 3 months for payment ; What do I gain per cent. ?

d. d. *£. £. s. d.* *Mo. £. Mo. £. s.*
 As 18 : 24 :: 100 : 133 6 8 ; then by Discount. As 12 : 6 :: 3 : 1 10

£. s. *£. s.* *£. s. d.* *£. s. d.*
 Then, As 101 10 : 1 10 :: 133 6 8 : 1 19 $4\frac{3}{4}$, which taken from 133l. 6s. 8d. leaves 131l. 7s. $3\frac{1}{4}d.$ therefore, *Ans. £31 7 3 $\frac{1}{4}$.*
9. If

9. If I buy cloth at 6s. per yard, ready money, and sell it again at 6s. 8d. per yard on 3 months credit; What is gained per cent.?

s. d. s. s. d. £. £. s. d. Mo. £. Mo. £. s.
 6 8—6=8d. As 6 : 8 :: 100 : 11 2 $\frac{1}{2}$ As 12 : 6 :: 3 : 1 10
£. s. £. £. s. d. £. s. d.

As 101 10 : 100 :: 111 2 $\frac{1}{2}$: 109 9 $4\frac{1}{2}$; therefore

£. s. d. £. £. s. d.
 109 9 $4\frac{1}{2}$ —100=9 9 $4\frac{1}{2}$ *Ans.*

10. If I buy cloth at 13s. per yard, on 8 months credit, and sell it again at 12s. ready money, Do I gain or lose, and what per cent.?

Mo. £. Mo. £. £. s. £. s. d.

As 12 : 6 :: 8 : 4 As 104 : 13 :: 100 : 12 6: So that 13s. on 8 months credit at 6l. per cent. is equal to 12s. 6d. ready money; then,

s. d. d. £. £.

As 12 6 : 6 :: 100 : 4

Prime cost 12 6 ready money,

Sold at 12 0 ready money,

Lost 0 6 in the yard.

Ans. lost £4 per cent.

11. If I buy gloves at 1D. 25c. per pair; How long credit must I have, to gain 13D. per cent. when I sell them at 1D. 36c. per pair?

D. c.

D. c.

c.

D.

D. c.

Sold at 1,36

As 1,25 : 11 : 100 : 8,80 gain per cent.

Prime cost 1,25

D. D. c. D. c. [ready money.

Then, 13—8,80=4,20 Now,

Gained ,11 per pair.

D. Mo. D. c. Mo. days.

As 6 : 12 :: 4,20 : 8 12 *Ans.*

In casting up the amount of goods bought, imported or exported; to the prime cost of such goods we must add all the charges upon them, in order to fix the price they stand us in.

12. Suppose I import from France, 12 bales of cloth, containing 10 pieces each, which, with the charges there, amounted to 360D.; I pay duty here 92c. per piece, for freight 12D. and portage 1D. 25c.; What does it stand me in per piece, and how must I sell it per piece to gain 10D. per cent.?

12 bales.

D. c.

10

First cost 360

Duty 110,40

120 pieces.

Freight 12

Pieces. D. c. Piece. D. c.

Porterage 1,25

As 120 : 483,65 :: 1 : 4,03 cost per piece.

Whole cost, 483,65

D. Pieces. D. c. c. m.

Again, As 100 : 10 :: 4,03 : 40,3 gain.

40,3

D. 4,43,3 the price, at which it must be sold per piece.

CASE

C A S E II.

To know how a commodity must be sold, to gain or lose so much per cent.

RULE.—As 100*l.* is to the price ; so is 100*l.* with the profit added, or loss subtracted, to the gaining or losing price.

E X A M P L E S.

1. If I buy a quantity of serge, at 5*s.* per yard ; How must I sell it per yard, to gain 13*l.* 6*s.* 8*d.* per cent. ?

$$\begin{array}{rcl} \text{As } 100 : 5 :: 113 \text{ } 6 \text{ } 8 \\ \underline{20} \end{array}$$

$$\begin{array}{rcl} 2000 & & 2266 \\ \underline{12} & & \underline{12} \end{array}$$

$$\begin{array}{rcl} 24000 & & 27200 \\ & & \underline{5} \end{array}$$

$$\begin{array}{rcl} 24,000)136,000(5 & & \\ & & \underline{120} \end{array}$$

$$\begin{array}{rcl} & & \underline{16} \\ & & \underline{12} \end{array}$$

$$\begin{array}{rcl} 24,000)192,000(8 & & \\ & & \underline{192} \\ & & \underline{\hspace{1cm}} \\ & & 000 \end{array}$$

$$\begin{array}{rcl} \text{Or thus :} & & \\ \text{£. } & \text{s. } & \text{d.} \\ 113 & 6 & 8 \\ \underline{5} & & \end{array}$$

$$\begin{array}{rcl} \text{s. } 5 & | & 66 \text{ } 13 \text{ } 4 \\ \underline{12} & & \end{array}$$

$$\begin{array}{rcl} & & \underline{79^2} \\ \text{Add } 8 = \frac{2}{3} \text{ of } 12 = 13\text{s. } 4\text{d.} & & \\ & & \underline{\hspace{1cm}} \\ & & \text{d. } 8 | 00 \end{array}$$

$$\begin{array}{rcl} & \text{s. } & \text{d.} \\ \text{Ans. } 5 & 8 & \text{per yard.} \end{array}$$

2. If a barrel of powder cost 4*l.* how must it be sold to lose 10*l.* per cent. ?

$$\begin{array}{rcl} \text{As } 100 : 4 :: 90 \\ & & \underline{4} \end{array}$$

$$\begin{array}{rcl} 100)360(3 & & \\ & & \underline{300} \end{array}$$

$$\begin{array}{rcl} & & \underline{60} \\ & & \underline{20} \end{array}$$

$$\begin{array}{rcl} 100)1200(12 & \text{Ans. } \text{£ } 3 \text{ } 12\text{s.} \\ & & \underline{1200} \end{array}$$

$$\begin{array}{rcl} \text{Or thus :} & & \\ 90 & & \\ \underline{4} & & \end{array}$$

$$\begin{array}{rcl} \text{£ } 3 & | & 60 \\ & & \underline{20} \end{array}$$

$$\text{s. } 12 | 00$$

3. Bought cloth, at 15*s.* per yard, which not proving so good as I expected, I am content to lose 17½ per cent. by it ; How must I sell it per yard ?

£. s. £. s. d. Or thus :
 As 100 : 15 :: 82½ : 12 4½ *Ans.* 12s. 4½d. 82 10

$$3 \times 5 = 15$$

$$247 \ 10$$

$$5$$

$$s. \ 12 | 37 \ 10$$

$$2 + 6 = \frac{1}{2} \text{ of } 12 \times 10$$

$$d. \ 4 | 50$$

$$4$$

$$qrs. \ 2 | 0$$

4. If 120lb of steel cost 7l. How must I sell it per lb to gain 15½% per cent. ?

£. s. d. £. s. d. £. s. d.
 As 120 : 7 :: 1 : 1 2 As 100 : 1 2 :: 115½ : 1 4 per lb *Ans.*

5. A gentleman bought 10 tons of iron for 200l. the freight and duties came to 25l. and his own charges to 8l. 6s. 8d. ; How must he sell it per lb to gain 20l. per cent. by it ?

£. £. £. s. d. £. s. d. £. s. d. £. s. d. £.
 As 100 : 20 :: 233 6 8 : 46 13 4 Then, 233 6 8 + 46 13 4 = 280
 Tons. £. lb d.

As 10 : 280 :: 1 : 3 per lb *Ans.*

6. If a bag of cotton, weighing 8Cwt. 0qr. 20lb cost 45D. 55c. How must it be sold per Cwt. to lose 8D. per cent ?

Cwt. qrs. lb D. c. Cwt. D. c. m. D. c. m. D. c. m.

As 8 0 20 : 45,55 :: 1 : 5,56,9 As 100 : 92 :: 5,56,9 : 5,12,3 *Ans.*

7. Bought fish in Newburyport, at 10s. per quintal, and sold it at Philadelphia, at 17s. 6d. per quintal ; now, allowing the charges at an average, or one with another, to be 2s. 6d. per quintal, and considering I must lose 20l. per cent. by remitting my money home ; What do I gain per cent. ?

s. d.
 Selling price 17 6 Philadelphia currency, per quintal.
 Charges 2 6 ditto.

$$15 \ 0 \text{ ditto.}$$

£. s. £. s.
 As 100 : 15 :: 80 : 12 Newengland currency.

Sold at 12s. per quintal.

Bought at 10s. per quintal.

Gained 2s. per quintal.

s. s. £. £.
 As 10 : 2 :: 100 : 20 per cent. gained, *Ans.*

8. Bought

8. Bought 50 gallons of brandy, at 4s. per gallon, but, by accident, 10 gallons leaked out; At what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of 10l. per cent. ?

50 galls. at 4s. per gall. = £ 10
10 gallons leaked out.

Galls. £. Gall. s.

As 40 : 10 :: 1 : 5

£. s. £. s. d.

As 100 : 5 :: 110 : 5 6 Ans.

40 gallons remain.

C A S E III.

When there is gained or lost per cent. to know what the commodity cost.

RULE.—As 100l. with the gain per cent. added, or loss per cent. subtracted, is to the price : So is 100l. to the prime cost.

E X A M P L E S.

1. If 1 yard of cloth be sold, at 5s. 8d. and there is gained 13l. 6s. 8d. per cent. What did the yard cost ?

£.	s. d.	s. d.	£.
As 113	6 8	: 5 8	:: 100
20	12		68
<hr/>			<hr/>
2266	68		800
12			600
<hr/>			<hr/>
27200	272,00	68	100(0
		20	

272,00)1360|00(5s. Ans. 5s. prime cost.

1360

2. If 12 yards of cloth are sold at 15s. per yard, and there is 7l. 10s. loss per cent. in the sale; What is the prime cost of the whole? Yd. s. Yds. £. £. s. £. £. s. d.

As 1 : 15 :: 12 : 9 As 92 10 : 9 :: 100 : 9 14 7 Ans.

3. If 40lb of chocolate be sold at 1s. 6d. per lb, and I gain 9l. per cent. What did the whole cost me ?

lb	s. d.	lb	£.	£.	£.	£.	s. d.
As 1	: 1 6	:: 40	: 3	As 109	: 3	:: 100	: 2 15 0½ Ans.

4. If 19½ Cwt. sugar be sold at 14D. 50c. per Cwt. and I gain 15D. per cent. What did it cost per Cwt. ?

D.	D.	D. c.	D. c. m.
As 114	: 100	:: 14,50	: 12,60,8 Ans.

C A S E IV.

If by wares sold at such a rate, there is so much gained or lost per cent. to know what would be gained or lost per cent. if sold at another rate.

RULE.—As the first price is to 100l. with the profit per cent. added, or loss per cent. subtracted; so is the other price, to the gain or loss per cent. at the other rate.

N. B.

N. B. If your answer exceed 100%, the excess is your gain per cent. but if it be less than 100%, the deficiency is your loss per cent. ?

E X A M P L E S.

1. If cloth, sold at 5s. 8d. per yard, be 13l. 6s. 8d. profit per cent. What gain or loss per cent. shall I have, if I sell the same at 5s. per yard ?

$$\begin{array}{r} \text{s. d.} \quad \text{£. s. d.} \quad \text{s.} \\ \text{As } 5 \text{ } 8 : 113 \text{ } 6 \text{ } 8 :: 5 \end{array}$$

$$\begin{array}{r} 12 \quad 20 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \quad 2266 \\ \hline \end{array}$$

$$12$$

$$\begin{array}{r} 27200 \\ \hline \end{array}$$

$$5$$

$$\begin{array}{r} 2,0 \\ \hline \end{array}$$

68) 136000 (200,0 100—100=0 *Ans.* I neither gain nor lose.

$$\begin{array}{r} 136 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£ } 100 \\ \hline \end{array}$$

$$,000$$

2. If cloth, sold at 4s. per yard, be 10% per cent. profit ; What shall I gain or lose per cent. if sold at 3s. 6d. per yard ?

$$\begin{array}{r} \text{s.} \quad \text{£.} \quad \text{s. d.} \\ \text{As } 4 : 110 :: 3 \text{ } 6 \end{array}$$

$$\begin{array}{r} 12 \quad 12 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \quad 42 \\ \hline \end{array}$$

$$110$$

$$\text{Then, } \frac{\text{£.}}{100} - \frac{\text{£.}}{96\frac{1}{4}} = 3\frac{1}{4}$$

48) 4620 (96 $\frac{1}{4}$ *Ans.* I lost £ 3 $\frac{1}{4}$ per cent. by the last sale.

$$432$$

$$\begin{array}{r} 300 \\ \hline \end{array}$$

$$288$$

$$12$$

3. If I sell a gallon of wine for 1D. 50c. and thereby lose 12 per cent. What shall I gain or lose per cent. if I sell 4 gallons of the same wine for 6D. 75c. ?

$$\text{D. D.} \quad \text{D. c.} \quad \text{D.}$$

$$\text{As } 6 : 88 :: 6,75 : 99 \quad \text{And } 100 - 99 = 1 \text{ per cent. loss.}$$

4. I sold a watch for 50l. and by so doing, lost 17% per cent. whereas I ought in trading to have cleared 20% per cent. How much was it sold under its real value ?

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \quad \text{£. s. d.} \quad \text{£.} \quad \text{£. s. d.} \quad \text{£.} \quad \text{£. s. d.} \\ \text{As } 83 : 50 :: 100 : 60 \text{ } 4 \text{ } 9\frac{1}{4} \quad \text{As } 100 : 60 \text{ } 4 \text{ } 9\frac{1}{4} :: 120 : 72 \text{ } 5 \text{ } 9\frac{1}{4} \end{array}$$

$$\begin{array}{r} \text{£. s. d.} \quad \text{£.} \quad \text{£. s. d.} \\ \text{Then, } 72 \text{ } 5 \text{ } 9\frac{1}{4} - 50 = 22 \text{ } 5 \text{ } 9\frac{1}{4} \text{ } \text{Ans.} \end{array}$$

EQUATION OF PAYMENTS

Is the finding of a time to pay, at once, several debts due at different times, so that no loss shall be sustained by either party.

R U L E I.*

Multiply each payment by the time at which it is due ; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time, or that required.

E X A M P L E S.

1. A owes B 380*l.* to be paid as follows, viz. 100*l.* in 6 months, 120*l.* in 7 months, and 160*l.* in 10 months ; What is the equated time for the payment of the whole debt ?

$$100 \times 6 = 600$$

$$120 \times 7 = 840$$

$$160 \times 10 = 1600$$

$$100 + 120 + 160 = 380 \quad 3040 \text{ (8 months, } \textit{Ans.})$$

$$3040$$

2. A owes B 104*l.* 15*s.* to be paid in $4\frac{1}{2}$ months, 161*l.* to be paid in $3\frac{1}{2}$ months, and 152*l.* 5*s.* to be paid in 5 months ; What is the equated time for the payment of the whole ?

Ans. 4 months and 8 days.

3. There is owing to a merchant 698*l.* to be paid 178*l.* ready money, 200*l.* at 3 months, and 320*l.* in 8 months ; I demand the indifferent time for the payment of the whole ?

Ans. $4\frac{1}{2}$ months.

4. The sum of 164*D.* 16*c.* 6*m.* is to be paid $\frac{1}{2}$ in 6 months, $\frac{1}{3}$ in 8 months, and $\frac{1}{6}$ in 12 months ; What is the mean time for the payment of the whole ?

Ans. $7\frac{2}{3}$ months.

R U L E II.

See, by rule 1st, at what time the first man, mentioned, ought to pay in his whole money ; then, as his money is to his time, so is the other's money, to his time, inversely, which, when found, must be added to, or subtracted from, the time at which the second ought to have paid in his money, as the case may require, and the sum, or remainder, will be the true time of the second's payment.

EXAMPLES.

* This rule is founded upon a supposition, that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Some, who defend this principle, have endeavoured to prove it to be right by this argument ; that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due ; but this cannot be the case ; for though, by keeping a debt after it is due, there is gained the interest of it for that time ; yet by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest, and therefore the rule is not accurately true ; however, in most questions, which occur in business, the error is so trifling, that it will always be made use of as the most eligible method.

The true rule will be given in Equation of Payments by Decimals.

EXAMPLES.

1. A is indebted to B 150*l.* to be paid, 50*l.* at 4 months, and 100*l.* at 8 months : B owes A 250*l.* to be paid at 10 months : It is agreed between them, that A shall make present pay of his whole debt, and that B shall pay his so much the sooner, as to balance that favour ; I demand the time at which B must pay the 250*l.* reckoning simple interest.

$$50 \times 4 = 200$$

$$100 \times 8 = 800$$

$$50 + 100 = 150 \mid 0 \quad 100 \mid 0 \quad (6\frac{2}{3} \text{ months, A's equated time.}$$

$$90$$

$$10$$

$\begin{array}{cc} \text{£.} & \text{mo.} \end{array}$ $\begin{array}{cc} \text{£.} & \text{mo.} \end{array}$ $\begin{array}{ccc} \text{mo.} & \text{mo.} & \text{mo.} \end{array}$
As 150 : 6 $\frac{2}{3}$:: 250 : 4 Then, 10—4=6 time of B's payment.

2. A merchant has 120*l.* due to him, to be paid at 7 months ; but the debtor agrees to pay $\frac{1}{2}$ ready money, and $\frac{1}{3}$ at 4 months ; I demand the time he must have to pay in the rest, at simple interest, so that neither party may have the advantage of the other ?

Debt £120

$$\frac{1}{2} = 60 \text{ must be paid down.}$$

$$\frac{1}{3} = 40 \text{ must be paid at 4 months.}$$

$$\frac{1}{6} = 20 \text{ unpaid.}$$

Now, as he pays 60*l.* 7 months, and 40*l.* 3 months before they are respectively due : Say, As the interest of 20*l.* for 1 month, is to 1 month, so is the sum of the interest of 60*l.* for 7 months, and of 40*l.* for 4 months, to a 4th number, which, added to the 7 months, will give the time for which the 20*l.* ought to be retained.

Ans. 2 years and 10 months.

3. A merchant has 1200*l.* due to him, to be paid $\frac{1}{6}$ at 2 months, $\frac{1}{3}$ at 3 months, and the rest at 6 months ; but the debtor agrees to pay $\frac{1}{2}$ down ; How long may the debtor detain the other half, so that neither party may sustain loss ?

$\begin{array}{cc} \text{mo.} & \text{mo.} \end{array}$

$$\frac{1}{6} \times 2 = 0\frac{1}{3}$$

$$\frac{1}{3} \times 3 = 1$$

$$\frac{1}{2} \times 6 = 3$$

$$\text{Equated time} = 4\frac{1}{3}$$

Now, as $\frac{1}{2}$ was paid 4 $\frac{1}{3}$ months before it was due, it is reasonable that he should detain the other $\frac{1}{2}$ 4 $\frac{1}{3}$ months after it became due, which, added, gives 8 $\frac{2}{3}$ months, the true time for the second payment.

EQUATION

EQUATION OF PAYMENTS BY DECIMALS.

R U L E.*

1. To the sum of both payments add the continual product of the first payment, the ratio, and the time between the payments, and call this the first number.

2. Multiply twice the first payment by the ratio, and call this the second number.

3. Divide the first number by the second, and call the quotient the third number.

4. Call the square of the third number the fourth number.

5. Divide the product of the second payment and time between the payments by the product of the first payment and the ratio, and call the quotient the fifth number.

6. From the fourth number take the fifth, and call the square root of the difference the sixth number.

7. Then the difference of the third and sixth numbers is the equated time, after the first payment.

E X A M P L E.

There are 100*l.* payable in 2 years, and 106*l.* at 6 years hence ; What is the equated time, allowing simple interest, at 6 per cent. per annum ?

1st payment=100

Ratio=.06

6,00

Time between the payments=4 years. Multiply by the ratio=.06

24

Add both payments= $\begin{cases} 100 \\ 106 \end{cases}$

Div. by the 2d. numb.=12)230=1st number.

19,166+=3d number.

19,166+

3d number squared=367,345556=4th number.

1st payment 100

Multiply by 2

200

12,00=2d. number

2d payment

* Suppose a sum of money be due immediately, and another sum at the expiration of a certain given time forward, and it is proposed to find a time, so that neither party shall sustain loss.

Now, it is plain that the equated time must fall between the two payments ; and that what is gotten by keeping the first debt after it is due, should be equal to what is lost by paying the second debt before it is due ; but the gain arising from the keeping of a sum of money after it is due, is evidently equal to the *interest* of the debt for that time : And the loss, which is sustained by the paying of a sum of money before it is due, is evidently equal to the *discount* of the debt for that time : Therefore it is obvious that the debtor must retain the sum immediately due, or the first payment, till its *interest* shall be equal to the *discount* of the second sum for the time it is paid before due ; because in that case the gain and loss will be equal, and consequently neither party can be a loser.

2d payment = 106
Multiplied by the time = 4

1st payment mult. by the ratio = 6) 424 = { product of the 2d product and time
between the payments.

70,666 + = 5th number.

From the 4th number = 367,345,556

Take the 5th number = 70,666,666

296,678890 (17,224 square root = 6th number.

From the 3d number = 19,166

Take the 6th number 17,224

1,942 = equated time from the first payment; therefore 3,942 years = 3y. 11m. 14d. = whole equated time.

$$\text{Or, } \frac{100 + 106 + \frac{100 \times .06 \times 4}{100 \times 2 \times .06}}{100 \times 2 \times .06} - \frac{100 + 106 + \frac{100 \times .06 \times 4}{100 \times 2 \times .06}}{100 \times 2 \times .06} \Bigg| \frac{1}{2} = 1,942.$$

COMMISSION or FACTORAGE*

Is an allowance of so much per cent. to a factor, or correspondent, abroad, for buying and selling goods, for his employer.

EXAMPLES.

1. What comes the commission on 539*l.* 12*s.* 9*d.* to, at $4\frac{1}{2}$ per cent. ?

$$\begin{array}{r} \text{£. s. d.} \\ 539 \quad 12 \quad 9 \\ \quad \quad 4\frac{1}{2} \\ \hline 2158 \quad 11 \quad 0 \\ \text{Product of } \frac{1}{2} = 269 \quad 16 \quad 4\frac{1}{2} \\ \hline \text{£} 24 \quad 28 \quad 7 \quad 4\frac{1}{2} \\ \quad \quad 20 \\ \hline \text{s. } 5 \quad 67 \\ \quad \quad 12 \\ \hline \text{d. } 8 \quad 08 \end{array}$$

2. My factor receives 100*l.* to lay out, after having deducted his commission of 5*l.* per cent. What does his commission amount to ?

Here,

* The method of working questions in this rule, Brokage, and the first case of Insurance, is the same as in Simple Interest.

Here, as his commission is to be deducted from the given sum, it is evident that I ought not to pay him commission on his own money, (which, however, is often unjustly practised) therefore,

$$\begin{array}{cccc} \text{£.} & \text{£.} & \text{£.} & \text{£.} \\ \text{As } 105 : 5 :: 1008 : 48 \text{ Ans.} \end{array}$$

3. My correspondent writes me, that he has purchased goods to the value of 673*l.* 12*s.* ; What does his commission on that sum amount to, at $3\frac{1}{2}\%$ per cent. ? *Ans.* £23 11*s.* 6*d.*

4. What must I allow my correspondent, at $2\frac{1}{4}\%$ per cent. for disbursing on my account 395*l.* 15*s.* 5*d.* ? *Ans.* £8 18*s.* 1*d.*

B R O K E R A G E

Is an allowance of so much per cent. to a person called a broker, for assisting merchants or factors in purchasing or selling goods.

E X A M P L E S.

1. What is the brokerage upon 523*l.* 10*s.* at 5*s.* or $\frac{1}{4}\%$ per cent. ?

$$\begin{array}{r} \text{£. } \text{s.} \\ 15\text{s. } | \frac{1}{4} | 525 \text{ } 10 \\ \hline 1 | 31 \text{ } 7 \text{ } 6 \\ \text{ } 20 \\ \hline 6 | 27 \\ \text{ } 12 \\ \hline 3 | 30 \\ \text{ } 4 \\ \hline 1 | 20 \end{array}$$

5 shillings being an aliquot part of 1*l.* ; divide by that part, and the quotient is the answer.

Ans. £1 6*s.* 3 $\frac{1}{4}$ *d.*

2. What is the brokerage upon 673*l.* 16*s.* at $\frac{5}{8}\%$ per cent. ?

Ans. £4 4*s.* 2 $\frac{1}{2}$ *d.*

3. If a broker sell goods to the amount of 2864*D.* 46*c.* What is his demand at $1\frac{1}{2}\%$ per cent. ?

Ans. 42*D.* 95*c.* 6 $\frac{2}{10}$ *m.*

B U Y I N G A N D S E L L I N G S T O C K S.

Stock is a general name for the capitals of trading companies, banks, &c. and the buying and selling certain sums of money in those funds, is not unusual.

E X A M P L E S.

1. What is the purchase of 275*l.* 15*s.* bank stock, at $7\frac{1}{2}\%$ per cent. ?

O o

£711 $\frac{1}{2}$

74½*l.* want 25½*l.* of 100*l.* therefore, take parts for the deficiency, and subtract the sum of those parts from the given sum.

£.		£.	s.	d.
20	½	275	15	0
5	¼	55	3	0
½	⅒	13	15	9
		1	7	6¼
<hr/>				
Subtract 70 6 3¼				

£205 8 8¼ *Ans.*

Or, If I had taken parts for 74½*l.* the rate per cent. then the sum of the several quotients would have been the answer as above.

When the price is above 100, take parts for the surplus of the price above 100, and add them to the given sum for the answer.

2. What is the purchase of 102*g*l. 10*s.* 6*d.* bank stock, at 110¼*l.* per cent. ?

£.		£.	s.	d.	
10	⅓	1029	10	6	at 100 per cent.
¼	⅔	102	19	0½	at 10 per cent.
		2	11	5½	at ¼ per cent.

Ans. £1135 1 0 at £110¼ per cent.

3. What is the purchase of 1058*l.* 12*s.* bank stock, at 115¾*l.* per cent. ?

Ans. £1225 6*s.* 6½*d.*

4. What does 1600*l.* capital stock, in the Massachusetts bank, come to, at 120⅓*l.* per cent. ?

Ans. £1922.

POLICIES OF INSURANCE.

Insurance is a security, or assurance, by mean of a writ called a *Policy*, to indemnify the insured of such losses as shall be specified in the policy subscribed by the insurer, or insurers, by which the underwriters oblige themselves to make good and effectual the property insured, in consideration of a certain premium at a stipulated rate per cent. (which varies according to the risque) to be immediately paid down, or otherwise secured according to the tenor of the agreement.

The average loss is 10 per cent. ; that is, if the insured suffer any damage or loss, not exceeding 10 per cent. he bears it himself, and the insurers are free.

A policy should be taken out for a sum sufficient to cover the principal and premium, and the business of this rule is, in general, to calculate what that sum should be.

C A S E I.

When the premium, at a certain rate per cent. for insuring a sum, is required, the operation is the same as in interest, or commission.

1. What

1. What is the premium upon 537*l.* 15*s.* 9*d.* at $6\frac{1}{2}$ per cent. ?

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 537 \quad 15 \quad 9 \\
 \quad 6\frac{1}{2} \\
 \hline
 3226 \quad 14 \quad 6 \\
 \frac{1}{2} = 268 \quad 17 \quad 10\frac{1}{2} \\
 \hline
 34|95 \quad 12 \quad 4\frac{1}{2} \\
 \quad 20 \\
 \hline
 19|12 \\
 \quad 12 \\
 \hline
 1|48 \\
 \quad 4 \\
 \hline
 1|94
 \end{array}$$

Ans. £34 19*s.* 1*½d.*

C A S E II.

To find the sum for which a policy should be taken out to cover a given sum.

RULE.—Take the premium from 100*l.* and say, As the remainder is to 100 ; so is the sum adventured, to the policy.*

1. It is required to cover 759*l.* premium 8 per cent. ; For what sum must the policy be taken ?

$$\begin{array}{r}
 100 \\
 8 \\
 \hline
 92 : 100 :: 759 \\
 100 \\
 \hline
 92)75900(825 \text{ Ans.} \\
 736 \\
 \hline
 230 \\
 184 \\
 \hline
 460 \\
 460 \\
 \hline
 0
 \end{array}$$

2. A

* Now it is plain, that if I want to recover 92*l.* I must, in this case, insure upon 100*l.* ; therefore, to recover 759*l.* I must insure upon 825*l.* ; for when 8 per cent. for premium is deducted, I shall have 759*l.* remaining nett.

For £825= sum insured upon at 8 per cent.

66=premium to be deducted.

759=the first outset.

In this and the following cases, let $x=100$, p =premium, a =amount to be insured upon, and s =sum to be covered ; then $x-p : x :: s : a$, or $\frac{xs}{x-p}=a$.

2. A merchant sent a vessel and cargo to sea, valued at 1525*l.*; What sum must the policy be taken out for, to cover his property, premium 19½ per cent. ?

$$\begin{array}{r} 100 \\ 19.5 \\ \hline \end{array}$$

$$80.5 : 100 :: 1525 : \text{£}1894 \text{ } 8\text{s. } 2\frac{1}{4}\text{d. Ans.}$$

C A S E III.

When a policy is taken out for a certain sum in order to cover a given sum :

To find the premium, say, As the policy is to the covered sum; so is 100*l.* to a fourth number, which, being taken from 100, will leave the premium.*

If a policy be taken out for 1250*l.* to cover 500*l.* What is the premium per cent. ?

$$\begin{array}{r} 1250 : 500 :: 100 \\ 100 \\ \hline \end{array}$$

$$1250)50000(40 \text{ and } \text{£}100 - 40 = \text{£}60 \text{ Ans.}$$

C A S E IV.

When the policy for covering any sum and the premium per cent. are given, to find the sum to be covered.

RULE.—Deduct the premium per cent. from 100, and say, As 100 is to the remainder ; so is the policy to the sum required to be covered.

If a policy be taken out for 1250*l.* at 60 per cent. ; What is the adventure, or sum to be covered ?†

$$\begin{array}{r} 100 \\ 60 \\ \hline \end{array}$$

$$100 : 40 :: 1250$$

$$\begin{array}{r} 40 \\ \hline \text{£} \\ 100)125000(500 \text{ Ans.} \end{array}$$

C A S E V.

When a given sum is adventured several voyages round from one place to another, either at the same, or different risques, from place to place, and it is required to take out a policy for such a sum as will cover the adventure all round, supposing the risque out and home to be equal and tantamount to the several given risques.

RULE.

$$* a : s :: x : x - p, \text{ or } x - \frac{sx}{a} = p.$$

$$† x : x - p :: a : s, \text{ or, } \frac{a \times x - p}{x} = s.$$

R U L E.

1. Raise 100*l.* to that power denoted by the number of risques, and multiply the said power by the sum adventured, (or to be covered) for a dividend.

2. Subtract the several premiums, each, from 100*l.* and multiply the several remainders continually together for a divisor, and the quotient, arising from this division, will give the policy to cover the adventure the voyage round.*

1. A merchant adventured 480*l.* 10*s.* from Newburyport to Southcarolina, from thence to Jamaica, and from thence, home, and the premium was 5 per cent. from port to port ; What sum must he take out a policy for, to cover his adventure the voyage round, supposing the risque to be equal out and home, and tantamount to the several given risques ?

$$100 \times 100 \times 100 \times 480,5 = 480500000 = \text{Dividend.}$$

$$\begin{array}{r} 100 \\ -5 \\ \hline 95 \end{array} \quad \begin{array}{r} 100 \\ -5 \\ \hline 95 \end{array} \quad \begin{array}{r} 100 \\ -5 \\ \hline 95 \end{array}$$

$$95 \times 95 \times 95 = 857375 = \text{Divisor.}$$

$$857375 \overline{) 480500000} (560*l.* 8*s.* 7\frac{1}{2}*d.* Ans.$$

2. A merchant adventured 500*l.* from Boston to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4, from thence to Nantz, at 5, and from thence home, at 6 per cent. ; For what sum must he take out a policy to cover his adventure the voyage round, supposing the risque to be equal out and home, and tantamount to the several given risques ?

$$\frac{100 \times 100 \times 100 \times 100 \times 500}{100-3 \times 100-4 \times 100-5 \times 100-6} = £601,278, \text{ Ans.}$$

CASE

* For the first Voyage.

$$x-p : x :: s : a.$$

$$\frac{xs}{x-p} = a.$$

$$\frac{a \times x-p}{x} = s.$$

$$x - \frac{ax}{a} = p.$$

Second Voyage.

$$x-p : x :: \frac{xs}{x-p} : a.$$

$$\frac{xxs}{x-p^2} = a.$$

$$\frac{a \times x-p^2}{x^2} = s.$$

$$x - \sqrt{\frac{xxs}{a}} = p.$$

Third Voyage.

$$x-p : x :: \frac{xxs}{x-p^2} : a.$$

$$\frac{xxxs}{x-p^3} = a.$$

$$\frac{a \times x-p^3}{x^3} = s.$$

$$x - \frac{\sqrt[3]{xxxs}}{a} = p. \text{ and so on}$$

for as many voyages as may be required. Hence, making m =exponent of any given

power, $\frac{x^m s}{x-p \times x-p \times x-p, \text{ \&c.}} = \text{sum to be insured upon, all round :—And}$

$x - \sqrt[m]{\frac{x^m s}{a}} = \text{the premium all round, tantamount to the several given premiums ;}$

s , in this Theorem, being equal to the first adventure, and a =amount which will cover that adventure the voyage round.

C A S E VI.

When a given sum is adventured several voyages round, as in the last case, either at the same, or different risques, from port to port, and the premium for the voyage round is required, tantamount to the several given rates per cent.

R U L E.

1. Find the sum for which the policy must be taken, by the last case.
2. Multiply the sum adventured by 100, and divide that product by the policy.
3. Take the quotient from 100% and the remainder will be the premium per cent. on the policy, tantamount to the several premiums given in the question.

1. A merchant adventured 480*l.* 10*s.* from Newburyport to Southcarolina, from thence to Jamaica, and from thence, home, and the premium was 5 per cent. from port to port : What will be the premium tantamount to the several given premiums, (allowing the risques out and home to be equal) on the policy which will cover the first adventure of 480*l.* 10*s.*

In question 1, Case 5, we found the policy to be £560 8*s.* 7½*d.* = £560,43125
 $480,5 \times 100 = 48050$ and $560,43125 \div 48050 = 0,0000085,7375$ and $100 - 85,7375 = 14,2625$ the premium required : Or thus,

$$100 - \frac{480,5 \times 100}{560,43125} = 14,2625.$$

2. A merchant adventured 500*l.* from Boston to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4 ; from thence to Nantz, at 5 ; and from thence, home, at 6 per cent. ; What will be the premium, tantamount to those given in the question, on a policy for covering the first adventure, the voyage, supposing the risques out and home equal ?

In question 2d, Case 5, we found the policy, which would cover the adventure the voyage round, to be £601,278.

Then, $100 - \frac{500 \times 100}{601,278} = 16,844$ = the premium per cent. on the policy the voyage round, and tantamount to the several given premiums.

C A S E VII.

If a policy be taken out for a given sum, to cover a certain adventure from one port to another, on to several ports, at equal premiums from one place to the other, to find what that equal premium is.

R U L E.

1. Involve 100 to that power denoted by the number of risques, and multiply this power by the sum adventured, (or covered.)
2. Divide the last product by the policy.
3. Extract that root of the quotient denoted by the number of risques.
4. Take

4. Take this root from 100*l.* and the remainder will be the equal premium from one port to the other.

1. A merchant adventured 480*l.* 10*s.* from Newburyport to Southcarolina, from thence to Jamaica, and from thence home; to cover which all round he took out a policy for 560*l.* 8*s.* 7½*d.* and the premium was equal from one place to the other; What was the premium per cent.?

$$100 - \sqrt[3]{\frac{100 \times 100 \times 100 \times 480,5}{560,43125}} = 5 \text{ per cent.}$$

C A S E VIII.

When an adventure is insured out and home at one risque, at a given rate per cent. and the voyage terminates short of what was at first intended: To find what the underwriter must receive per cent.

RULE 1.—If just half the voyage is performed, it must be considered as *two* equal risques: If one third, then, as *three* equal risques; if but one fourth, then, as *four* risques, and so on; and by Case 2d must be found the amount which will cover the adventure the voyage round.

2. Involve 100 to that power denoted by the number of risques, and multiply this power by the sum adventured.

3. Divide this product by the aforesaid amount.

4. Extract that root of the quotient denoted by the number of risques.

5. Take this root from 100*l.* and the remainder will be the sum per cent. which the underwriter must receive.

1. A merchant covers 200*l.* at 6 per cent. from Newburyport to the Westindies and home again; but the voyage terminating in the Westindies, what must the insurer receive per cent.?

$$\begin{array}{r} 100 \\ 6 \\ \hline 94 : 100 :: 200 : 212,765957 = \text{amount to cover } \pounds 200 \text{ voyage round.} \end{array}$$

$$100 \times 100 \times 200 = 2000000 \text{ and } \frac{2000000}{212,765957} = 9400$$

and $100 - \sqrt[3]{9400} = 3,0465$ to be paid the insurer per cent. upon the above amount.

2. A merchant insures 350*l.* to the Westindies, from thence, to France, and from thence home, at 10 per cent. the voyage round; but there is but one third part of the voyage performed; What must the insurer receive per cent.?

There being but ⅓ of the voyage performed, we must suppose three equal risques.

$$\begin{array}{r} 100 \\ 10 \\ \hline 90 : 100 :: 350 : 388,8889, \text{ \& } 100 \times 100 \times 100 \times 350 = 350000000 \\ \frac{350000000}{388,8889} = 899999,9999 \text{ \& } 100 - \sqrt[3]{899999,9999} = \pounds 3,4515 \text{ Ans.} \end{array}$$

COMPOUND INTEREST.

COMPOUND INTEREST

Is that which arises from the interest being added to the principal, and (continuing in the hands of the borrower) becomes a part of the principal, at the end of each stated time of payment.

METHOD I.

RULE.*

Find the *amount* of the given principal, for the time of the first payment, by Simple Interest; next, find the interest of that sum, or principal, and add it as before, and thus proceed for any number of years, still accounting the last *amount* as the principal for the next payment. The given principal being subtracted from the last *amount*, the remainder will be the compound interest.

EXAMPLES.

1. What will 480*l.* amount to in 5 years, at 6 per cent. per annum?

Principal 480	Principal for the 1st year 480	0
Rate of interest 6	Interest of ditto	28 16
<hr/>	Principal for the 2d year 508	16
28 80		6
20		
<hr/>		
16 00		30 52 16
		20
		<hr/>
Prin. for the 2d year 508		10 56
Interest for ditto 30 10 6½		12
<hr/>		<hr/>
Prin. for the 3d year 539		6 72
		6
		<hr/>
		4
		<hr/>
32 35 19 3		2 88
20		
<hr/>	Principal for the 3d year £539	6 6½
7 19	Interest for ditto 32 7 2¼	
12		
<hr/>	Principal for the 4th year 571	13 8½
2 31		
4		
<hr/>		
1 24		Principal

* As all the computations relating to Simple Interest, are founded upon Arithmetical Progression, the simple interest of one pound being a series of terms in Arithmetical Progression increasing; whose first term and common difference is the interest of one pound for one year, and the number of years shewing the number of all the terms; therefore, the last term will always be equal to the product of the time and rate, equal to the interest of one pound for any given time: So those relating to Compound Interest are founded upon a series of terms increasing in Geometrical Progression, wherein the number of years assigns the the index of the last and highest term: Therefore, as one pound is to the amount of one pound, for any given time; so is any proposed principal, or sum, to its amount for the same time.

Prin. for the 4th year $\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 571 \quad 13 \quad 8\frac{1}{4} \\ \underline{\hspace{1cm}} \\ 6 \end{array}$

$\begin{array}{r} 34 \mid 30 \quad 2 \quad 4\frac{1}{2} \\ \underline{\hspace{1cm}} \\ 20 \end{array}$

$\begin{array}{r} 6 \mid 02 \\ \underline{\hspace{1cm}} \\ 12 \end{array}$

$\begin{array}{r} 0 \mid 28 \\ \underline{\hspace{1cm}} \\ 4 \end{array}$

$\begin{array}{r} 1 \mid 14 \\ \underline{\hspace{1cm}} \end{array}$

Prin. for the 5th year $\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 605 \quad 19 \quad 9 \\ \text{Interest for ditto} \quad 36 \quad 7 \quad 2 \\ \underline{\hspace{1cm}} \end{array}$

Amount for 5 years $\begin{array}{r} 642 \quad 6 \quad 11 \\ \text{Subtract the first prin.} \quad 480 \quad 0 \quad 0 \\ \underline{\hspace{1cm}} \end{array}$

Comp. int. for 5 years $\begin{array}{r} 162 \quad 6 \quad 11 \end{array}$

$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ \text{Prin. for the 4th yr.} \quad 571 \quad 13 \quad 8\frac{1}{4} \\ \text{Interest for ditto.} \quad 34 \quad 6 \quad 0\frac{1}{4} \\ \underline{\hspace{1cm}} \end{array}$

Prin. for the 5th year $\begin{array}{r} 605 \quad 19 \quad 9 \\ \underline{\hspace{1cm}} \\ 6 \end{array}$

$\begin{array}{r} 36 \mid 35 \quad 13 \quad 6 \\ \underline{\hspace{1cm}} \\ 20 \end{array}$

$\begin{array}{r} 7 \mid 18 \\ \underline{\hspace{1cm}} \\ 12 \end{array}$

$\begin{array}{r} 2 \mid 22 \\ \underline{\hspace{1cm}} \end{array}$

2. What is the compound interest of 740*l.* for 6 years, at 4*l.* per cent. per annum ? *Ans.* £ 195 6*s.* 8½*d.*

3. What will 400*l.* amount to in 5 years, at 4*l.* per cent. per annum ? *Ans.* £ 486 13*s.* 2½*d.*

4. What will 150*l.* amount to in a year, at 2*l.* per cent. per month ? *Ans.* £ 190 4*s.* 5*d.*

METHOD II.

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the first year; and the principal for the second.

2. In like manner find the amount for every succeeding year.

When the rate is at 6 per cent. per annum.

1. Divide the principal by 20, and that quotient by 5: These quotients, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner obtain the amount for every succeeding year.

EXAMPLES.

What is the amount of 480£ . Of the same sum at 5 per cent. per annum, for 5 years? at 6 per cent. per annum, for 5 years?

20)480

5) 24

4 16

20)508 16 amount of 1st year.

5) 25 8 $9\frac{1}{2}$

5 1 9

20)539 6 $6\frac{1}{2}$ ditto of 2d.5) 26 19 $3\frac{3}{4}$ 5 7 $10\frac{1}{4}$ 20)571 13 $8\frac{1}{2}$ ditto of 3d.

5) 28 11 8

5 14 4

20)605 19 $8\frac{1}{2}$ ditto of 4th.5) 30 5 $11\frac{1}{4}$ 6 1 $2\frac{1}{4}$ $\text{£}642$ 6 $10\frac{1}{2}$ do. of 5th, *Ans.*

20)480

24

20)504 amount of 1st year.

25 4

20)529 4 ditto of 2d.

26 9 $2\frac{1}{4}$ 20)555 13 $2\frac{1}{4}$ ditto of 3d.27 15 $7\frac{1}{4}$

20)583 8 10 ditto of 4th.

29 3 $5\frac{1}{4}$ $\text{£}612$ 12 $3\frac{1}{4}$ do. of 5th, *Ans.*

COMPOUND INTEREST BY DECIMALS.

A TABLE of the Amount of 1£ . at $\frac{1}{2}$ per Cent. per Month, as practised at the Massachusetts Bank.

Months.	£. Dec. pts.	Months.	£. Dec. pts.	Months.	£. Dec. pts.
1	1,005	5	1,025	9	1,045
2	1,01	6	1,03	10	1,05
3	1,015	7	1,035	11	1,055
4	1,02	8	1,04	12	1,06

A TABLE of the Amount of 1£ . from 1 Day to 31 Days, at 6 per Cent. per Annum.

Days.	£. Dec. parts.	Days.	£. Dec. parts.	Days.	£. Dec. parts.
1	1,00016	12	1,00197	22	1,00361
2	1,00032	13	1,00213	23	1,00378
3	1,00049	14	1,00230	24	1,00394
4	1,00065	15	1,00246	25	1,00410
5	1,00082	16	1,00263	26	1,00427
6	1,00098	17	1,00279	27	1,00443
7	1,00115	18	1,00295	28	1,00460
8	1,00131	19	1,00312	29	1,00476
9	1,00147	20	1,00328	30	1,00493
10	1,00164	21	1,00345	31	1,00509
11	1,00180				

C A S E I.*

When the principal, the rate of interest, and time, are given, to find either the amount or interest.

RULE 1.—Find the amount of 1*l.* for 1 year, at the given rate per cent.

2. Involve the amount, thus found, to such power, as is denoted by the number of years; or, in Table I, at the end of Annuities;

* Let r = amount of 1*l.* for 1 year, and p = principal, or given sum; then, since r is the amount of 1*l.* for 1 year, r^2 will be its amount for 2 years, r^3 for 3 years, and so on; therefore, it will be as $1 : r :: r : r^2$ = amount for the second year, or principal for the third: Again, As $1 : r :: r^2 : r^3$ = amount for the third year, or principal for the fourth, &c. to any number of years.—And if the time, or number of years, be denoted by t , the amount of 1*l.* for t years, will be r^t ; from hence it will appear that the amount of any other principal sum p , for t years, is pr^t ; for, as $1 : r^t :: p : pr^t$, the same as in the rule.

If the rate of interest be determined to any other time than a year, as $\frac{1}{2}$, $\frac{1}{4}$, &c. the rule is the same, and then t will represent that stated time.

Let $\begin{cases} r = \text{amount of 1}l. \text{ for 1 year, at the given rate per cent,} \\ p = \text{principal, or sum put out at interest.} \\ i = \text{interest.} \\ t = \text{time.} \\ m = \text{amount for the time } t. \end{cases}$

Then the following Theorems will exhibit the solutions of all the cases in Compound Interest.

$$\text{I. } pr^t = m. \quad \text{II. } prt - p = i. \quad \text{III. } \frac{m}{r^t} = p. \quad \text{IV. } \frac{m}{p} \bigg| t = r.$$

The most convenient way of giving the Theorems, especially that for the time, will be by Logarithms, as follows:

$$\begin{aligned} \text{I. } t \times \text{Log. } r + \text{Log. } p &= \text{Log. } m. & \text{II. } \text{Log. } m - t \times \text{Log. } r &= \text{Log. } p. \\ \text{III. } \frac{\text{Log. } m - \text{Log. } p}{\text{Log. } r} &= t. & \text{IV. } \frac{\text{Log. } m - \text{Log. } p}{t} &= \text{Log. } r. \end{aligned}$$

If the compound interest, or amount of any sum, be required for the parts of a year, it may be determined as follows:

I. *When the time is an aliquot part of a year.*

RULE 1.—Find the amount of 1*l.* for 1 year, as before, and that root of it, which is denoted by the aliquot part, will be the amount of 1*l.* for the time sought.

2. Multiply the amount, thus found, by the principal, and it will be the amount of the given sum required.

II. *When the time is not an aliquot part of a year.*

RULE 1.—Reduce the time into days, and the 365th root of the amount of 1*l.* for 1 year is the amount for 1 day.

2. Raise this amount to that power, whose index is equal to the number of days, and it will be the amount of 1*l.* for the given time.

3. Multiply this amount by the principal, and it will be the amount of the given sum required.

To avoid extracting very high roots, the same may be done by logarithms, thus: Divide the logarithm of the rate, or amount of 1*l.* for 1 year, by the denominator of the given aliquot part, and the quotient will be the logarithm of the root sought.

nities, under the rate, and against the given number of years, you will find the power.*

3. Multiply this power by the principal, or given sum, and the product will be the amount required, from which, if you subtract the principal, the remainder will be the interest.

E X A M P L E S.

1. What is the compound interest of 600*l.* for 4 years, at 6 per cent. per annum?

Multiply by $1.06 = \left\{ \begin{array}{l} \text{amount of 1*l.* for 1 year, at 6 per} \\ \text{cent. per annum.} \end{array} \right.$

Multiply by $1.1236 = 2d \text{ power.}$

Multiply by $1.26247696 = 4th \text{ power.}$

Subtract 600

$157,486176 = £157 \text{ 9s. } 8\frac{1}{2}d. = \text{interest required.}$

By TABLE I.

Tabular amount of 1*l.* for 4 years, at 6 per cent. per ann. = 1,26247696

Multiply by the principal = 600

Amount = 757,4861400

2. What is the amount of 570*l.* 10*s.* for 12 years, at $3\frac{1}{2}$ per cent. per annum?

Ans. £862 1*s.* $3\frac{1}{4}d.$

Another method of working Compound Interest for years, months and days, which is much more concise than the preceding method.

R U L E.

To the logarithm of the principal, found in any Table of logarithms, add the several logarithms, answering to the number of years,

* The amounts of 1*l.* in this Table, are so many powers of the amount of 1*l.* for 1 year, whose indices are denoted by the number of years.

Note. When the given time consists of years and months, or years, months and days; first seek the amount of 1*l.* in the Table for years, then in the Table of months, &c. multiply these several amounts and the principal continually together, and the last product will be the amount required.

Thus, if the amount of 480*l.* in $5\frac{1}{2}$ years, at 6 per cent. per annum, were required; the amount of 1*l.* for 5 years = £1,33822, ditto for 6 months = £1,02956, Now, $1,33822 \times 1,02956 \times 480 = £661,2341$ *Answer.*

COMPOUND INTEREST BY DECIMALS.

301

years, months and days, found in the following tables, and their sum will be the logarithm of the amount for the given time, which being found in any table of logarithms, the natural number corresponding thereto will be the answer.*

Logarithmic TABLES, at 6 per Cent. per Annum, for Years, Months and Days.

Years.	dec. pts.	Y.	dec. pts.	Y.	dec. pts.	Y.	dec. pts.	Months	dec. pts.
1	,025306	11	,278366	21	,531426	31	,784586	1	,002166
2	,050612	12	,303672	22	,556732	32	,809792	2	,004321
3	,075918	13	,328978	23	,582038	33	,835098	3	,006466
4	,101224	14	,354284	24	,607344	34	,860404	4	,0086
5	,12653	15	,37969	25	,63265	35	,88571	5	,010724
6	,151836	16	,404896	26	,657956	36	,911016	6	,012837
7	,177142	17	,430202	27	,683262	37	,936322	7	,01494
8	,202448	18	,455508	28	,708568	38	,961628	8	,017033
9	,227754	19	,480814	29	,733974	39	,986934	9	,019116
10	,25306	20	,50612	30	,75933	40	,10124	10	,021189
								11	,023352

Days.	D.	D.	D.	D.	D.
1	,000071	8	,000571	14	,000999
2	,000143	9	,000642	15	,00107
3	,000215	10	,000713	16	,001142
4	,000287	11	,000785	17	,001213
5	,000358	12	,000857	18	,001284
6	,000429	13	,000928	19	,001355
7	,0005			20	,001426
				21	,001497
				22	,001568
				23	,001639
				24	,00171
				25	,001781
				26	,001852
				27	,001923
				28	,001994
				29	,002065
				30	,002136
				31	,002207

6. What is the amount of 132*l.* 10*s.* at 6 per cent. per annum, for 9 years, 8 months, and 15 days?

$$\begin{array}{l} \text{To the log. of } £132,5 = 2,122216 \\ \text{Add } \left\{ \begin{array}{l} \text{Log. for 9 years} = ,227754 \\ \text{ditto for 8 months} = ,017033 \\ \text{ditto for 15 days} = ,00107 \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{Because 8 months are past, deduct 4 } \\ \text{per cent. upon the logarithm of 15 days } \end{array} \left. \vphantom{\begin{array}{l} \text{Because 8 months are past, deduct 4 } \\ \text{per cent. upon the logarithm of 15 days } \end{array}} \right\} = \begin{array}{l} 2,368073 \\ ,0000428 \end{array}$$

Remains 2,3680302, the nearest to which, in the Table of logarithms, is 2,368101, and the natural number answering thereto is 233,4 = £233 8*s.* *Ans.*

CASE

* Although there is a small error in the logarithms for days, yet they are exact enough for common use.—And if after the first month you deduct $\frac{1}{2}$ per cent. for each month past (that is, $\frac{1}{2}$ per cent. after 1 month, $1\frac{1}{2}$ per cent. after 3 months, &c.) from the logarithm of the number of days, it will give the true answer.

Note. That, after 1 month, $\frac{1}{2}$ per cent. on the logarithm of 1 day, is ,0000355, on 2 days, is ,00000715 : After 2 months, 1 per cent. on the logarithm of 1 day, is ,00000071, on 2 days, ,00000143 : After 10 months, 5 per cent. on the logarithm for 1 day, is ,00000355, on 6 days, is ,00002145, &c.

COMPOUND INTEREST BY DECIMALS.

C A S E II.

When the amount, rate and time, are given, to find the principal.

R U L E.

Divide the amount by the amount of 1*l.* for the given time, and the quotient will be the principal.

Or, If you multiply the present value of 1*l.* for the given number of years, at the given rate per cent. by the amount, the product will be the principal, or present worth.*

E X A M P L E S.

1. What is the present worth of 757*l.* 9*s.* 8½*d.* due 4 years hence, discounting at the rate of 6*l.* per cent. per annum?

By TABLE I.

Divide by the tabular } = 1,2624769)757,4861400 (£600 *Ans.*
amount of 1*l.* for 4 years,

By TABLE II.

Mult. by the present worth of 1*l.* } Amount = 757,48614
for 4 years, at 6 per cent. per ann. } = 79 20936

Ans. 599,999923582704 + = £600

2. What principal must be put to interest 6 years, at 5½ per cent. per annum, to amount to 965*l.* 3*s.* 9½*d.* 3616? *Ans.* £700.

C A S E III.

When the principal, rate and amount, are given, to find the time.

R U L E.

Divide the amount by the principal; then divide this quotient by the amount of 1*l.* for 1 year, this quotient by the same till nothing remain, and the number of the divisions will shew the time.

Or, Divide the amount by the principal, and the quotient will be the amount of 1*l.* for the given time, which seek under the given rate in Table 1, and, in a line with it, you will see the time.

E X A M P L E S.

1. In what time will 600*l.* amount to 757*l.* 9*s.* 8½*d.* at 6 per cent. per annum, compound interest?

Divide the amount } = 600)757,486176(1,26247696 = quotient
by the principal }
to be divided by 1,06 till it can be had no more; or you may find it in Table 1, under 6 per cent. and against 4 years.

Divide

* See Table II. shewing the present value of 1*l.* discounting at the rates of 4, 4½, &c. per cent. the construction of which is thus:

Amount. Pref. worth. Amount. Pref. worth.

As 1,06 : 1 :: 1 : ,9433962, and so on, for any other rate per cent. and time,

DISCOUNT BY COMPOUND INTEREST.

303

Divide by 1,06)1,26247696

1,06)1,191016

1,06)1,1236

1,06)1,06

1

Four divisions shew the time to be 4 years.

C A S E IV.

When the principal, amount and time, are given, to find the rate per cent.

RULE.—Divide the amount by the principal, and the quotient will be the amount of 1*l*. for the given time, then, extract such root as the time denotes, and that root will be the amount of 1*l*. for 1 year, from which subtract unity, and the remainder will be the ratio.

Or, Having found the amount of 1*l*. for the time, as above directed, look for it in Table 1, even with the given time, and directly over the amount you will find the ratio.

E X A M P L E.

At what rate per cent. per annum will 600*l*. amount to 757*l*. 9*s*. 8½*d*. in 4 years?

600)757,486¼(1,2624769. Now, the time being 4 years, the 4th root of this quotient minus 1 will be the ratio.

1,26247690(1,123599+ and 1,123599(1,05999+ and 1,05999—1 =,06 Answer.

DISCOUNT BY COMPOUND INTEREST.

C A S E I.*

The sum, or debt to be discounted, the time and rate, given, to find the present worth.

RULE.

* Let m = sum or debt to be discounted, and the other letters as before : Then the following Theorems will shew all the cases in Discount by Compound Interest.

I. $\frac{m}{r^t} = p$. II. $pr^t = m$. III. $\frac{m}{p} = r^t$ which being continually divided by r , till nothing remain, the number of these divisions will be equal to t .

IV. $\frac{m}{p} = r^t$ which being extracted, (the time, given in the question, shewing the power) will be equal to r .

Note. Case 2d may be wrought by Table 1, thus : Find that power of 1*l*. for 1 year, denoted by the time ; multiply the present worth by it, and the product will be the answer.

Or, by Table 2d, thus : Find the present worth of 1*l*. for the given time, by which divide the present worth, and the quotient will be the debt or principal.

Case 3d, thus : Divide the debt by its present worth, and seek the quotient in Table 1, under the given rate, and in a line with it you will see the time.

Case 4th is wrought in the same manner, only, seek the quotient in a line with the time, it will shew the rate at op.

RULE.—Divide the debt by that power of the amount of *1l.* for 1 year, denoted by the time, and the quotient will be the present worth, which, subtracted from the debt, will leave the discount.

EXAMPLES.

1. What is the present worth, and discount, of 600*l.* due 3 years hence, at 6*l.* per cent. per annum, compound interest?

Divide by $1.061^3 = 1.19101$ 600,00000 (503,7741 = £503 15*s.* 5¼*d.* present worth, and £600 — £503 15 5¼ = £96 4*s.* 6¼*d.* = Discount.

$$\text{Or, } \frac{600}{1.19101} = £503.7741, \text{ £ } 600 - \frac{600}{1.19101} = £96.2259$$

By TABLE II.

In this Table, corresponding to the time and rate, we have, 899619 = present worth of *1l.* for the time and rate. Multiply by 600 = debt, or principal.

503,771400 = present worth of the debt.

2. What is the present worth of 312*l.* 10*s.* due 2 years hence, at 4½ per cent. per annum, compound interest?

Ans. £286 3*s.* 3*d.* 2,97*qrs.*

3. What ready money will discharge a debt of 1000*D.* due 4 years hence, at 5*D.* per cent. per annum, compound interest?

Ans. 822*D.* 70*c.* 2*m.*

ANNUITIES OR PENSIONS IN ARREARS,
AT COMPOUND INTEREST.

CASE I.

When the annuity, or pension, the time it continues, and the rate per cent. are given, to find the amount.

RULE* 1.—Make 1 the first term of a Geometrical Progression, and the amount of *1l.* for 1 year at the given rate per cent. the ratio.

2. Carry

* It is plain that upon the first year's annuity there will be due so many year's compound interest, as the given number of years less 1, and gradually one year less, upon every succeeding year, to that preceding the last, which has but one year's interest, and the last bears none.

Let *r* = rate, or amount of *1l.* for 1 year, then the series of amounts of *1l.* annuity for several years, from the first to the last, is 1, *r*, *r*², *r*³, &c. to *r*^{*t*−1}; and the sum of this, according to the rule in Geometrical Progression, will be $\frac{r^t - 1}{r - 1} = a$ amount of *1l.* annuity for *t* years. And all annuities are proportional to their amounts;

therefore, $1 : \frac{r^t - 1}{r - 1} :: n : \frac{r^t - 1}{r - 1} \times n = \text{amount of any given annuity } n$.

Let *r* = rate, or amount of *1l.* for 1 year, and the other letters as before, then, $\frac{r^t - 1}{r - 1} \times n = a$, and $\frac{ar - a}{r^t - 1} = n$.

And from these equations, all the cases relating to annuities or pensions in arrears, may be conveniently exhibited in logarithmic terms, thus,

1. *L*_{*a*}

2. Carry the series to so many terms as the number of years, and find its sum.

3. Multiply the sum, thus found, by the given annuity, and the product will be the amount sought.

Or, Multiply the amount of 1*l.* for one year into itself so many times as there are years less by 1; then multiply this product by the annuity; and subtract the annuity therefrom: Lastly, Divide the remainder by the ratio less 1, and the quotient will be the amount.

EXAMPLES.

1. What will an annuity of 60*l.* per annum, payable yearly, amount to in 4 years, at 6*l.* per cent. ?

First Method.

$$1 + 1,06 + \overline{1,06}^2 + \overline{1,06}^3 = 4,374616 = \text{sum.}$$

Multiply by 60 = annuity.

$$\begin{array}{r} 252,476960 \\ 20 \end{array}$$

20

$$\begin{array}{r} 9,53920 \\ 12 \end{array}$$

12

$$\begin{array}{r} 6,4704 \\ 4 \end{array}$$

4

$$\begin{array}{r} 1,8816 \end{array}$$

Ans. £262 9*s.* 6¼*d.*

$$\text{Or, } 1 + 1,06 + \overline{1,06}^2 + \overline{1,06}^3 \times 60 = £262 \text{ } 9*s.* \text{ } 6\frac{1}{4}*d.*$$

Second

$$\text{I. } L.n + L.r^t - 1 + L.\overline{r-1} = L.a.$$

$$\text{II. } L.a - L.r^t - 1 + L.\overline{r-1} = L.n$$

$$\text{III. } \frac{L.ar - a + n - L.n}{L.r} = t. \quad \text{IV. } r^t - \frac{ar}{n} - 1 = 0.$$

$$\text{Or thus, I. } \frac{nr^t - n}{r - 1} = a. \quad \text{II. } \frac{ar - a}{r^t - 1} = n. \quad \text{III. } \frac{ar + n - a}{n} = r^t$$

which continually divided by *r* till nothing remain, the number of those divisions will be equal to *t*; Or, being extracted, (the given time shewing the power) will be equal to *r*.

Second Method.

$$1,06 \times 1,06 \times 1,06 \times 1,06 = 1,262,47$$

Multiply by 60 annuity.

$$\begin{array}{r} 75,74820 \\ \text{Subtract } 60 \end{array}$$

$$\text{Divide by } 1,06 - 1 = ,06 \quad 15,7482(262,47 = \text{£ } 262 \text{ } 9\text{s. } 4\frac{1}{4}\text{d. Ans.}$$

$$\begin{array}{r} 37 \\ 36 \\ \hline 14 \\ 12 \\ \hline 28 \\ 24 \\ \hline 42 \\ 42 \end{array}$$

$$\text{Or, } \frac{1,06 \times 1,06 \times 1,06 \times 1,06 \times 60 - 60}{1,06 - 1} = \text{£ } 262,47$$

*Or, By TABLE III.**

Multiply the tabular number under the rate, and opposite to the time, by the annuity, and the product will be the amount.

2. What will an annuity of 60*l.* per annum amount to in 20 years, allowing 6*l.* per cent. compound interest?

Under 6*l.* per cent. and opposite 20, in Table 3d, you will find,

Tabular number = 36,78559

Multiply by 60 = annuity.

$$2207,13540 = \text{£ } 2207 \text{ } 2\text{s. } 8\frac{1}{4}\text{d. Ans.}$$

3. What will a pension of 75*l.* per annum, payable yearly, amount to in 9 years, at 5*l.* per cent. compound interest.

Ans. £ 826 19s. 10d.

4. If a salary of 100*l.* per annum, to be paid yearly, be forborne 5 years, at 6*l.* per cent. What is the amount?

Ans. £ 563 14s. 2d.

CASE

* Table 3d is calculated thus : Take the first year's amount, which is 1*l.* multiply it by 1,06 + 1 = 2,06 = second year's amount, which also multiply by 1,06 + 1 = 3,1836 = third year's amount, &c. and in this manner proceed in calculating Tables at any other rates.

C A S E II.

When the amount, rate per cent. and time are given, to find the annuity, pension, &c.

RULE.—Multiply the whole amount by the amount of 1*l.* for 1 year, from which subtract the whole amount; divide the remainder by that power of the amount of 1*l.* for 1 year, signified by the number of years, made less by unity, and the quotient will be the answer.

E X A M P L E S.

1. What annuity, being forborne 4 years, will amount to £262,47696, at 6*l.* per cent. compound interest?

262,47696 = amount.

Multiply by 1,06 = amount of 1*l.* for 1 year.

157486176	1,06
262476960	1,06
278,2255776	636
Subtract 262,47696	1060
15,7486176 (£60 Ans.)	1,1286
26247696	1,06
157486176	67416
26247696	112860
15,7486176	1,191016
26247696	1,06
15,7486176	7146096
26247696	11910160
15,7486176	1,26247696
Subtract 1	1,26247696

Divisor = ,26247696

$$\text{Or, } \frac{262,47696 \times 1,06 - 262,47696}{1,06 \times 1,06 \times 1,06 \times 1,06 - 1} = 60.$$

Or, By TABLE III.

Divide the amount by the tabular number under the rate, and opposite to the time, and the quotient will be the annuity.

2. What annuity, being forborne 20 years, will amount to £2207,1354, at 6*l.* per cent. compound interest?

Tabular amount = 26,78559 2207,1354 (£60 Ans.)

2207,1354

C A S E III.

When the annuity, amount and ratio are given, to find the time.

RULE.—Multiply the amount by the ratio, to this product add the annuity, and from the sum subtract the amount; this remainder being divided by the annuity, the *quotient* will be that power of the ratio signified by the time, which being divided by the amount of 1*l.* for 1 year, and this *quotient* by the same, till nothing remain, the number of those divisions will be equal to the time. Or, look for this number under the given rate in Table 1. and, in a line with it, you will see the time.

E X A M P L E S.

1. In what time will 60*l.* per annum, payable yearly, amount to £262,47696, allowing 6*l.* per cent. compound interest, for the forbearance of payment?

262,47696 = amount.
Multiply by 1,06 = ratio.

157486176
262476960

278,2255776
Add 60 = annuity,

338,2255776
Subtract 262,47696

Divide by 60)75,7486176

Divide by 1,06)1,26247696

Divide by 1,06)1,191016

Divide by 1,06)1,1236

Divide by 1,06)1,06

1

The number of divisions by 1,06, being 4, gives the number of years = 4 the answer.

Or, In Table 1. under the given rate, you will find 1,262476, and in a line, under years, you will find 4.

2. In what time will an annuity of 60*l.* payable yearly, amount to £2207,1354, allowing 6*l.* per cent. for the forbearance of payment?

Ans. 20 years.
PRESENT

PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

C A S E I.

When the annuity, &c. rate and time are given, to find the present worth.

RULE* 1.—Divide the annuity by the ratio, or the amount of 1*l.* for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth for two years.

3. In like manner, find the present worth of each year by itself, and the sum of all these will be the present value of the annuity, sought.

Or, Divide the annuity, &c. by that power of the ratio signified by the number of years, and subtract the quotient from the annuity : This remainder being divided by the ratio less 1, the quotient will be the present worth.

EXAMPLES.

* The reason of this rule is evident from the nature of the question, and what was said upon the same subject in the purchasing of annuities by simple interest.

Let p = present worth of the annuity, and the other letters as before : Then,

$$n \times \frac{r^t - 1}{r^t + 1 - r^t} = p. \quad \text{And } p \times \frac{r^{t+1} - r^t}{r^t - 1} = n.$$

And from these Theorems, all the cases, where the purchase of annuities is concerned, may be exhibited in logarithmic terms, as follows :

$$\text{I. } L.n + L.1 - \frac{1}{r^t} - L.r - 1 = L.p.$$

$$\text{II. } L.p + L.r - 1 - L.1 - \frac{1}{r^t} = n.$$

$$\text{III. } \frac{L.n - L.n + p - pr}{L.r} = t.$$

$$\text{Or, thus, I. } \frac{n - \frac{n}{r^t}}{r - 1} = p. \quad \text{II. } \frac{pr^t \times r - pr^t}{r^t - 1} = n.$$

III. $\frac{n}{p + n - pr} = r^t$ which being continually divided by r till nothing remain, the number of those divisions will be equal to t .

Let t express the number of half years, or quarters, n the half years, or quarter's payment, and r the sum of 1*l.* and $\frac{1}{2}$ or $\frac{1}{4}$ year's interest, then all the preceding rules will be applicable to half yearly, and quarterly payments, the same as to whole years.

The amount of an annuity may also be found for years and parts of a year, thus :

1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a year.
3. Add this interest to the former account, and it will give the whole amount required.

The present worth of an annuity for years and parts of a year may be found thus :

1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

EXAMPLES.

1.* What ready money will purchase an annuity of 60*l.* to continue 4 years, at 6*l.* per cent. compound interest.

First Method.

Ratio = 1,06 | 60,00000 (56,603 = present worth for 1 year.

Ratio² | = 1,1236 | 60,00000 (53,399 = do. for 2 years.

Ratio³ | = 1,191016 | 60,00000 (50,377 = do. for 3 years.

Ratio⁴ | = 1,26247696 | 60,00000 (47,525 = do. for 4 years.
207,904 = £ 207 18*s.* 0¹/₄*d.* Ans.

Second Method.

4th power of the ratio } = 1,26247696 | 60,0000000 (47,525

From 60

Subtract 47,525

Or, $\frac{60}{,06 | 4} = 47,525$ 60 - 47,525 = 12,475

Divid. 1,06 - 1 = ,06 | 12,475

And $\frac{12,475}{,06} = 207,916.$

207,916 = £ 207 18*s.* 3¹/₄*d.* Ans.

By TABLE III.

Under 6*l.* per cent. and opposite 4, we find

4,37461 = amount of 1*l.* annuity for 4 years.

Multiply by 60 = annuity.

262,47660 = amount of 60*l.* for 4 years.

Then, opposite 4 years, and under 6*l.* per cent. in Table 2d,

We have ,797093

Multiply by 262,7466

```

      4752558
      4752558
      3168372
      5544651
      1584186
      4752558
      1584186
  
```

208,1157426338 = £ 208 2*s.* 4¹/₂*d.*

Or, Opposite 4 years, and under 6*l.* per cent. in Table 1st, we have 1,26247 = the amount of 1*l.* for 4 years :

Then, 262,7466 ÷ 1,26247 = 208,1209 = £ 208 2*s.* 5*d.* Ans.

By

* Questions in this Case may also be answered by first finding the amount of the given annuity by Case 1. of Annuities in Arrears, page 304, and then the present worth, or principal, by Case 2. of Compound Interest, page 302.

By TABLE IV.*

Multiply the tabular number, under the rate, and opposite the time, into the annuity, and the product will be the present worth.

Thus, in Example 1st, What ready money will purchase 60*l.* annuity, to continue 4 years, at 6*l.* per cent. compound interest ?

Under 6*l.* per cent. and even with 4 years,

We have 3,4651 = present worth 1*l.* for 4 years.
Multiply by 60 = annuity.

Ans. = 207,9060 = £ 207 18*s.* 1½*d.*

2. What is the present worth of an annuity of 60*l.* per annum, to continue 20 years, at 6*l.* per cent. compound interest ?

Ans. £ 688 3*s.* 10½*d.*

C A S E II.

When the present worth, time and rate are given, to find the annuity, rent, &c.

RULE 1.—From that power of the ratio, denoted by the number of years plus 1, subtract that power of it, denoted by the number of years.

2. Divide the remainder by that power of the ratio, signified by the time made less by unity.

3. Multiply the present worth into this quotient, and the product will be the annuity, pension, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the present worth.

2. Multiply that power of the ratio, denoted by the time, by the present worth, and subtract this product from the former.

3. Divide the remainder by that power of the ratio, denoted by the time made less by unity, and the quotient will be the annuity.

E X A M P L E S.

1. What annuity, to continue 4 years, will £ 207,904 purchase, compound interest, at 6*l.* per cent. ?

First Method.

From $1,06 \times 1,06 \times 1,06 \times 1,06 \times 1,06 = 1,3382255776$

Subt. $1,06 \times 1,06 \times 1,06 \times 1,06 = 1,26247696$

Div. by $1,06 |^4 - 1 = ,26247696),0757486176(,2885898$
,2885898

Multiply by 207,9 present worth.

25973082

20201286

57717060

Ans. 59,99781942 = £ 60.

Second

* Table 4th is thus made : Divide 1*l.* by 1,06 = ,94339 the present worth of the first year, which, divided by 1,06, is equal to ,88999, which, added to the first year's present worth, is = 1,83339, the second year's present worth, then ,88999 divided by 1,06, and the quotient added to 1,83339, gives 2,6701 for the third year's present worth, &c.

Second Method.

$$\text{From } 1,06 \times 1,06 \times 1,06 \times 1,06 \times 1,06 \times 207,9 = 278,217097573$$

$$\text{Take } 1,06 \times 1,06 \times 1,06 \times 1,06 \times 207,9 = 262,468959984$$

$$\text{Divide by } 1,06^4 - 1 = ,2624769615748137589(59,998 = £60.$$

*By TABLE V.**

Multiply the tabular number, corresponding with the rate and time, by the purchase money, and the product will be the annuity.

Under 6*l.* per cent. and opposite 4 years, you will find

,28859 = annuity which 1*l.* will purchase in 4 years.

Mult. by 207,9

$$\begin{array}{r} 259731 \\ 202013 \\ \hline 577180 \end{array}$$

$$59,997861 = £60.$$

2. What salary, to continue 20 years, will 688*l.* 3*s.* 10½*d.* purchase, at 6*l.* per cent. compound interest? *Ans.* £60.

C A S E III.

When the annuity, present worth, and ratio are given, to find the time.

R U L E.

Divide the annuity by the product of the present worth and ratio subtracted from the sum of the present worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which being divided by the ratio, and this quotient by the same, till nothing remain, the number of divisions will shew the time : *Or*, the above quotient being sought in Table 1st, under the given rate, in a line with it, you will see the time.

E X A M P L E S.

1. For how long may an annuity of 60*l.* per annum be purchased for £207,906336762, at 6*l.* per cent. compound interest? *Multiply*

* Table 5th is made in this manner : Divide 1*l.* by the present worth of 1*l.* for 1 year, and the quotient will be the annuity, which 1*l.* will purchase for 1 year ; divide 1*l.* by the present worth of 1*l.* for 2 years, and the quotient will be the annuity, which 1*l.* will purchase for 2 years, &c.

Multiply 207,906336762 by 1,06 To 207,906336762 = present worth.
Add 60 = annuity.

1247438020572 From 267,906336762
2079063367620 Subt. 220,380716967

220,38071696772 47,525619795 = divisor.
47,525619795)60,000000000(1,26247696
Divide by 1,06)1,26247696

1,06)1,191016

1,06)1,1236

1,06)1,06

1 { The number of divisions
= time = 4 years.

60

Or, $\frac{207,906336762 + 60 - 207,906336762 \times 1,06}{1,26247696} = 1,26247696,$

which being sought in Table 1. under the given rate, in a line with it, is 4 = 4 years.

2. How long may a lease of 75*l.* yearly rent be had for 533*l.* 1*s.* 8½*d.* allowing 5*l.* per cent. compound interest, to the purchaser? *Ans.* 9 years.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION AT COMPOUND INTEREST.

C A S E I.

When the annuity, time and ratio are given, to find the present worth of the annuity in reversion.

RULE* 1.—Divide the annuity by that power of the ratio denoted by the time of its continuance.

2. Subtract this quotient from the annuity; divide the remainder by the ratio less 1, and the quotient will be the present worth, to commence immediately.

3. Divide this quotient by that power of the ratio denoted by the time of reversion, (or, time to come, before the annuity commences)

* Let v denote the time in reversion, and the other letters as before. Then, the two cases under this rule will be expressed by the following Theorems.

I. $n - \frac{n}{r^t} = p.$ Then change p into m , and $\frac{m}{r^v} = p.$

II. $pr^v = m.$ Change m into p , and $\frac{pr^t \times r - pr^t}{r^t - 1} = n.$

Or, I. $\frac{r^t - 1 \times n}{r - 1 \times r^t \times r^v} = p.$ II. $\frac{r - 1 \times r^t \times r \times v p}{r^t - 1} = n.$

mences) and the quotient will be the present worth of the annuity in reversion.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend.

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it denoted by the time of reversion, and the ratio less 1, continually together for a divisor, and the quotient arising from the division of these two numbers will be the present worth of the annuity in reversion.

E X A M P L E S.

1. What is the present worth of 60*l.* payable yearly, for 4 years; but not to commence till 2 years hence at 6*l.* per cent.?

First Method.

$$\begin{array}{r} \text{Ratio} = 1,06 \\ 1,06 \\ \hline 636 \\ 1060 \\ \hline \end{array}$$

$$\begin{array}{r} \text{2d power} = 1,1236 \\ 1,1236 \\ \hline 67416 \\ 33708 \\ 22472 \\ 11236 \\ 11236 \\ \hline \end{array}$$

Or, In Table 4th, find the present value of 1*l.* at the given rate, both for the time in being and the time in reversion added together, and subtract the present worth of the time in being from the other, multiply the remainder by the annuity, and the product will be the answer.

$$\begin{array}{r} \text{Pres. worth of the time in being \& reversion} = 4,91732 \\ \text{Pres. worth of the time in being} = 1,8333 \\ \hline \end{array}$$

$$\begin{array}{r} 3,08402 \\ 60 \\ \hline \end{array}$$

$$\underline{\underline{£185,0412}}$$

$$\text{Div. by 4th pow.} = 1,26247696 \mid 60,000000 (47,525621378467$$

$$\text{Subtract the quotient} = 47,525621378467$$

$$\text{Divide by } 1,06 - 1 = ,06 \mid 12,474378621533$$

Divide by $1,06 \times 1,06 = 1,1236 \mid 207,9063103588 (185,035876 = £185 \text{ or } 8\frac{1}{2}d. \text{ the present worth of the annuity in reversion.}$

$$\text{Or, } \frac{60}{1,26247696} = 47,5256$$

$$\frac{60 - 47,5256}{1,06 - 1} = 207,906.$$

$$\text{And } \frac{207,906}{1,1236} = 185,035876$$

Second Method.

$$\begin{array}{l} \text{Multiply by } 1,26247696 = 4\text{th power} - 1 \\ 60 = \text{annuity.} \end{array}$$

$$\begin{array}{l} 15,74861760 = \text{dividend.} \\ 1,26247696 = 4\text{th power.} \\ 1,1236 = 2\text{d power.} \end{array}$$

$$,08511115 \mid 15,74861760 (185,036 \text{ Ans.}$$

$$\begin{array}{r} 757486176 \\ 378743088 \\ 252495392 \\ 126247696 \\ 126247696 \\ \hline \end{array}$$

$$\begin{array}{r} 1,418519112256 \\ ,06 = \text{ratio} - 1 \end{array}$$

$$,08511115 \mid 15,74861760 (185,036 \text{ Ans.}$$

$$\text{Or, } \frac{1,06^4 - 1 \times 60}{1,06^4 \times 1,06^2 \times 1,06 - 1} = 185,036$$

2. What

2. What is the present worth of a reversion of a lease of 60*l.* per annum, to continue 20 years, but not to commence till the end of 8 years, allowing 6*l.* per cent. to the purchaser?

Ans. £ 431 15*s.* 7*d.* 2,781*qrs.*

An annuity, several times in reversion, and rate being given, to find the several present values.

Find the present value of 1*l.* per Table 4. at the given rate, and for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders will answer the question.

3. A has a term of 6 years in an estate of 60*l.* per annum. B has a term of 14 years in the same estate, in reversion, after the 6 years are expired; and C hath a further term of 16 years, after the expiration of 20 years. I demand the present values of the several terms, at 6 per cent.?

	<i>£.</i>	<i>s.</i>	<i>d.</i>
Pres. val. of 1 <i>l.</i> for 36 years	14,617.22	×	60 = 877 0 7 $\frac{3}{4}$
Ditto of ditto for 20	11,469.92	×	60 = 688 3 10 $\frac{3}{4}$
Ditto of ditto for 6	4,917.32	×	60 = 295 0 9 $\frac{1}{4}$ = A's term.
Therefore, 877 0 7 $\frac{3}{4}$ — 688 3 10 $\frac{3}{4}$	£ 188	16	9 C's term, and
688 3 10 $\frac{3}{4}$ — 295 0 9 $\frac{1}{4}$	£ 393	3	1 $\frac{1}{2}$ = B's term.

4. For a lease of certain profits for 7 years, A offers to pay 300*l.* gratuity, and 300*l.* per annum, B offers 800*l.* gratuity and 250*l.* per annum, C bids 1300*l.* gratuity and 200*l.* per annum, and D bids 2500*l.* for the whole purchase, without any yearly rent; which is the best offer, computing at 6 per cent.?

By Table 4. the present worth of 300 <i>l.</i> per annum, for 7 years, at 6 per cent. is	} £.
	1674.714
To which add	300

Value of A's offer = 1974.714

Present worth of 250 <i>l.</i> per annum, for 7 years	= 1395.595
To which add	800

Value of B's offer = 2195.595

Present worth of 200 <i>l.</i> per annum for 7 years	= 1116.476
To which add	1300

Value of C's offer = 2416.476

D's offer = 2500

Hence it appears that D's offer is the best.

The above questions may be answered by the 4th and 2d Tables.

Take question 1st for Example.

1. Multiply the tabular number in Table 4, corresponding to the rate and the time of continuance, into the annuity, and the product will be the present worth, to commence immediately.

2. Multiply this present worth by the tabular number in Table 2, corresponding to the rate and the time of reversion, and the product will be the present worth of the annuity in reversion.

In Table 4th we have 3,4651

Multiply by 60 = annuity.

207,9060

In Table 2d we have ,889996

1247436

1871154

1871154

1871154

1663248

1663248

185,035508376 = pref. worth of the revers.

C A S E II.

When the present worth of the reversion, rate and time are given, to find the annuity.

RULE 1.—Multiply that power of the ratio signified by the time of reversion, by the present worth, and the product will be the amount of the present worth for the time before the annuity commences.

2. Multiply that power of the ratio signified by the time of continuance plus 1 by the last product.

3. Multiply that power of the ratio, signified by the time, by the aforesaid product, and this last product, divided by that power of the ratio denoted by the time, minus unity, will give the annuity.

Or, Divide the continual product of the present worth, that power of the ratio denoted by the time of continuance, that power of it denoted by the time of reversion, and the ratio minus 1, by that power of the ratio denoted by the time of continuance minus 1, and the quotient will be the annuity.

E X A M P L E S.

1. What annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for £185,035876, at 6l. per cent. ?

First

First Method.

$$1,06 \times 1,06 = 1,1236 = 2d \text{ power of the ratio.}$$

$$\text{Multiply by } 185,036 = \text{present worth.}$$

$$\begin{array}{r} 67416 \\ 33708 \\ 561800 \\ 89888 \\ \hline 11236 \end{array}$$

$$\text{Multiply by } 1,33822 \text{ } 207,9064496 \text{ amount for the time of reversion.} \\ = 5th \text{ power of the ratio.}$$

$$\begin{array}{r} 415812 \\ 415812 \\ 1663248 \\ 623718 \\ 623718 \\ \hline 207906 \end{array}$$

$$\begin{array}{r} \text{From } 278,22396732 \\ \text{Take } 262,47508782 \\ \hline \end{array}$$

$$4th \text{ power of the ratio} = 1,26247 \\ \text{Multiply by } 207,906$$

$$\begin{array}{r} 757482 \\ 11362230 \\ 883729 \\ \hline 2524940 \\ 262,47508782 \end{array}$$

$$\text{Divide by } 1,06^4 - 1 = ,26247) 15,74887950 (60 \text{ the annuity required.}$$

$$\text{Or, } 185,036 \times 1,1236 = 207,906$$

$$\text{Then, } \frac{207,906 \times 1,33822 - 207,906 \times 1,26247}{1,26247 - 1} = \text{£}60 \text{ Ans.}$$

Second Method.

$$185,036 = \text{present worth of the reversion.}$$

$$1,26247 = 4th \text{ power of the ratio.}$$

$$\begin{array}{r} 1295252 \\ 740144 \\ 370072 \\ 1110216 \\ 370072 \\ \hline 185036 \end{array}$$

Or by Table 4th, divide the present worth of the reversion by the difference between the present worth of 1*l.* for the time both in being and reversion, and the time in being and the quotient will be the annuity.

$$\begin{array}{r} 233,6024 \\ \hline 1,1236 \end{array}$$

$$= 2d \text{ power of the ratio.}$$

$$\begin{array}{r} 14016144 \\ 7008072 \\ 4672048 \\ \hline 2336024 \end{array}$$

$$\begin{array}{r} 4,91732 \\ \hline 1,8333 \end{array}$$

$$\begin{array}{r} 2336024 \\ \hline 2336024 \end{array}$$

$$3,08402) 185,0412 (60 \text{ Ans.}$$

$$262,47565664$$

$$,06 = \text{ratio} - 1.$$

$$1,06^4 - 1 = ,26247) 15,7485393984 (60.$$

$$\text{Or, } \frac{185,036 \times 1,26247 \times 1,1236 \times 1,06 - 1}{1,26247 - 1} = 60.$$

2. The present worth of a lease of an house is. 43*l.* 15*s.* 7*d.* 2,7819 *qrs.* taken in reversion for 20 years ; but not to commence till the end of 8 years, allowing 6*l.* per cent. to the purchaser ; What is the yearly rent ?

Ans. £60.

PURCHASING

PURCHASING ANNUITIES FOREVER, or FREEHOLD ESTATES, at COMPOUND INTEREST.

C A S E I.

When the annuity, or yearly rent, and the rate are given, to find the present worth, or price.

RULE.*—As the rate per cent. is to 100l.; so is the yearly rent, to the value required.

Or, Divide the yearly rent by the ratio less 1, and the quotient will be the value required.

E X A M P L E S.

1. What is the worth of a freehold estate of 60l. per annum, allowing 6l. per cent. to the purchaser?

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ 6 : 100 :: 60 \\ \hline 60 \end{array}$$

$$\text{Or, } 1,06 - 1 = ,06 \quad 60,00 \\ \hline 1000$$

$$\hline 6)6000$$

£1000 Ans.

2. An estate brings in yearly, 75l.; What would it sell for, allowing the purchaser 5l. per cent. compound interest?

Ans. £1500.

C A S E II.

When the price, or present worth, and rate are given, to find the annuity, or yearly rent.

RULE.—As 100l. is to the rate, so is the present worth to its rent.

Or, Multiply the present worth by the ratio less 1, and the product will be the yearly rent.

E X A M P L E S.

1. If a freehold estate be bought for 1000l. allowing 6l. per cent. to the purchaser; What is the yearly rent?

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ 100 : 6 :: 1000 \\ \hline 6 \end{array}$$

$$\text{Or, } 1000 \times ,06 = \text{£}60.$$

$$\begin{array}{r} 100)6000(\text{£}60 \text{ Ans.} \\ \hline 600 \\ \hline 0 \end{array}$$

2. IF

* The reason of this rule is obvious; for since a year's interest of the price, which is given for it, is the annuity, there can neither more nor less be made of that price, than of the annuity, whether it be employed at simple or compound interest.

The following Theorems shew all the varieties of this rule.

$$I. \frac{n}{r-1} = p. \quad II. r-1 \times p = n. \quad III. \frac{n}{p} + 1 = r, \text{ or } \frac{n}{p} = r-1; \text{ or } \frac{1+n}{p} = r.$$

FREEHOLD ESTATES, AT COMPOUND INTEREST. 319

2. If an estate be sold for 1500*l.* and 5 per cent. allowed to the buyer; What is the yearly rent? *Ans.* £75.

C A S E III.

When the present worth, or price, and yearly rent are given, to find the rate.

RULE.—As the present worth is to the rent; so is 100*l.* to the rate.

Or, Divide the rent by the present worth; add 1 to the quotient, and the sum will be the ratio of the rate per cent.

Or, Divide the sum of the present worth and rent by the present worth, and the quotient will be the ratio.

E X A M P L E S.

1. If an estate of 60*l.* per annum be bought for 1000*l.*; What rate of interest was allowed the purchaser for his money.

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ 1000 : 60 :: 100 \\ \quad \quad 100 \end{array}$$

$$\text{Or, } 1000)60,00(.06 + 1 = 1,06 \\ \quad \quad 6000$$

$$\begin{array}{r} 1000)6000(\text{£}6 \text{ Ans.} \\ \quad \quad 6000 \end{array}$$

$$\begin{array}{r} \text{Or, to } 1000 = \text{present worth,} \\ \text{Add } 60 = \text{rent.} \end{array}$$

$$\begin{array}{r} 1000)1060(1,06 \\ \quad \quad 1000 \end{array}$$

$$\begin{array}{r} 6000 \\ 6000 \end{array}$$

2. An estate of 75*l.* per annum was purchased for 1500*l.*; What rate of interest had the buyer for his money? *Ans.* £5.

To find at how many year's purchase an estate may be bought.

C A S E I.

When the rate of interest is given, to find the number of years.

RULE.—Divide 100*l.* by the rate, and the quotient will be the years.

E X A M P L E S.

1. How many years' purchase should a gentleman offer for the purchase of an estate, to have 6*l.* per cent. for his money?

$$\begin{array}{r} 6)100 \\ \hline \end{array}$$

$$16,666 + = 16\frac{2}{3} \text{ years.}$$

2. How many years' purchase is an estate worth, allowing 5*l.* per cent. to the purchaser? *Ans.* 20 years.

C A S E II.

When the number of year's purchase, at which an estate is bought, or sold, is given, to find the rate of interest.

RULE.—Divide 100*l.* by the number of years, and the quotient will be the rate.

EXAMPLES,

E X A M P L E S.

1. A gentleman gives $16\frac{2}{3}$ years' purchase for a farm ; What interest is he allowed ?

$$16\frac{2}{3} = 16,666 +) 100,000 (\text{£} 6 \text{ Ans.}$$

2. A gentleman gives 20 years' purchase for an estate ; What interest has he ?

Ans. £5.

PURCHASING FREEHOLD ESTATES IN REVERSION.

C A S E I.

The rate and rent of a freehold estate being given, to find the present worth of reversion.

RULE 1.*—Find the present worth of the annuity or rent, (by Case 1. of purchasing Freehold Estates, page 318,) as though it were to be entered on immediately.

2. Divide the last present worth by that power of the ratio denoted by the time of reversion (by Case 1. of Discount by Compound Interest) and the quotient will be the answer required.

Or, 1. Having found the present value of the estate, supposing it to be immediate : Multiply the annuity, or rent, by the present worth of 1*l.* corresponding with the time of reversion and rate in Table 4th, and the product will be the present worth of the annuity or rent, for the time of reversion ; or the value of the present possession.

2. Subtract the value of the possession from the value of the estate, and the remainder will be the value of reversion.

E X A M P L E S.

1. Suppose a freehold estate of 60*l.* per annum to commence 2 years hence, be put up to sale ; What is its value, allowing the purchaser 6*l.* per cent. ?

First Method.

$$1,06 - 1 = ,06) 60,00 = \text{rent per annum.}$$

$$1000 = \text{present worth, if entered on immediately.}$$

$$1,06|^2 = 1,1236) 1000,000 (889,996 = \text{£} 889 \text{ } 19\text{s. } 11\text{d.} = \text{present worth of } 1000\text{i}l. \text{ for 2 years, or the whole present worth required.}$$

Second

* The following Theorems express all the Cases under this rule.

I. $\frac{n}{r-1} = p$; then change p into m , and $\frac{m}{r^v} = p$.

II. $p r^v = m$; then change m into p , and $\frac{p r r - p r}{r} = n$.

Second Method.

$$1,06-1 = ,06)60,00$$

1000 = present worth, for immediate possession.

In Table 4th we have, 1,83339 = value of 1*l.* for 2 years.

Multiply by 60 = rent.

110,00340 = value of possession.

From 1000,0000

Subtract 110,0034

889,9966 = value required.

2. Suppose an estate of 75*l.* per annum, to commence 10 years hence, were to be sold, allowing the purchaser 5*l.* per cent. ; What is its worth ?

Ans. £920 17*s.* 5*d.*

C A S E II.

The Value of a Reversion, the Time prior to its Commencement, and Rate of Interest given, to find the Annuity or Rent.

RULE 1.—Multiply the price of the reversion by that power of the amount of 1*l.* for 1 year, denoted by the time of reversion, and the product will be its amount (by Case 1, of Compound Interest.)

2. Find the interest of the amount (by Case 1st, Simple Interest) and it will be the annuity, or yearly rent.

E X A M P L E S.

1. A freehold estate is bought for £889,9966 which does not commence till the end of 2 years ; the buyer being allowed 6*l.* per cent. for his money ; I desire to know the yearly income ?

Multiply by $\frac{889,9966}{1,06^2} = 1,1236$ denoted by the time of reversion.

$$\begin{array}{r} 53399796 \\ 26699898 \\ 17799933 \\ 8899966 \\ 8899966 \end{array}$$

1000,00017976 = amount of the reversion.

,06

Ans. £60,00

2. If a freehold estate, to commence 10 years hence, be sold for £920 17*s.* 5*d.* allowing the purchaser 5*l.* per cent. ; What is the yearly income ?

Ans. £75.
TABLE

TABLE I. *Shewing the amount of £1 from 1 year to 50.*

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000	1,0550000	1,0600000
2	1,0609000	1,0712250	1,1816000	1,0920250	1,1025000	1,1130250	1,1236000
3	1,0927270	1,1087178	1,1248640	1,1411661	1,1576950	1,1742413	1,1910160
4	1,1255088	1,1475230	1,1698585	1,1925186	1,2155062	1,2388245	1,2624796
5	1,1592740	1,1876863	1,2166529	1,2461819	1,2762815	1,3069598	1,3382256
6	1,1940523	1,2292553	1,2653190	1,3022601	1,3400956	1,3788426	1,4185191
7	1,2298738	1,2722792	1,3159317	1,3608618	1,4071004	1,4546789	1,5036302
8	1,2667700	1,3168090	1,3685690	1,4221006	1,4774554	1,5346862	1,5938480
9	1,3047731	1,3628973	1,4233118	1,4860051	1,551382	1,6190939	1,6894789
10	1,3439163	1,4105987	1,4802842	1,5529694	1,6288946	1,7081440	1,7908476
11	1,3842338	1,4599697	1,5394540	1,6228530	1,7103393	1,8020919	1,8982985
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563	1,9012066	2,0121964
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8856491	2,0057732	2,1329282
14	1,5125897	1,6180945	1,7316764	1,8519449	1,9799316	2,1160907	2,2609039
15	1,5579674	1,6753488	1,8009435	1,9352824	2,0789281	2,2324756	2,3965558
16	1,6047064	1,7330886	1,8729312	2,0223701	2,1828745	2,3552617	2,5472716
17	1,6528476	1,7946755	1,9479005	2,1133768	2,2920183	2,4848011	2,6927727
18	1,7024330	1,8574892	2,0258161	2,2084787	2,4066192	2,6214652	2,8543391
19	1,7535060	1,9225013	2,1068491	2,3078603	2,5269502	2,7656458	3,0255995
20	1,8061112	1,9897888	2,1911231	2,4117140	2,6532977	2,9177563	3,2071305
21	1,8602945	2,0594314	2,2787680	2,5202411	2,7859625	3,0782329	3,3995636
22	1,9161034	2,1315115	2,3699187	2,6336520	2,9252607	3,2475357	3,6035374
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715237	3,4261502	3,8197496
24	2,0327941	2,2833284	2,5633041	2,8760138	3,2250999	3,6145885	4,0489346
25	2,0937779	2,3637449	2,6658363	3,0054344	3,3863549	3,8133910	4,2918707
26	2,1565912	2,4469585	2,7724697	3,1406790	3,5556726	4,0231279	4,5493829
27	2,2212890	2,5315671	2,8823685	3,2820099	3,7334563	4,2443999	4,8223459
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9010291	4,4778419	5,1116867
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161356	4,7241232	5,4183879
30	2,4270024	2,8067937	3,2422975	3,7453181	4,3219423	4,9839499	5,7434912
31	2,5000803	2,9050314	3,3731334	3,9138574	4,5380394	5,2580671	6,0881007
32	2,5750827	2,0067075	3,5080587	4,0899810	4,7649414	5,5472608	6,4533867
33	2,6522352	3,1110423	3,6483811	4,2740301	5,0031885	5,8523600	6,8405899
34	2,7319052	3,2208603	3,7943163	4,4663615	5,2533479	6,1742398	7,2510253
35	2,8138624	3,3335004	3,9460889	4,6673478	5,5160152	6,5138230	7,6860868
36	2,8982783	3,4502661	4,1039315	4,8773784	5,7918101	6,8720832	8,147252
37	2,9852266	3,5710254	4,2680898	5,0968604	6,0814069	7,2500478	8,6360871
38	3,0747824	3,6960113	4,4388134	5,3262192	6,3854772	7,6488004	9,1542523
39	3,1670265	3,8253717	4,6163659	5,5658990	6,7047511	8,0694844	9,7035074
40	3,2620377	3,9592597	4,8010206	5,8164645	7,0399887	8,5133060	10,2857178
41	3,3590828	4,0978337	4,9930614	6,0782054	7,3919881	8,9815378	10,9028608
42	3,4606958	4,2412579	5,1927838	6,3517246	7,7615875	9,4755224	11,5570325
43	3,5645167	4,3897020	5,4004952	6,6375522	8,1496666	9,9966761	12,2504547
44	3,6714522	4,5433415	5,616515	6,9362421	8,5571502	10,5464933	12,9854817
45	3,7815057	4,7023585	5,8411756	7,248373	8,9850077	11,1265504	13,7626109
46	3,8950436	4,8669411	6,0748236	7,5745497	9,4342531	11,7385217	14,5883673
47	4,0118949	5,0372840	6,3168166	7,9154045	9,9059710	12,3841404	15,4636693
48	4,1322518	5,2135889	6,5694802	8,2715977	10,4012696	13,0652681	16,3914894
49	4,2562193	5,3960645	6,8322688	8,6438196	10,9213331	13,7838579	17,3749788
50	4,3830059	5,5849268	7,1055596	9,0327015	11,4673697	14,5440000	18,2174775

TABLE II. Shewing the present value of £1, due at the end of any number of years, from 1 to 40.

yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	yrs.
1	,901538	,956938	,952381	,947867	,943396	1
2	,924556	,91573	,90703	,898513	,889996	2
3	,888996	,876297	,863838	,851728	,839619	3
4	,854804	,838561	,822702	,807397	,790993	4
5	,821927	,802451	,782526	,765392	,747258	5
6	,790314	,767891	,746215	,725587	,70496	6
7	,759918	,734828	,710681	,687869	,665057	7
8	,730690	,703185	,676839	,652125	,627412	8
9	,702587	,672904	,644609	,618253	,591898	9
10	,675564	,643928	,612913	,586153	,558394	10
11	,649581	,616199	,584679	,557373	,52787	11
12	,624597	,589664	,556837	,526903	,496969	12
13	,600574	,564271	,530321	,49958	,468839	13
14	,577475	,539973	,505068	,473684	,442301	14
15	,555264	,516720	,481017	,449141	,417265	15
16	,533900	,494409	,458311	,425979	,393647	16
17	,513373	,473176	,436297	,40383	,371364	17
18	,493628	,4528	,415521	,382932	,350343	18
19	,474642	,433302	,395744	,363123	,330513	19
20	,456387	,414643	,376880	,344346	,311804	20
21	,438833	,396787	,358042	,326568	,294155	21
22	,421955	,379701	,34185	,30677	,277505	22
23	,405726	,36335	,325571	,293684	,261797	23
24	,390121	,347703	,310068	,278523	,246978	24
25	,375117	,332731	,305203	,26915	,230998	25
26	,360689	,318402	,281241	,250525	,21581	26
27	,346816	,304691	,267848	,237608	,207368	27
28	,333477	,291571	,255094	,225362	,19563	28
29	,320651	,279015	,242946	,213715	,184556	29
30	,308309	,267	,231377	,202743	,17411	30
31	,296460	,255502	,220359	,192307	,164255	31
32	,285058	,2445	,209860	,182411	,154057	32
33	,274094	,233971	,199872	,173029	,146186	33
34	,263552	,223896	,190355	,164133	,137912	34
35	,254415	,214251	,18129	,155692	,130105	35
36	,243009	,205028	,172057	,147399	,122741	36
37	,234297	,196299	,164436	,140114	,115793	37
38	,225285	,18775	,156605	,132893	,109182	38
39	,216671	,179665	,149148	,126075	,103002	39
40	,208280	,171929	,142046	,119608	,9717	40

TABLE III. *Shewing the amount of £1 annuity for any number of years, from 1 to 40.*

Yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	Yrs.
1	1,	1,	1,	1,	1,	1
2	2,04	2,045	2,05	2,055	2,06	2
3	3,1216	3,137025	3,1525	3,16802	3,1836	3
4	4,246464	4,278191	4,310125	4,34226	4,374616	4
5	5,416322	5,47071	5,525631	5,58109	5,637093	5
6	6,632975	6,716892	6,801913	6,888051	6,975318	6
7	7,898294	8,019152	8,142008	8,266894	8,393837	7
8	9,214266	9,380014	9,549109	9,721573	9,897467	8
9	10,582795	10,802114	11,026564	11,256259	11,491315	9
10	12,006107	12,2882	12,577892	12,875354	13,180794	10
11	13,486351	13,841172	14,206787	14,583498	14,971642	11
12	15,025805	15,464032	15,917126	16,38559	16,86994	12
13	16,626838	17,159912	17,712983	18,286798	18,882132	13
14	18,291911	18,932109	19,598632	20,292572	21,015064	14
15	20,023588	20,784054	21,578563	22,408603	23,275968	15
16	21,824531	22,719337	23,657492	24,64114	25,672527	16
17	23,697512	24,741707	25,840366	26,996402	28,212879	17
18	25,645413	26,855084	28,132385	29,481205	30,905652	18
19	27,671229	29,063562	30,529004	32,102671	33,759991	19
20	29,778078	31,371423	33,065954	34,868318	36,78559	20
21	31,969202	33,783137	35,719252	37,786075	39,992725	21
22	34,24797	36,303378	38,505214	40,864309	43,392289	22
23	36,617888	38,93703	41,430475	44,111846	46,995826	23
24	39,081604	41,689196	44,501999	47,537998	50,815576	24
25	41,645008	44,56521	47,727099	51,152588	54,86451	25
26	44,311745	47,570645	51,113454	54,96598	59,156381	26
27	47,084214	50,711324	54,669126	58,989109	63,705763	27
28	49,967583	53,993333	58,402583	63,23351	68,528109	28
29	52,966286	57,423033	62,322712	67,711353	73,639796	29
30	56,084938	61,007067	66,438847	72,435478	79,058183	30
31	59,328335	64,752388	70,76079	77,419429	84,801674	31
32	62,701469	68,666245	75,298829	82,677498	90,889775	32
33	66,209527	72,756226	80,063771	88,22476	97,343161	33
34	69,87908	77,030256	85,066959	94,077122	104,183751	34
35	73,652225	81,496618	90,320307	100,251363	111,434776	35
36	77,598314	86,103966	95,836323	106,705188	119,120863	36
37	81,702246	91,041344	101,628139	113,637274	127,268114	37
38	85,970336	96,138205	107,709546	120,887324	135,904201	38
39	90,40915	101,464424	114,095025	128,536127	145,058453	39
40	95,025516	107,030323	120,799774	136,605614	154,761961	40

TABLE IV. *Shewing the present worth of £1, annuity, for any number of years, from 1 to 40.*

Yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	0,96154	0,95694	0,95238	0,94786	0,94339
2	1,88609	1,87267	1,85941	1,8463	1,83339
3	2,77509	2,74896	2,72325	2,6979	2,67301
4	3,62989	3,58752	3,54595	3,49862	3,4651
5	4,45182	4,38997	4,32948	4,25759	4,21236
6	5,24214	5,15787	5,07569	4,97699	4,91732
7	6,00205	5,8927	5,78637	5,65888	5,58238
8	6,73274	6,59589	6,46321	6,30522	6,20979
9	7,43533	7,26879	7,10782	6,91786	6,80169
10	8,11089	7,91272	7,72173	7,49856	7,36008
11	8,76048	8,52892	8,30641	8,04898	7,88687
12	9,38507	9,11858	8,86325	8,5707	8,38384
13	9,98565	9,68285	9,39357	9,06522	8,85268
14	10,56312	10,22282	9,85864	9,53395	9,29498
15	11,11839	10,73954	10,37966	9,97824	9,71225
16	11,65229	11,23401	10,83777	10,39936	10,10589
17	12,16567	11,70719	11,27407	10,79852	10,47726
18	12,65929	12,15099	11,68958	11,17687	10,8276
19	13,13394	12,59329	12,08532	11,53549	11,15811
20	13,59032	13,00793	12,46221	11,87541	11,46992
21	14,02916	13,40472	12,82115	12,1976	11,76407
22	14,45111	13,79442	13,163	12,50299	12,04158
23	14,85684	14,14777	13,48807	12,79245	12,30338
24	15,24696	14,49548	13,79864	13,06682	12,55035
25	15,62208	14,82821	14,09391	13,3688	12,78335
26	15,98277	15,14661	14,37518	13,57338	13,00316
27	16,32959	15,4513	14,64303	13,80702	13,21053
28	16,66306	15,74287	14,89813	14,02848	13,40616
29	16,98371	16,02189	15,14107	14,23838	13,59072
30	17,20202	16,28889	15,37245	14,43723	13,76483
31	17,58849	16,54439	15,59281	14,6259	13,92908
32	17,87355	16,78889	15,80268	14,80463	14,08398
33	18,14764	17,02285	16,00255	14,97404	14,22917
34	18,4112	17,24676	16,1929	15,13461	14,36613
35	18,66461	17,46101	16,37419	15,2868	14,49533
36	18,90828	17,66604	16,54685	15,43105	14,61722
37	19,14258	17,86224	16,71129	15,56779	14,73211
38	19,36787	18,04999	16,86789	15,6974	14,84048
39	19,58448	18,22965	17,01704	15,82024	14,9427
40	19,79277	18,40158	17,1509	15,93667	15,03913

TABLE V. *The annuity which £1 will purchase for any number of years to come, from 1 to 40.*

Yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1,04	1,045	1,05	1,055	1,05
2	,5302	,534	,5378	,54162	,54544
3	,36035	,36377	,36721	,37065	,37411
4	,27549	,27874	,28201	,28582	,28859
5	,22463	,22779	,23097	,23487	,23739
6	,19076	,19388	,19702	,20092	,20336
7	,16661	,1697	,17282	,17671	,17918
8	,14853	,15161	,15473	,15859	,16103
9	,13449	,13757	,14059	,14455	,14702
10	,12329	,12638	,1295	,13331	,13587
11	,11415	,11725	,12039	,12424	,12679
12	,10655	,10967	,11282	,11667	,11927
13	,10014	,10327	,10645	,11031	,11290
14	,09467	,09782	,10102	,10489	,10758
15	,08994	,09311	,09624	,10022	,10296
16	,08582	,08901	,09227	,0962	,09895
17	,0822	,08542	,0887	,0926	,09544
18	,07899	,08224	,08555	,08947	,09235
19	,07614	,07941	,08274	,08699	,08962
20	,07359	,07688	,08024	,08427	,08718
21	,07128	,0746	,078	,08198	,085
22	,0692	,07254	,07597	,07998	,08303
23	,06731	,07068	,07414	,07825	,08128
24	,06559	,06897	,07247	,07653	,07968
25	,06401	,06744	,07095	,07503	,07823
26	,06257	,06602	,06956	,07367	,0769
27	,06124	,06472	,06829	,07242	,0757
28	,06001	,06352	,06712	,07128	,07459
29	,05888	,06241	,06604	,07023	,07358
30	,05783	,06139	,06505	,06926	,07272
31	,05685	,06044	,06413	,06837	,07179
32	,05595	,05956	,06328	,06754	,071
33	,0551	,05874	,06249	,06678	,07027
34	,05431	,05798	,06175	,06607	,06959
35	,05358	,05727	,06107	,06541	,06899
36	,05289	,0566	,06043	,0648	,06839
37	,05224	,05524	,05984	,06423	,06785
38	,05163	,0554	,05928	,0637	,06735
39	,05106	,05485	,05876	,06321	,06689
40	,05052	,05434	,05828	,06274	,06646

CIRCULATING DECIMALS

Are produced from Vulgar Fractions, whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. The circulating figures are called *repetends*; and, if one figure only repeats, it is called a *single repetend*: As, $\dot{1}111$ &c. $\dot{6}666$ &c.

2. A *compound repetend* has the same figures circulating alternately: As, $\dot{0}1\dot{0}101$ &c. $\dot{3}79379379$ &c.

3. If other figures arise before those which circulate, the decimal is called a *mixed repetend*; thus, $\dot{3}75555$ &c. is a *mixed single repetend*. and $\dot{3}78123123$ &c. a *mixed compound repetend*.

4. A single repetend is expressed by writing only the circulating figure with a point over it; thus, $\dot{1}111$ &c. is denoted by

$\dot{1}$, and $\dot{6}666$ &c. by $\dot{6}$.

5. Compound repetends are distinguished by putting a point over the first and last repeating figures; thus, $\dot{0}1\dot{0}101$ &c. is

written $\dot{0}1$ and $\dot{3}79379379$ &c. thus $\dot{3}79$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after

the decimal point; thus, $\dot{3}$ and $\dot{5}$ are similar circulates; as are

also $\dot{3}, \dot{5}4$ and $\dot{7}, \dot{3}6$, &c.

7. *Dissimilar repetends* consist of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $\dot{4}7, \dot{3}457\dot{6}$, $\dot{9}, \dot{7}352\dot{8}$ and $\dot{0}, \dot{5}46\dot{3}$, &c.

REDUCTION of CIRCULATING DECIMALS.

C A S E I.

To reduce a simple Repetend to its equivalent Vulgar Fraction.

RULE* 1.—Make the given decimal the numerator, and let the denominator be a number, consisting of so many nines as there are recurring places in the repetend.

2. If

* If unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; that is, if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate

$\dot{1}$, and since $\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $\dot{2}$ will = $\frac{2}{9}$, $\dot{3}$ = $\frac{3}{9}$, and so on till $\dot{9}$ = $\frac{9}{9}$ = 1. Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9.

Again, $\frac{1}{99}$ or $\frac{1}{999}$, being reduced to decimals, make $\dot{0}1\dot{0}101$ &c. and $\dot{0}01\dot{0}01001$ &c. *ad infinitum* = $\dot{0}1$ and $\dot{0}01$; that is, $\frac{1}{99} = \dot{0}1$, and $\frac{1}{999} = \dot{0}01$, consequently $\frac{2}{99} = \dot{0}2$, $\frac{3}{99} = \dot{0}3$, &c. and $\frac{2}{999} = \dot{0}2\dot{0}2$, $\frac{3}{999} = \dot{0}3\dot{0}3$, &c. and the same will hold universally.

328 REDUCTION OF CIRCULATING DECIMALS.

2. If there be integral figures in the circulate, so many cyphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

EXAMPLES.

1. Required the least vulgar fractions equal to $\dot{,}3$ and $\dot{,}324$.

$$\dot{,}3 = \frac{3}{9} = \frac{1}{3}; \text{ and } \dot{,}324 = \frac{324}{999} = \frac{12}{37} \text{ Ans. } \frac{1}{3} \text{ and } \frac{12}{37}.$$

2. Reduce $\dot{,}7$ to its equivalent vulgar fraction. *Ans.* $\frac{7}{9}$.

3. Reduce $\dot{,}37$ to its equivalent vulgar fraction. *Ans.* $\frac{37}{99}$.

4. Required the least vulgar fraction equal to $\dot{,}384615$. *Ans.* $\frac{5}{13}$.

CASE II.

To reduce a mixed Repetend to its equivalent Vulgar Fraction.

RULE* 1.—To so many nines as there are figures in the repetend, annex so many cyphers as there are finite places, (that is, as there are decimal places before the repetend) for a denominator.

2. Multiply the nines in the said denominator by the finite part, and add the repeating decimals to the product for the numerator.

3. If the repetend begins in some integral place, the finite value of the circulating part must be added to the finite part.

EXAMPLES.

1. What is the vulgar fraction equivalent to $\dot{,}153$?

There being 1 figure in the repetend, and 2 finite places, I annex 2 cyphers to 9 for a denominator, viz. 900; then I multiply the 9 in the denominator by the two figures in the finite part, and add the repeating figure for a numerator; thus, $9 \times 15 + 3 = 138$ numerator.

Therefore, $\dot{,}153 = \frac{138}{900} = \frac{23}{150}$ the *Ans.*

2. What is the least vulgar fraction equal to $\dot{,}4123$? *Ans.* $\frac{4079}{9990}$.

CASE III.

To make any number of dissimilar repetends similar and conterminous; that is, of an equal number of places.

RULE

* In like manner for a mixed circulate; consider it as divisible into its finite and circulating parts, and the same principle will be seen to run through them also;

thus the mixed circulate $\dot{,}1\dot{3}$ is divisible into the finite decimal, 1, and the repetend

$\dot{,}03$; but, $1 = \frac{1}{10}$, and $\dot{,}03$ would be equal to $\frac{3}{90}$ provided the circulation began immediately after the place of units; but as it begins after the place of tenths, it is $\frac{3}{900}$

of $\frac{1}{10} = \frac{3}{90}$, and so the vulgar fraction = $\dot{,}1\dot{3}$ is $\frac{1}{10} + \frac{3}{90} = \frac{9}{90} + \frac{3}{90} = \frac{12}{90}$, and is the same as by the rule.

REDUCTION OF CIRCULATING DECIMALS. 329

R U L E.*

Change them into other repetends, which shall each consist of so many figures, as the least common multiple of the sums of the several numbers of places, found in all the repetends, contains units.

E X A M P L E S.

1. Make $6,3\dot{1}7$; $3,4\dot{5}$; $52,3$; $191,0\ddot{3}$; $\dot{,}057$; $5,3$ and $1,359$ similar and conterminous.

Here, in the first repetend, there are three places, in the second, one, in the third, none, in the fourth, two, in the fifth, three, in the sixth, one, and in the seventh, one.

Now find the least common multiple of these several sums, thus :

$3 \overline{) \begin{smallmatrix} 3, & 1, & 2, & 3, & 1, & 1 \\ 1, & 1, & 2, & 1, & 1, & 1 \end{smallmatrix}}$ and $2 \times 3 = 6$ units ; therefore, the similar and conterminous repetends must contain 6 places.†

Dissimilar made similar and conterminous.

$$6,3\dot{1}7 = 6,31731731$$

$$3,4\dot{5} = 3,45555555$$

$$52,3 = 52,30000000$$

$$191,0\ddot{3} = 191,03030303$$

$$\dot{,}057 = .05705705$$

$$5,3 = 5,33333333$$

$$1,359 = 1,35999999$$

2. Make $\dot{,}531$, $\ddot{,}7348$, $\ddot{,}07$ and $\dot{,}0503$ similar and conterminous.

C A S E IV.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will consist of.

R U L E

* Any given repetend whatever, whether single, compound, pure, or mixed, may be transformed into another repetend, which shall consist of an equal or greater number of figures at pleasure ; thus $\dot{,}3$ may be transformed into $\dot{,}33$, or $\dot{,}333$, &c. also $\dot{,}79 = 7979 = 797$, and so on.

† The learner may observe that the similar and conterminous repetends begin just so far from unity, as is the farthest among the dissimilar repetends ; and is so in all cases.

330 ADDITION OF CIRCULATING DECIMALS.

RULE* 1.—Reduce the given fraction to its least terms, and divide the denominator by 2, 5 or 10, as often as possible.

2. Divide 9999, &c. by the former result, till nothing remain, and the number of 9s used will shew the number of places in the repetend ; which will begin after so many places of figures as there were 10s, 2s, or 5s, divided by.

If the whole denominator vanish in dividing by 2, 5 or 10, the decimal will be finite, and will consist of so many places as you perform divisions.

EXAMPLES,

1. Required to find whether the decimal equal to $\frac{475}{2800}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

$$\text{First } 25 \overline{) \frac{475}{2800}} = \frac{19}{112} \quad 2 \overline{) 112} = \frac{(2)}{56} = \frac{(2)}{28} = \frac{(2)}{14} = 7.$$

Then, $7 \overline{) 999999}$; therefore, because the denominator 112 did not vanish in dividing by 2, the decimal is infinite, and, as six 9s were used, the circulate consists of 6 places, beginning at the fifth place, because four 2s were used in dividing.

2. Let $\frac{2}{11}$ be the fraction proposed.

3. Let $\frac{3}{5}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE 1.—Make the repetends *similar and conterminous*, and find their sum as in common addition.

2. Divide this sum (of the repetends only) by so many nines as there are places in the repetend, and the remainder is the repetend of their sum ; which must be set under the figures added, with cyphers on the left hand, when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest, as infinite decimals.

EXAMPLES.

* In dividing 1,000, &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as soon as the remainder is 1 : And since 999, &c. is less than 1000, &c. by 1, therefore 999, &c. divided by any number whatever, will, when the repeating figures are at their period, leave 0 for a remainder.

Now, whatever number of repeating figures we have, when the dividend is 1, there will be exactly the same number, when the dividend is any other number whatever.

Thus, let 390539053905 &c. be a circulate, whose repeating part is 3905. Now every repetend (3905,) being equally multiplied, must give the same product : For although these products will consist of more places, yet the overplus in each, being alike, will be carried to the next, by which mean, each product will be equally increased, and consequently every four places will continue alike. And the same will hold for any other number.

Now from hence it appears that the dividend may be altered at pleasure, and the number of places in the repetend will still be the same ; thus $\frac{1}{11} = .\dot{9}0$ and $\frac{4}{11}$; or $\frac{1}{11} \times 4 = \dot{3}6$, whence the number of places in each are alike,

SUBTRACTION OF CIRCULATING DECIMALS. 331

EXAMPLES.

1. Let $5,3 + 59,435\dot{6} + 397,\dot{6} + 519 + ,39 + 217,\dot{5}$ be added together.

$$\begin{array}{rcl} 5,3 & = & 5,3333333 \\ 59,435\dot{6} & = & 59,435\dot{6}35\dot{6} \\ 397,\dot{6} & = & 397,666666\dot{6} \\ 519 & = & 519,0000000 \\ ,39 & = & ,393939\dot{3} \\ 217,\dot{5} & = & 217,555555\dot{5} \end{array}$$

1199,3851303 the sum.

In this question, the sum of the repetends is 2851303, which, divided by 999999, gives 2 to carry to the next column 5,3,0 &c. and the remainder is 851305.

2. Let $3275,319 + 36,45 + 123,19 + 5,317\dot{3} + 112,351\dot{3} + 11,131 + ,125 + 29,1005\dot{3}$ be added together. *Ans.* 3593,00042.

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.—Make the repetends *similar* and *conterminous*, and subtract as usual, observing, that if the repetend of the number to be subtracted be greater than the repetend of the number it is to be taken from, then the right hand of the remainder must be less by unity than it would be if the expressions were finite.

EXAMPLES.

1. From $57,0\dot{3}$ take $29,7358\dot{7}$

$$\begin{array}{rcl} 57,0\dot{3} & = & 57,03030 \\ 29,7358\dot{7} & = & 29,73587 \end{array}$$

27,29442 the difference.

2. From $325,17$ take $137,5819$ —

Ans. 187,5957.
MULTIPLICATION

MULTIPLICATION OF CIRCULATING DECIMALS.

RULE 1.—Turn both the terms into their equivalent vulgar fractions, and find the product of those fractions as usual.

2. Turn the vulgar fraction, expressing the product, into an equivalent decimal one, and it will be the product required.

E X A M P L E S.

1. Multiply $\ddot{54}$ by $\dot{15}$. $\ddot{54} = \frac{54}{99} = \frac{6}{11}$ and $\dot{15} = \frac{14}{90} = \frac{7}{45}$
 $\frac{6}{11} \times \frac{7}{45} = \frac{42}{495} = ,084$ the product.

2. Multiply $378.\dot{5}$ by $23.\dot{6}$ — *Ans.* $8959,\dot{148}$.

DIVISION OF CIRCULATING DECIMALS.

RULE 1.—Change both the divisor and dividend into their equivalent vulgar fractions, and find their quotient as usual.

2. Turn the vulgar fraction, expressing the quotient, into its equivalent decimal, and it will be the quotient required.

E X A M P L E S.

1. Divide $\ddot{54}$ by $\dot{15}$.

$$\ddot{54} = \frac{54}{99} = \frac{6}{11} \text{ and } \dot{15} = \frac{14}{90} = \frac{7}{45}$$

$$\frac{6}{11} \div \frac{7}{45} = \frac{6}{11} \times \frac{45}{7} = \frac{270}{77} = 3\frac{39}{77} = 3,506493 \text{ the quotient.}$$

2. Divide $345.\dot{8}$ by $\dot{6}$. *Ans.* $518,\dot{83}$.

A L L I G A T I O N

Is the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality: It consists of two kinds, viz. Alligation Medial and Alligation Alternate.

A L L I G A T I O N M E D I A L

Is, when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

R U L E.

As the sum of the quantities, or the whole composition, is to their total value; so is any part of the composition to its mean price or value.

E X A M P L E S.

E X A M P L E S.

1. A tobacco-nist would mix 60lb of tobacco, at 6d. per lb, with 50lb at 1s. 40lb at 1s. 6d. and 30lb at 2s. per lb : What is 1lb of this mixture worth ?

lb	s. d.	£. s.	lb	£.	lb
60	at 0 6	is 1 10	As 180 :	10 :: 1	
50	— 1 0	— 2 10		1	
40	— 1 6	— 3 0		—	
30	— 2 0	— 3 0		10	
Sum of the				20	
simples, } 180				—	
Total val.				180)200(1s.	
				180	
				20	
				12	
				180)240(1½d. s. d.	
				180	
				60	
					Ans. 1 1½ per lb.

2. A farmer would mix 20 bushels of wheat at 6s. per bushel, 16 bushels of rye at 4s. per bushel, 12 bushels of barley at 3s. per bushel, and 8 bushels of oats at 2s. per bushel ; What is the value of one bushel of this mixture ?

Ans. 4s. 2½d.

3. A wine merchant mixes 12 gallons of wine, at 75c. per gallon, with 24 gallons, at 90c. and 16 gallons at 1D. 10c. ; What is a gallon of this composition worth ?

Ans. 92c. 6m.

4. A goldsmith melted together 8oz. of gold of 22 carats fine, 1lb 8oz. of 21 carats fine, and 10oz. of 18 carats fine : Pray what is the quality, or fineness of the composition ?

$$\frac{8 \times 22 + 20 \times 21 + 10 \times 18}{8 + 20 + 10} = 20\frac{8}{19} \text{ carats fine, } \textit{Ans.}$$

5. A refiner melts 5lb of gold of 20 carats fine with 8lb of 18 carats fine ; How much alloy must he put to it, to make it 22 carats fine ?

$$22 - \frac{5 \times 20 + 8 \times 18}{5 + 8} = 3\frac{3}{13}$$

Answer, It is not fine enough by $3\frac{3}{13}$ carats, so that no alloy must be added, but more gold.

ALLIGATION ALTERNATE*

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate : So that it is the reverse of Alligation Medial, and may be proved by it.

CASE

* *Demon.* By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities resulting are such that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss, upon the whole, are equal, and are exactly the proposed rate.

In

C A S E I.

The whole work of this case consists in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities sought.

RULE 1.—Place the several prices of the simples, being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase, with a line of connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those, which are greater than the compound, and each greater rate or price with one, or any number of the less.

3. Place the difference, between the mean price (or mixture rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

E X A M P L E S.

1. A merchant has spices, some at 1s. 6d. per lb, some at 2s. some at 4s. and some at 5s. per lb; How much of each sort must he mix, that he may sell the mixture at 3s. 4d. per lb?

d.	lb	s. d.		d.	lb	s. d.	
Mean rate 40d.	$\left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\}$	$\left\{ \begin{array}{l} 20 \text{ at } 1 \text{ } 6 \\ 8 - 2 \text{ } 0 \\ 16 - 4 \text{ } 0 \\ 22 - 5 \text{ } 0 \end{array} \right\}$	per lb.	40d.	$\left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\}$	$\left\{ \begin{array}{l} 8 \text{ at } 1 \text{ } 6 \\ 20 - 2 \text{ } 0 \\ 22 - 4 \text{ } 0 \\ 16 - 5 \text{ } 0 \end{array} \right\}$	per lb.
d.	lb	s. d.		d.	lb	s. d.	
40	$\left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\}$	$\left\{ \begin{array}{l} 20 + 8 \\ 8 - 2 \text{ } 0 \\ 16 + 22 \\ 22 - 5 \text{ } 0 \end{array} \right\}$	per lb.	40	$\left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\}$	$\left\{ \begin{array}{l} 20 + 8 \\ 8 + 20 \\ 16 - 4 \text{ } 0 \\ 22 + 16 \end{array} \right\}$	per lb.
						40d.	

In like manner, let the number of simples be what it may, and with how many soever each one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

It is obvious from the rule, that questions of this sort admit of a great variety of answers; for having found one answer, we may find as many more as we please by only multiplying or dividing each of the quantities found by 2, 3, 4, &c. the reason of which is evident; for if two quantities of two simples make a balance of loss and gain with respect to the mean price, so must also the double or triple, the half or third part, or any other ratio of these quantities, and so on *ad infinitum*.

If any one of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

40d. $\left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\} \begin{array}{l} 20+8 \\ 8+20 \\ 16+22 \\ 22+16 \end{array} \left| \begin{array}{l} 28 \text{ at } 16 \\ 28 - 20 \\ 38 - 40 \\ 38 - 50 \end{array} \right\} \text{per lb.}$

Note. These five answers arise from as many various ways of linking the rates of the ingredients together.

2.* A merchant has Canary wine, at 3s. per gallon, Sherry, at 2s. 1d. and claret at 1s. 5d. per gallon; How much of each fort must he take, to sell it at 2s. 4d. per gallon?

Mean rate 28d. $\left\{ \begin{array}{l} 36 \\ 25 \\ 17 \end{array} \right\} \begin{array}{l} 3+11 \\ 8 \\ 8 \end{array} \left| \begin{array}{l} 14 \text{ at } 30 \\ 8 - 21 \\ 8 - 15 \end{array} \right\} \text{per gallon.}$

3. How much barley at 2s. 4d. rye at 3s. 9d. and wheat at 5s. per bushel, must be mixed together, that the compound may be worth 4s. 4d. per bushel?

Ans. 8 bushels of barley, 8 of rye, and 31 of wheat.

4. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine; What quantity of each must he take?

21 $\left\{ \begin{array}{l} 16 \\ 18 \\ 19 \\ 23 \\ 24 \end{array} \right\} \left\{ \begin{array}{l} \text{Ans. 50z. of 16 carats} \\ \text{fine, 50z. of 18, 50z.} \\ \text{of 19, 100z. of 23,} \\ \text{and 100z. of 24 ca-} \\ \text{rats fine.} \end{array} \right.$

5. It is required to mix several sorts of wine, at 60c. 90c. and 1d. 15c. per gallon, with water, that the mixture may be worth 75c. per gallon; Of how much of each fort must the composition consist?

75 $\left\{ \begin{array}{l} 0 \\ 60 \\ 90 \\ 115 \end{array} \right\} \left\{ \begin{array}{l} \text{Ans. 40galls. of water,} \\ 15galls. of wine, at 60c.} \\ 15galls. do. at 90c. and} \\ 75galls. do at 1d. 15c. \end{array} \right.$

C A S E II.

When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixtures are given, to find the several quantities of the rest, in proportion to the quantity given.

RULE.—Take the differences between each price, and the mean rate, and place them alternately, as in Case 1. Then, As the difference standing against that simple, whose quantity is given, is to that quantity; so is each of the other differences, severally, to the several quantities required.

E X A M P L E S.

1. A merchant has 40lb of tea, at 6s. per lb, which he would mix with some at 5s. 8d. some at 5s. 2d. and some at 4s. 6d. How much

* *Note,* the 2d and 3d questions admit but of one way of linking, and so but of one answer; yet all numbers in the same proportion between themselves, as the numbers, which compose the answer, will likewise satisfy the condition of the question.

much of each sort must he take, to mix with the 40lb, that he may sell the mixture at 5s. 5d. per lb?

$$\begin{array}{rcl}
 & \text{lb} & \\
 65 \left\{ \begin{array}{l} 54 \\ 62 \\ 68 \\ 72 \end{array} \right. & \begin{array}{l} 7+3 \\ 3+7 \\ 3+11 \\ 11+3 \end{array} & \begin{array}{l} 10 \\ 10 \\ 14 \\ 14 \end{array} \text{ stands against the given quantity.} \\
 & \text{lb} &
 \end{array}$$

$$\text{As } 14 : 40 :: \left\{ \begin{array}{l} 10 : 28\frac{8}{14} \text{ at } 4 \text{ } 6 \\ 10 : 28\frac{8}{14} - 5 \text{ } 2 \\ 14 : 40 - 5 \text{ } 8 \end{array} \right\} \text{ per lb.}$$

2. A farmer being determined to mix 20 bushels of oats, at 2s. per bushel, with barley, at 2s. 6d. rye at 4s. and wheat, at 5s. 6d. per bushel; I demand the quantity of each, which must be mixed with the 20 bushels of oats, that the whole quantity may be worth 4s. 6d. per bushel? *Ans.* 20 of barley, 20 of rye, and 100 of wheat.

3. How much gold of 16, 20 and 24 carats fine, and how much alloy, must be mixed with 100oz. of 18 carats fine, that the composition may be 22 carats fine?

Ans. 100z. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

ALTERNATION TOTAL.*

C A S E III.

When the rates of the several ingredients, the quantity to be compounded; and the mean rate of the whole mixture are given, to find how much of each sort will make up the quantity.

RULE.—Place the differences between the mean rate, and the several prices alternately, as in Case 1;—then, As the sum of the quantities, or differences thus determined, is to the given quantity,

* To this Case belongs that curious question concerning king Hiero's crown.

Hiero, king of Syracuse, gave orders for a crown to be made, entirely of pure gold; but suspecting the workmen had debased it by mixing with it silver or copper, he recommended the discovery of the fraud to the famous Archimides, and desired to know the exact quantity of alloy in the crown.

Archimides, in order to detect the imposition, procured two other masses, the one of pure gold, and the other of silver, or copper, and each of the same weight with the former; and, by putting each separately into a vessel full of water, the quantity of water expelled by them, determined their specifick bulks; from which and their given weights it is easier to determine the quantities of gold and alloy in the crown by this case of Alligation, than by an Algebraic process.

Suppose the weight of each mass to have been 5lb, the weight of the water expelled by the alloy, 23oz. by the gold 13oz: and by the crown 16oz. that is, that their specifick bulks were as 23, 13, and 16, then, What were the quantities of gold and alloy respectively in the crown?

Here, the rates of the simples are 23 and 13, and of the compound 16; whence,

$$16 \left\{ \begin{array}{l} 13 \text{ — } 7 \text{ of gold} \\ 23 \text{ — } 7 \text{ of alloy} \end{array} \right\} \text{ And the sum of these is } 7+3=10, \text{ which should have been but } 5, \text{ whence, by the rule,}$$

$$10 : 5 :: \left\{ \begin{array}{l} 7 : 3\frac{1}{2} \text{ lb of gold} \\ 3 : 1\frac{1}{2} \text{ lb of alloy} \end{array} \right\} \text{ the } \textit{Ans.}$$

Gal. s. Gal.

$$\text{Limited simples } \left\{ \begin{array}{l} 6 \text{ gallons at } 4s. = 24 \\ 6 \text{ gallons at } 3s. = 18 \end{array} \right\} \text{ As } 12 : 42 :: 1 : 3 \text{ f } 6 \text{ per gal.}$$

$$\begin{array}{r} 24 \\ 18 \\ \hline 42 \end{array}$$

Now, having found the rate of the limited simples, the question may stand thus : How much wine at 4s. 6d. and 5s. per gallon, must be mixed with 12 gallons, at 3s. 6d. per gallon, that the mixture may be worth 4s. 4d. per gallon ?

$$52 \left\{ \begin{array}{l} 42 \\ 54 \\ 60 \end{array} \right\} \begin{array}{l} 2+8 \\ 10 \\ 10 \end{array} \left| \begin{array}{l} 10 \\ 10 \\ 10 \end{array} \right. \text{ As } 10 : \left\{ \begin{array}{l} 10 \\ 10 \end{array} \right\} :: 12 : \left\{ \begin{array}{l} 12 \text{ gal. at } 4/6 \\ 12 \text{ gal. at } 5s. \end{array} \right\} \text{ per gal.}$$

Ans.

2. How much gold, of 14 and 16 carats fine, must be mixed with 6oz. of 19, and 12 of 22 carats fine, that the composition may be 20 carats fine ?

Ans. $1\frac{6}{10}$ oz. of each sort.

P O S I T I O N.

Position is a rule, which, by false or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts ; *single* and *double*.

S I N G L E P O S I T I O N.

Single Position teaches to resolve those questions, whose results are proportional to their suppositions : Such are those which require the multiplication or division of the number sought by any proposed number ; or when it is to be increased or diminished by itself a certain proposed number of times.

RULE* 1. Take any number, and perform the same operations with it, as are described to be performed in the question.

2. Then say, As the sum of the errors is to the given sum ; so is the supposed number, to the true one required.

Proof. Add the several parts of the sum together, and if it agrees with the sum, it is right.

E X A M P L E S.

1. A schoolmaster, being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half

* The reason of this rule is obvious, it being evident that the results are proportional to the suppositions.

$$\text{Thus, } \left\{ \begin{array}{l} nx : x :: na : a \\ \frac{x}{n} : x :: \frac{a}{n} : a \\ \frac{x}{n} \pm x \&c. : x :: \frac{a}{n} \pm \frac{a}{m} \&c. : a, \text{ and so on,} \end{array} \right.$$

half as many, one fourth and one eighth as many, I should then have 435 ; Of what number did his school consist ?

Suppose he had 80 As 290 : 435 :: 80

As many = 80	80	
$\frac{3}{4}$ as many = 60	—	120
$\frac{1}{2}$ as many = 40	29 0)3480 0(120 <i>Answer.</i>	120
$\frac{1}{4}$ as many = 20	29	90
$\frac{1}{8}$ as many = 10	—	60
	58	30
290	58	15
	—	—
	0	435 <i>Proof.</i>

2. A person lent his friend a sum of money unknown, to receive interest for the same at 6%. per cent. per annum, simple interest, and, at the end of 12 years, received for principal and interest 860l. What was the sum lent ? *Ans.* £500.

3. A, B and C joined their stocks, and gained 350l. of which A took up a certain sum, B took up four times so much as A, and C, eight times so much as B ; What share of the gain had each ?

£.	s.	d.	grs.	
<i>Ans.</i> {	9	9	2	$1\frac{3}{4}$ A's share.
	37	16	9	$0\frac{12}{37}$ B's ditto.
	302	14	0	$2\frac{22}{37}$ C's ditto.

4. A, B, C, and D spent 35s. at a reckoning, and, being a little dipped, they agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$; What did each pay in the above proportion ?

	s.	d.
<i>Ans.</i> {	A,	$13\frac{4}{5}$
	B,	10 0
	C,	6 8
	D,	5 0

5. A certain sum of money is to be divided between 5 men, in such a manner as that A shall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is 40l. ; What is the sum ?

Suppose 200l. then $\frac{1}{4} + \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = 120$.

200—120 = 80. As 80 : 40 :: 200 : 100 *Ans.*

6. A person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had $26\frac{2}{3}$ l. left ; What had he at first ? *Ans.* £160.

7. A and B talking of their ages, B said his age was once and an half the age of A ; C said his was twice and one tenth the age of both, and that the sum of their ages was 93 ; What was the age of each ? *Ans.* A's 12, B's 18, and C's 63 years.

8. A vessel has 3 cocks, A, B and C ; A can fill it in $\frac{1}{2}$ an hour, B in $\frac{1}{4}$ of an hour, and C in $\frac{1}{3}$ of an hour ; In what time will they all fill it, together ? *Ans.* $\frac{1}{7}$ hour.

9. A person having about him a certain number of dollars, said that

that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 57; Pray, how many had he? *Ans.* 60.

10. A gentleman bought a chaise, horse and harness for 100*l*. the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{3}$ more than the horse; What was the price of each?

Ans. Harness $\pounds 25\frac{2}{3}$. Horse $\pounds 31\frac{1}{3}$. Chaise $\pounds 42\frac{2}{3}$.

11. A and B, having found a purse of money, disputed who should have it: A said that $\frac{1}{5}$, $\frac{1}{4}$ and $\frac{1}{6}$ of it amounted to 35*l*. and if B could tell him how much was in it, he should have the whole, otherwise he should have nothing; How much did the purse contain?

Ans. $\pounds 100$.

12. A gentleman divided his fortune among his sons; to A he gave 9*l*. as often as to B 5*l*. and to C, but 3*l*. as often as to B 7*l*. yet C's portion came to 1050*l*.; What was the whole estate?

Ans. $\pounds 7916\frac{2}{5}$.

13. Seven eighths of a certain number exceeds four fifths by 6; What is that number?

Ans. 80.

14. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itself, the sum will be 234*l*.?

Ans. 90.

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

Those questions, in which the results are not proportional to their positions, belong to this rule; Such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

RULE* 1.—Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the result or errors against their positions or supposed

Pos. *Err.*

numbers, thus, $\begin{array}{r} 30 \\ 20 \end{array} \times \begin{array}{r} 12 \\ 6 \end{array}$ and if the error be too great, mark it with +; and if too small with —.

3. Multiply

* The rule is founded on this supposition, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number: When that is not the case, the exact answer to the question cannot be found by this rule.

That the rule is true, according to the supposition, may be thus demonstrated.

Let A and B be any two numbers produced from a and b by similar operations, it is required to find the number from which N is produced by a like operation.

Put x = number required, and let $N - A = r$, and $N - B = s$. Then, according to the supposition on which the rule is founded, $r : s :: x - a : x - b$, whence, by multiplying means and extremes, $rx - rb = sx - sa$; and by transposition, $rx - sx$

$= rb - sa$; and by division, $x = \frac{rb - sa}{r - s}$ = number sought; and if r and s be both negative, the Theorem is the same, and if r or s be negative, x will be equal to $\frac{rb + sa}{r - s}$ which is the rule.

3. Multiply them crosswise ; that is, the first position by the last error, and the last position by the first error.

4. If the errors be alike, that is, both too small, or both too great, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

5. If the errors be unlike ; that is, one too small, and the other too great, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Note, When the errors are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

1. A lady bought damask for a gown, at 8s. per yard, and, lining for it at 3s. per yard ; the gown and lining contained 15 yards, and the price of the whole was 3*l.* 10s. ; How many yards were there of each ?

Suppose 6 yards damask, value 48s.
Then she must have 9 yards of lining, value 27s.

Sum of their values = 75s.

So that the first error is 5 too much, or +5

Again, suppose she had 4 yards of damask, value 32s.

Then she must have 11 yards of lining, value 33s.

Sum of their values = 65s.

So that the second error is 5 too little, or -5s.

$$\begin{array}{r} \text{Then } \begin{array}{c} 6 \\ 4 \end{array} \times \begin{array}{c} 5+ \\ 5- \end{array} \\ \hline 20 \quad 30 \\ \hline 20 \end{array}$$

$$\begin{array}{r} \text{£. s. d.} \\ 5 \text{ yds. at } 8s. = 2 \quad 0 \quad 0 \\ 10 \text{ yds. at } 3s. = 1 \quad 10 \quad 0 \\ \hline \end{array}$$

3 10 0 proof.

$$\text{Sum of err.} = 5 + 5 = 10 \text{) } 50$$

Ans. 5 yds. damask, and 15 - 5 = 10 yds. lining.

Or, $6 + 4 \div 2 = 5$ as before.

2. A and B have the same income ; A saves $\frac{1}{5}$ of his ; but B, by spending 30*l.* per annum more than A, at the end of 8 years finds himself 40*l.* in debt ; What is their income, and what does each spend per annum ?

$$\text{Suppose } \begin{array}{c} 80 \\ 160 \end{array} \times \begin{array}{c} 120+ \\ 40+ \end{array}$$

Ans. Their income is 200*l.* per ann.

Also, A spends 175*l.* and B 205*l.* per annum. Then $80 - 10 = 70$ A's expense per annum, and $70 + 30 = 100$, B's expense per annum. Then $100 \times 8 - 80 \times 8 = 160$, which should have been 40 ; therefore, $160 - 40 = 120$ more than it should be, for the first error. In like manner proceed for the second error.

3. A and B laid out equal sums of money in trade : A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost 225*l*. then A's money was double that of B ; What did each lay out ?

$$\text{Suppose } \begin{array}{r} 300 \\ 900 \end{array} X \begin{array}{r} 225+ \\ 225- \end{array} \quad \text{Ans. } £600.$$

4. A labourer was hired for 60 days, upon this condition, that, for every day he wrought, he should receive 3*s*. 4*d*. ; and for every day he was idle, should forfeit 1*s*. 8*d*. ; at the expiration of the time he received 3*l*. 15*s*. How many days did he work, and how many was he idle ?

$$\text{Suppose he worked } \begin{array}{r} 20 \\ 40 \end{array} X \begin{array}{r} 900- \\ 300+ \end{array}$$

Ans. He was employed 35 days, and was idle 25.

5. A gentleman has two horses of considerable value, and a carriage worth 100*l*. ; now, if the first horse be harnessed in it, he and the carriage together will be triple the value of the second ; but if the second be put in, they will be 7 times the value of the first ; What is the value of each horse ?

$$\text{Suppose } \begin{array}{r} 32 \\ 44 \end{array} X \begin{array}{r} 80- \\ 160- \end{array}$$

Ans. One £20, and the other £40.

6. There is a fish, whose head is 10 feet long ; his tail is as long as his head and half the length of his body ; and his body as long as the head and tail ; What is the whole length of the fish ?

$$\begin{array}{l} \text{First, suppose the body } 20 \\ \text{and suppose it } 30 \end{array} X \begin{array}{r} 10- \\ 5- \end{array} \quad \begin{array}{l} \text{Head} = 10 \\ \text{Tail} = 30 \\ \text{Body} = 40 \\ \hline \end{array}$$

Ans. 80 feet.

7. What number is that, which, being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be doubled ?

$$\text{Suppose } \begin{array}{r} 8 \\ 16 \end{array} X \begin{array}{r} 3+ \\ 1+ \end{array}$$

Ans. 20.

8. A farmer having driven his cattle to market, received for them all 80*l*. being paid at the rate of 6*l*. per ox, 4*l*. per cow, and 1*l*. 10*s*. per calf ; there were as many oxen as cows, and 4 times as many calves as cows ; How many were there of each sort ?

$$\text{Suppose cows } \begin{array}{r} 6 \\ 12 \end{array} X \begin{array}{r} 16+ \\ 112+ \end{array}$$

Ans. 5 oxen, 5 cows, and 20 calves.

9. A, B and C built a ship, which cost them 1000*l*. of which A paid a certain sum, B paid 100*l*. more than A, and C 100*l*. more

more than both ; having finished her, they fixed her for sea with a cargo worth twice the value of the ship : The outfits and charges of the voyage amounted to $\frac{1}{5}$ of the ship ; upon the return of which, they found their clear gain to be $\frac{2}{3}$ of $\frac{3}{4}$ of the vessel, cargo and expenses ; Please to inform me what the ship cost them, severally ; what share each had in her, and what, upon the final adjustment of their accompts, they had severally gained ?

Suppose it cost A $\overset{100}{\underset{200}{X}} \overset{300-}{\underset{100+}{}}$

A owned $\frac{2}{40}$ of the ship, which cost him 175*l.* and his share of the gain was 218*l.* 15*s.* B owned $\frac{11}{40}$, which cost 275*l.* and his gain was 343*l.* 15*s.* C owned $\frac{11}{40}$, which cost 550*l.* and his gain was 687*l.* 10*s.*

PERMUTATIONS and COMBINATIONS.

The permutation of quantities is, the shewing how many different ways any given number of things may be changed.

This is also called *variation*, *alternation*, or *changes* ; and the only thing to be regarded here is the order they stand in ; for no two parcels are to have all their quantities placed in the same situation.

The Combination of Quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

This is, sometimes, called *election*, or *choice* ; and here every parcel must be different from all the rest, and no two are to have precisely the same quantities, or things.

The Composition of Quantities is the taking of a given number of quantities out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places ; and it differs from Combination only as that admits but of one row of things.

Combinations of the same form are those in which there are the same number of quantities, and the same repetitions ; thus, *abcc*, *bbad*, *deef*. &c. are of the same form ; but *abbc*, *abbb*, *aaac* are of different forms.

PROBLEM I.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

RULE.*—Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

EXAMPLES.

* The reason of this rule may be shewn thus, any one thing *a* is capable of one position only, as *a*.

Any

EXAMPLES.

1. Christ's church, in Boston, has 8 bells; How many changes may be rung on them?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320 \text{ Ans.}$$

2. Nine gentlemen met at an inn, and were so pleased with their host, and with each other, that in a frolic they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner; Pray how long, had they kept their agreement, would their frolic have lasted?

$$\text{Ans. } 9941 \frac{335}{365} \text{ years.}$$

3. How many changes, or variations, will the alphabet admit of?

$$\text{Ans. } 620448401733239439360000.$$

PROBLEM II.

Any number of different things being given, to find how many changes can be made out of them, by taking any given number of quantities at a time.

RULE.*—Take a series of numbers, beginning at the number of things given, and decreasing by 1, to the number of quantities to be taken at a time: The product of all the terms will be the answer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array}$$

$$\text{Or, } 8 \times 7 \times 6 \times 5 (= 4 \text{ terms}) = 1680 \text{ Ans.}$$

2. How many words can be made with 6 letters of the alphabet, admitting a number of consonants may make a word?

$$\text{Ans. } 96909120.$$

PROBLEM

Any two things a and b are capable of two variations only; as ab, ba ; whose number is expressed by 1×2 .

If there be three things a, b and c ; then any two of them, leaving out the third, will have 1×2 variations; and consequently when the third is taken in, there will be $1 \times 2 \times 3$ variations; and so on, as far as you please.

* This Rule, expressed in terms, is as follows; $m \times m-1 \times m-2 \times m-3$ &c. to n terms; whence m = number of things given, and n = quantities to be taken at a time.

PROBLEM III.

Any number of things being given, whereof there are several things of one sort, several of another, &c. to find how many changes may be made out of them all.

RULE* 1.—Take the series $1 \times 2 \times 3 \times 4$, &c. up to the number of things given, and find the product of all the terms.

2. Take the series $1 \times 2 \times 3 \times 4$, &c. up to the number of the given things of the first sort, and the series, $1 \times 2 \times 3 \times 4$, &c. up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series by the joint product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zaphnathpaaneah*?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15$ (= number of letters in the word) = 1307674368000.

$1 \times 2 \times 3 \times 4 \times 5$ (= number of as) = 120

1×2 (= number of ps) = 2

1 (= number of ts) = 1

$1 \times 2 \times 3$ (= number of hs) = 6

1×2 (= number of ns) = 2

$2 \times 6 \times 1 \times 2 \times 120 = 2880$) 1307674368000 (454053600 *Ans.*

2. How many different numbers can be made of the following figures, 1223334444? *Ans.* 12600.

PROBLEM IV.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE

* This Rule is expressed in terms thus; $\frac{1 \times 2 \times 3 \times 4 \times 5, \&c. \text{ to } m.}{1 \times 2 \times 3, \&c. \text{ to } p. \times 1 \times 2 \times 3, \&c. \text{ to } q \&c.}$ whence m = number of things given, p = number of things of the first sort, q = number of things of the second sort, &c.

Any 2 quantities, a, b , both different, admit of 2 changes; but if the quantities are the same, or ab become aa , there will be only one alteration, which may be expressed by $\frac{1 \times 2}{1 \times 2} = 1$.

Any 3 quantities, a, b, c , all different from each other, admit of 6 variations; but if the quantities are all alike, or, a, b, c become aaa , then the 6 variations will be reduced to 1, which may be expressed by $\frac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. Again, if two quantities out of three are alike, or abc become aac ; then the 6 variations will be reduced to these 3, aac, caa, aca , which may be expressed by $\frac{1 \times 2 \times 3}{1 \times 2} = 3$, and so of any greater number.

346 PERMUTATIONS AND COMBINATIONS.

RULE* 1.—Take the series 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$ (=the number to be taken at a time) = 5040.

$12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6$ (=same number from 12) = 3991680.
5040)3991680(792 *Ans.*

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet? *Ans.* 134596.

3. A general was asked by his king, what reward he should confer on him for his services; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men; what did it amount to?

Ans. £23836022841 7s. 11- $\frac{65}{1134}$ d.

4. A farmer bargained with a gentleman for a dozen sheep, (at 2 dollars per head) which were to be picked out of 2 dozen; but, being long in choosing them, the gentleman told him that if he would give him a penny for every different dozen which might be chosen out of the two dozen, he should have the whole, to which the farmer readily agreed; Pray what did they cost him? *Ans.* £11267 6s. 4d.

5. How many locks, whose wards differ, may be unlocked with a key of 6 several wards? *Ans.* 63: 6 of which may have one single ward, 15 double wards, 20 triple wards, 15 four wards, 6 five wards, and 1 lock, 6 wards.

Wards.		Locks.	Wards.		Locks.
$\left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \right\}$	in 6 =	$\left\{ \begin{array}{c} 6 \\ 15 \\ 20 \\ 15 \\ 6 \\ 1 \end{array} \right\}$	$\left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \right\}$	in 5 =	$\left\{ \begin{array}{c} 5 \\ 10 \\ 10 \\ 5 \\ 1 \end{array} \right\}$

PROBLEM

* This Rule, expressed algebraically, is $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c. to n terms; where m is the number of given quantities, and n , those to be taken at a time.

Note, In any given number of quantities, the number of Combinations increases gradually till you come about the mean numbers, and then gradually decrease. If the number of quantities be *even*, half the number of places will shew the greatest number of Combinations, that can be made of those quantities; but if odd, then those 2 numbers which are the middle, and whose sum is equal to the given number of quantities, will shew the greatest number of Combinations.

PROBLEM V.

To find the number of combinations of any given number of things, by taking any given number at a time ; in which there are several things of one sort, several of another, &c.

RULE.—Find the number of different forms, which the things, to be taken at a time, will admit of, in the following manner :

1. Place the things so that the greatest indices may be first, and the rest in order.
2. Begin with the first letter, and join it to the second, third, fourth, &c. to the last.
3. Join the second letter to the third, fourth, &c. to the last ; and so on till they are all done, always rejecting such combinations as have occurred before ; and this will give the combinations of all the twos.
4. Join the first letter to every one of the twos ; then join the second, third, &c. as before ; and it will give the combinations of all the threes.
5. Proceed in the same manner to get the combinations of all the fours, fives, &c. and you will at last get all the several forms of combination, and the number in each form.
6. Having found the number of combinations in each form, add them all together, and the sum will be the number required.

EXAMPLE.

1. Let the things proposed be $aaabbc$; It is required to find the number of combinations of every 2, of every 3, and of every 4 of these quantities.

Combinations at large.	Forms.	Comb. in each form.
aa, aa, ab, ab, ac	a^2, b^2	2
aa, ab, ab, ac	ab, ac, bc	3
ab, ab, ac		—
bb, bc		5 = sum of the twos.
bc		
	a^3	1
aaa, aab, aab, aac	a^2b, a^2c, b^2a, b^2c	4
aab, aab, aac	abc	1
abb, abc		—
bbc		6 = sum of the threes.
$aaab, aaab, aaac$	a^3b, a^3c	2
$aabb, aabc$	a^2b^2	1
$abbc$	a^2bc, b^2ac	2
		—
		5 = sum of the fours.

Ans. 5 Combinations of every 2 ; 6, of every 3, and 5 of every 4 quantities.

MISCELLANEOUS MATTERS.

A short method of reducing a Vulgar Fraction into its equivalent Decimal, by Multiplication.

R U L E.

DIVIDE unity or 1 by the denominator, till the remainder is a single figure, 10, 100, &c. if convenient, then multiply the whole quotient, including the remainder after division, by the remainder (which is now the numerator, and the divisor, the denominator) and annex the product to the quotient, then multiply the quotient, thus increased, by the last numerator, and annex the product to the increased quotient; and thus it may be reduced to what exactness you please. But if the numerator of the given fraction exceed 1, you must finally multiply the last product by the said numerator.

E X A M P L E S.

1. Reduce $\frac{7}{26}$ to its equivalent decimal.

$$\begin{array}{r} 26 \overline{) 1,00(,03846\frac{4}{26}} \\ \underline{78} \\ 220 \\ \underline{208} \\ 120 \\ \underline{104} \\ 160 \\ \underline{156} \\ 4 \end{array}$$

This multiplied by 4 (the numerator) is, $15384\frac{16}{26} = \frac{8}{13}$
Which annexed to the quotient, .03846 is, .0384615384 $\frac{8}{13}$
And, .0384615384 $\frac{8}{13}$ $\times 8$ and annexed to the last product
= .03846153843076923076 $\frac{12}{13}$ &c.

2. Reduce $\frac{5}{246}$.

$$246 \overline{) 1,000000(,004065\frac{10}{246} \text{ and } ,0040650\frac{10}{246} \times 10 = ,0040650\frac{100}{246}}$$

and this annexed to the quotient is, .00406540650 $\frac{100}{246}$, and this multiplied by the given numerator 5, is, .02032703252 $\frac{500}{246}$.

For any number of pounds, avoirdupois, under 28, multiply the decimal, .00892857 by the given number of pounds, which generally gives the decimal true to the sixth place.

A short method of finding the duplicate, triplicate, &c. Ratio of any two numbers, whose difference is small, compared with the two numbers.

For the Duplicate Ratio.

RULE.—Assume two numbers, whose difference is small; subtract half their difference from the least, and add it to the greatest, and the two numbers, thus found, will be in the same proportion nearly as the squares of the assumed numbers.

E X A M P L E.

Let the assumed numbers be 10 and 11: Then $11 - 10 = 1$.
 $10 - .5 = 9,5$ and $11 + .5 = 11,5$.

Proof, As $10^2 : 11^2 :: 9,5 : 11,5$ nearly.

For a Triplicate Ratio.

RULE.—Subtract the difference of the assumed numbers from the

the least, and add it to the greatest, and the numbers, thus obtained, will be in the same proportion nearly as the cubes of the assumed numbers.

Let the numbers be 164 and 165: Then $165 - 164 = 1$. $164 - 1 = 163$ & $165 + 1 = 166$. *Proof*, As $164^3 : 165^3 :: 163 : 166$ nearly.

For a quadruplicate proportion subtract, and add once and a half the difference, and so on, for each higher power, increasing the number to be subtracted and added by .5.

To reduce a Ratio, consisting of large numbers, to its least Terms, and very nearly of the same value.

RULE 1.—Divide the greater of the terms by the less, and the last divisor by the remainder, and so on continually, till nothing remain, in the same manner as we get the greatest common measure for reducing a vulgar fraction: This will give a number of ratios, from which we can choose one that will suit our purpose.

2. Place the first quotient under unit for the first ratio; multiply that by the next quotient, adding nothing to the numerator, and 1 to the product of the denominator for a new denominator, and it will give a second ratio, nearer than the first: Then, multiply the last ratio by the next quotient, adding the preceding ratio, and so on continually till you have gone through.

EXAMPLES.

1. Sir Isaac Newton has demonstrated, in his Principia, that the velocity of a comet, moving in a parabola, is to that of a planet, moving in a circular orb, at the same distance from the sun, as $\sqrt{2}$ to 1. Let this be taken for an example.

$\sqrt{2} = 1,4142$; Those motions, then, are as 1,4142 to 1; or as 14142 to 10000?

$$\begin{array}{r} 10000 \overline{) 14142 (1} \\ 10000 \end{array}$$

10000

$$\begin{array}{r} 4142 \overline{) 10000 (2} \\ 8284 \end{array}$$

$$\begin{array}{r} 1716 \overline{) 4142 (2} \\ 3432 \end{array}$$

$$\begin{array}{r} 710 \overline{) 1716 (2} \\ 1420 \end{array}$$

$$\begin{array}{r} 296 \overline{) 710 (2} \\ 592 \end{array}$$

$$\begin{array}{r} 118 \overline{) 296 (2} \\ 236 \end{array}$$

$$\begin{array}{r} 60 \overline{) 118 (1} \\ 60 \end{array}$$

$$\begin{array}{r} 58 \overline{) 60 (1} \\ 58 \end{array}$$

$$\begin{array}{r} 2 \overline{) 58 (29} \\ 58 \end{array}$$

Then $\frac{1}{1} =$ first ratio.

$$\frac{1 \times 2 + 0}{1 \times 2 + 1} = \frac{2}{3} = \text{second.}$$

$$\frac{2 \times 2 + 1}{3 \times 2 + 1} = \frac{5}{7} = \text{third.}^*$$

$$\frac{5 \times 2 + 2}{7 \times 2 + 3} = \frac{12}{17} = \text{fourth.}$$

$$\frac{12 \times 2 + 5}{17 \times 2 + 7} = \frac{29}{41} = \text{fifth, \&c.}$$

2. Geometers

* The late Professor Winthrop chose 7 to 5 for a proportion.

2. Geometers have found the proportion of the circumference of a circle to its diameter, to be as 3,1416 to 1 : Let this ratio be reduced.

$$\begin{array}{r}
 10000 \overline{) 31416(3} \\
 \underline{30000} \\
 1416 \overline{) 10000(7} \\
 \underline{9912} \\
 88 \overline{) 1416(15} \\
 \underline{88} \\
 536 \\
 \underline{528} \\
 8 \overline{) 88(11} \\
 \underline{88}
 \end{array}$$

Then $\frac{1}{3} =$ first ratio.

$$\frac{1 \times 7 + 0}{3 \times 7 + 1} = \frac{7}{22} = \text{second.}$$

$$\frac{7 \times 16 + 1}{22 \times 16 + 3} = \frac{113}{355} = \text{third :—This is the ratio generally made use of, and is sufficiently exact for very nice calculations.}$$

3. The area of a circle is to its circumscribing square, as, 7854 to 1, very nearly : Let this be reduced.

$$\begin{array}{r}
 7854 \overline{) 10000(1} \\
 \underline{7854} \\
 2146 \overline{) 7854(3} \\
 \underline{6438} \\
 1416 \overline{) 2146(1} \\
 \underline{1416} \\
 730 \overline{) 1416(1} \\
 \underline{730} \\
 686 \overline{) 730(1} \\
 \underline{686} \\
 44 \overline{) 686(15} \\
 \underline{44} \\
 246 \\
 \underline{220} \\
 26 \text{ &c.}
 \end{array}$$

Then $\frac{1}{1} =$ first ratio.

$$\frac{1 \times 3 + 0}{1 \times 3 + 1} = \frac{3}{4} = \text{second.}$$

$$\frac{3 \times 1 + 1}{4 \times 1 + 1} = \frac{4}{5} = \text{third.}$$

$$\frac{4 \times 1 + 3}{5 \times 1 + 4} = \frac{7}{9} = \text{fourth.}$$

$$\frac{7 \times 1 + 4}{9 \times 1 + 5} = \frac{11}{14} = \text{fifth.—This is very exact, and the proportion generally used.}$$

Therefore, As 14 : 11 :: the square of the diameter of a circle to its area.

To estimate the Distance of Objects on level ground, or at sea, having only the height given.

RULE 1.—To the earth's diameter (viz. 42056462 feet,) add the height of the eye, and multiply the sum by that height, then the square root of the product is the distance, at which an object on the surface of the earth or water, can be seen by an eye so elevated.

2. As objects are seen in a straight line, and that line is a tangent to the earth's surface ; therefore, To find the distance of two elevated objects, when the right line joining them touches the earth's surface between those objects, (for instance, the line from the eye of the observer to the distance found by the first part of the rule, and from thence to the object :) work for each object separately, and the sum of the square roots of the products is the distance of the two objects from each other.

How

354 MISCELLANEOUS QUESTIONS.

How far may a mountain be seen on level ground, or at sea, which is a mile high, supposing the eye of the observer elevated 5 feet above the surface?

$$\sqrt{42056462 + 5 \times 5} = 2,746 \text{ miles.}$$

$$\sqrt{42056462 + 5280 \times 5280} = 89,253 \text{ miles.}$$

Ans. 91,999 miles.

To estimate the Height of Objects on level ground, or at sea, having only the distance given.

RULE 1.—From the given distance take the distance, which the elevation of your eye above the surface will give, found by the last Problem.

2. Divide the square of the remainder in feet by 42056462 feet, and the quotient will be the height required.

Being on my return from a foreign voyage, and finding by my reckoning I was about $5\frac{1}{2}$ leagues from Boston lighthouse, it being in the dusk of the evening, with my telescope I descried the lamp of the light house in the horizon, at which time my eye was elevated 6 feet above the surface of the water: Now, supposing my reckoning to be true; What is the height of the light-house above the water?

$5\frac{1}{2}$ leagues = 16,5 miles; then $16,5 - \sqrt{42056462 + 6 \times 6} = 13,943$ miles = 73619 feet nearly, and $73619 \times 73619 \div 42056462 = 129$ feet nearly, *Ans.*

MISCELLANEOUS QUESTIONS, with the METHOD of SOLUTION.

1. What part of 9d. is $\frac{2}{3}$ of 7d.?

$$\frac{2}{3} \text{ of } \frac{7}{1} = \frac{14}{3}, \text{ and } \frac{9}{1} \div \frac{14}{3} = \frac{1 \times 14}{9 \times 5} = \frac{14}{45} \text{ } \textit{Ans.}$$

2. What number is that, from which $\frac{3}{7}$ being taken, the remainder will be $\frac{1}{5}$?

$$\frac{1}{5} + \frac{3}{7} = \frac{1 \times 7 + 3 \times 5}{5 \times 7} = \frac{22}{35} \text{ } \textit{Ans.}$$

3. What number is that, to which if $\frac{3}{7}$ of $\frac{12}{5}$ of $\frac{129}{313}$ be added, the total will be 1?

$$\frac{3}{7} \text{ of } \frac{12}{5} \text{ of } \frac{129}{313} = \frac{4644}{10955}, \text{ and } \frac{1}{1} - \frac{4644}{10955} = \frac{1 \times 10955 - 1 \times 4644}{1 \times 10955} = \frac{6311}{10955} \text{ } \textit{Ans.}$$

4. What number is that, of which $19\frac{3}{13}$ is $\frac{5}{7}$?

$$19\frac{3}{13} = \frac{250}{13}; \text{ then, As } \frac{5}{13} : \frac{250}{13} :: \frac{7}{1} : 26\frac{12}{13} \text{ } \textit{Ans.}$$

5. In

5. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plumbs, 60 of them peaches, and 40 cherries ; How many trees does the orchard contain ?

$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$, and $\frac{12}{12} - \frac{11}{12} = \frac{1}{12}$; therefore, As $\frac{1}{12} : \frac{60+40}{1}$:: $\frac{12}{12} : 1200$ *Ans.*

6. A person, who was possessed of $\frac{2}{5}$ of a vessel, sold $\frac{3}{8}$ of his interest for 375*l.* ; What was the ship worth at that rate ?

$\frac{5}{8}$ of $\frac{2}{5} = \frac{1}{4}$. As $\frac{1}{4} : 375 :: \frac{1}{1} : \text{£} 1500$ *Ans.*

7. If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of the cargo, valued at 1000*l.* ; What did both ship and cargo cost ?

$\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5} = \frac{6}{28}$, and $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of $\frac{1000}{1} = \frac{7000}{39}$, then,
As $\frac{6}{28} : \frac{7000}{39} :: \frac{28}{8} : \frac{28 \times 7000 \times 28}{6 \times 39 \times 8} = \text{£} 837 \text{ } 12s. \text{ } 12\frac{5}{9}d.$ the cost of the ship ; and $\text{£} 1000 + \text{£} 837 \text{ } 12s. \text{ } 12\frac{5}{9}d. = \text{£} 1837 \text{ } 12s. \text{ } 12\frac{5}{9}d.$ value of the ship and cargo, *Ans.*

8. Two ships, A and B, sailed from a certain port at the same time ; A sailed north 8 miles an hour, and B east 6 miles an hour ; Required, by an easy method, their distance asunder at every hour's end ?

$\sqrt{8 \times 8 + 6 \times 6} = 10$ miles distant in 1 hour, and $10 \times 2 = 20$ miles in 2 hours, &c. *Ans.*

9. If a body be weighed in each scale of a balance, whose beam is unequally divided, and those different weights of the body be multiplied together, the square root of the product will be the true weight of that body.

Suppose the weight of a bar of silver, in one scale, to be 10oz. and in the other scale 12oz. ; Required the true weight of the bar ?

$\sqrt{12 \times 10} = 10,954 = 10 \text{ } 19 \text{ } 1,92$ *Ans.*
oz. oz. pwt. gr.

10. A younger brother received 156*ol.* which was just $\frac{7}{12}$ of his elder brother's fortune ; and $5\frac{3}{8}$ times the elder's money, was $1\frac{2}{3}$ the value of the father's estate ; Pray, what was the father worth ?

As $7 : 1560 :: 12 : 2674\frac{2}{3}$ the elder brother's fortune ; then, $2674\frac{2}{3} \times 5\frac{3}{8} \div 1\frac{2}{3} = \text{£} 8624 \text{ } 11s. \text{ } 5\frac{1}{7}d.$ *Ans.*

11. A gentleman divided his fortune among his sons, giving A 9*l.* as often as B 5*l.* and to C but 3*l.* as often as to B 7*l.* and yet C's dividend was 1537*l.* ; What did the whole estate amount to ?

As $7 : 5 :: 3 : 2\frac{1}{7}$; then, As $2\frac{1}{7} : 1537\frac{5}{8} :: 9+5+2\frac{1}{7} : \text{£} 11583 \text{ } 8s. \text{ } 10d.$ *Ans.*

12. A gentleman left his son a fortune : $\frac{5}{16}$ of which he spent in 3 months ; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only 537*l.* left ; Pray, what did his father bequeath him ?

$\frac{1}{16} =$ whole legacy, $\frac{1}{16} - \frac{5}{16} = \frac{11}{16}$ left at 3 months, then, $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{16} = \frac{165}{384}$, and $\frac{11}{16} - \frac{165}{384} = \frac{1584}{384} = \text{£} 537$, therefore, As $\frac{1584}{384} : \frac{537}{1} :: \frac{1}{1} : \text{£} 2082 \text{ } 18s. \text{ } 2\frac{1}{4}d.$ *Ans.*

13. A gay young fellow soon got the better of $\frac{2}{7}$ of his fortune; he then gave 1500*l.* for a commission, and his profusion continued till he had but 450*l.* left, which he found to be just $\frac{6}{16}$ of his money, after he had purchased his commission; What was his fortune at first?

As 6 : 450 :: 16 : 1200, and $1200 + 1500 = \text{£}2700 = \frac{2}{7}$ of his fortune, and, As 5 : 2700 :: 7 : $\text{£}3780$ *Ans.*

14. A merchant begins the world with 1500*l.* and finds that by his distillery he clears 1500*l.* in 7 years; by his navigation 1500*l.* in 9 years, and that he spends in gaming 1500*l.* in $3\frac{1}{2}$ years; How long will his estate last?

As $\left\{ \begin{array}{l} 7 : 1500 :: 1 : 214\frac{2}{7} \\ 9 : 1500 :: 1 : 166\frac{2}{3} \\ 3\frac{1}{2} : 1500 :: 1 : 428\frac{4}{7} \end{array} \right\}$ As $428\frac{4}{7} - 214\frac{2}{7} + 166\frac{2}{3} : 1 :: 1500 : 31\frac{1}{2}$ *Y. £. Y.*

15. A has 100*l.* of B's money in his hands, for the remittance of which B allows him 9 per cent.; What sum must he remit, to discharge himself of the 100*l.*?

As $100 + 9 : 100 :: 100 : \text{£}91\frac{81}{109}$; or, $\frac{100 \times 100}{100 + 9} = \text{£}91\frac{81}{109}$ to be remitted, and $\frac{100 \times 9}{100 + 9} = \text{£}8\frac{28}{109}$ his commission.

16. Said Harry to Edmund, I can place four 1s so that, when added, they shall make precisely 12; Can you do so too?

17. A and B are on opposite sides of a circular field 268 poles about; they begin to go round it, both the same way, at the same instant of time; A goes 22 rods in 2 minutes, and B 34 rods in 3 minutes; How many times will they go round the field, before the swifter overtakes the slower?

Pol. min. Pole. min. Pol. min. Pol. min.

As $22 : 2 :: 1 : \frac{1}{11}$ and, As $34 : 3 :: 1 : \frac{3}{34}$, then, $\frac{1}{11} - \frac{3}{34} = \frac{1}{374}$ minutes.

Pol. min. Pol. min. Time. Round. Time.

As $1 : \frac{1}{374} :: 22 : \frac{11}{187}$, As $\frac{11}{187} : \frac{1}{1} :: \frac{1}{1} : 17$ times round, *Ans.*

18. If 15 men can perform a piece of work in 11 days; How many men will accomplish another piece of work four times so large, in a fifth part of the time?

Work. Men. Works. Men. Time. Men. Time. Men.

As $1 : 15 :: 4 : 60$ As $\frac{1}{1} : \frac{60}{1} :: \frac{1}{5} : 300$ *Ans.*

19. If A can do a piece of work alone in 7 days, and B in 12; let them both about it together; In what time will they finish it?

Days. Work. Day. Work.

As $\left\{ \begin{array}{l} 7 : 1 :: 1 : \frac{1}{7} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array} \right\}$ Then, $\frac{1}{7} + \frac{1}{12} = \frac{19}{84}$ As $\frac{19}{84} : \frac{1}{1} :: \frac{1}{1} : 4\frac{8}{19}$ *Ans.*

20. A and B together can build a boat in 20 days; with the assistance of C they can do it in 12; In what time would C do it by himself?

As

D. W. D. W. W. W. W. D. W. D.

As $\left\{ \begin{array}{l} 20 : 1 :: 1 : \frac{1}{20} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array} \right\}$ Then, $\frac{1}{12} - \frac{1}{20} = \frac{8}{240}$, & As $8 : 1 :: 240 : 30$ *Ans.*

21. A can do a piece of work alone in 13 days, and A and B together in 8 days ; In what time can B do it alone ?

D. W. D. W. W. W. W. D. W. D.

As $\left\{ \begin{array}{l} 13 : 1 :: 1 : \frac{1}{13} \\ 8 : 1 :: 1 : \frac{1}{8} \end{array} \right\}$ Then, $\frac{1}{8} - \frac{1}{13} = \frac{5}{104}$, and, As $5 : 1 :: 104 : 20\frac{4}{5}$

22. A, B, and C, can complete a piece of work in 12 days ; A can do it alone in 23 days, and B in 37 days ; In what time can C do it by himself ?

D. W. D.

As $\left\{ \begin{array}{l} 12 : 1 :: 1 : \frac{1}{12} \\ 23 : 1 :: 1 : \frac{1}{23} \\ 37 : 1 :: 1 : \frac{1}{37} \end{array} \right\}$ Then, $\frac{1}{12} - \frac{1}{23} + \frac{1}{37} = \frac{131}{10212}$
 W. D. W. As $131 : 1 :: 10212 : 77\frac{125}{131}$ days, *Ans.*

23. A cistern, for water, has two cocks to supply it ; by the first it may be filled in 45 minutes, and by the second, in 55 minutes ; it has likewise a discharging cock, by which it may, when full, be emptied in 30 minutes ; Now, if these three cocks be all left open when the water comes in, in what time will the cistern be filled ?

Min. Cist. Min. Cist.

45 : 1 :: 60 : 1,3333

55 : 1 :: 60 : 1,0909

Cist. Hour. Cist. h. m. s.

As, $4242 : 1 :: 1 : 2\ 21\ 26\frac{1}{2}$ *Ans.*

2,4242

30 : 1 :: 60 : 2

Gains in an hour, 4242 of a cistern.

24. A water tub holds 73 gallons ; the pipe which conveys the water to it, usually admits 7 gallons in 5 minutes ; and the tap discharges 20 gallons in 17 minutes ; Now, supposing these both to be carelessly left open, and the water to be turned on at 4 o'clock in the morning ; a servant, at 6, finding the water running, puts in the tap ; In what time, after this accident, will the tub be full ?

Min. Gal. Min. Gal.

As $\left\{ \begin{array}{l} 5 : 7 :: 60 : 84 \\ 17 : 20 :: 60 : 70\frac{10}{17} \end{array} \right\}$ $84 - 70\frac{10}{17} \times 2 = 26\frac{14}{17}$ gal. & $73 - 26\frac{14}{17} = 46\frac{3}{17}$ gal. which now remain to be filled.

Gal. Min. Gal. M. s.

Therefore, As $7 : 5 :: 46\frac{3}{17} : 32\ 58\frac{118}{119}$, and therefore the tub will be full at $32\ 58\frac{118}{119}$ after 6.

25. A has a chest of tea, weighing $3\frac{1}{2}$ cwt. the prime cost of which is 60*l.* ; now, allowing interest at 6 per cent. per annum,

Ho.

356 MISCELLANEOUS QUESTIONS.

How must he rate it per lb to B, so that by taking his note of hand, payable at 6 months, he may clear 50 dollars by the bargain?

Intérest £2 5s. Then, As $3\frac{1}{2}$ cwt. : £60 + £15 + £2 5s. :: 1 lb : 3s. $11\frac{2}{3}$ d. *Ans.*

26. Suppose the American continental debt to be 18 millions; What annuity, at 6 per cent. per annum, will discharge it in 25 years?

By Table 5, of annuities, p. 326, .07823 is the annuity, which 1l. will purchase in 25 years, then, $.07823 \times 18000000 = \text{£}1408140$ *Ans.*

The annual interest of the debt = 1080000

Therefore, there must be a sinking fund of £328140 *pr. ann.*

27. The hour and minute hands of a watch are exactly together at 12 o'clock; When are they next together?

The velocities of the two hands of a watch, or clock, are to each other, As 12 to 1; therefore, the difference of velocities is $12 - 1 = 11$.

h. m. s.

As 11 : 1 :: $\left\{ \begin{array}{l} 12 \times 1 : 1 \quad 5 \quad 27\frac{3}{11} \\ 12 \times 2 : 2 \quad 10 \quad 54\frac{6}{11} \\ 12 \times 3 : 3 \quad 16 \quad 21\frac{9}{11} \end{array} \right\}$ *Ec. Ans.*

28. A hare starts 12 rods before a hound; but is not perceived by him till she has been up 45 seconds; she scuds away at the rate of 10 miles an hour, and the dog, on view, makes after, at the rate of 16 miles an hour; How long will the course hold, and what space will be run over, from the spot where the dog started?

$\frac{16-10}{16} = \frac{6}{16} = \frac{3}{8}$, or, as 8 to 3 against the hare, 1 hour = 3600 seconds.

Sec.	Feet.	Sec.	Feet.	10 miles = 52800 feet.
As 3600	: 52800	:: 45	: 660	distance the hare had run before the
		Add 12 rods = 198		(dog discovered her.

858 = the distance of the hare when the
8 dog started.

3)6864

Feet 2288 = the ground run over by the dog.

Miles. Feet. Sec. Feet. Sec.

Now, As 16 = 84480 : 3600 :: 2288 : $97\frac{1}{2}$

29. In a series of proportional numbers, the first is 4, the third 12, and the product of the second and third is 112,8; What is the difference of the second and fourth?

$112,8 \div 12 = 9,4$ the second. As 4 : 9,4 :: 12 : 28,2, and $28,2 - 9,4 = 18,8$ *Ans.*

30. A fellow said that when he counted his nuts, two by two, three by three, four by four, five by five, and six by six, there was

was still an odd one ; but when he told them seven by seven, they came out even ; How many had he ?

$2 \times 3 \times 4 \times 5 \times 6 = 720$, and $720 + 1 \div 7 = 103$ even, *Ans.* 721.
 $\frac{721}{2.3.4.5}$ and 6 respectively, will leave an odd one.

31. There is an island, 50 miles in circumference, and 3 men start together to travel the same way about it : A goes 7 miles per day, B 8, and C 9 ; when will they all come together again, and how far will each travel ?

$50 \times 7 + 50 \times 8 + 50 \times 9 \div 7 + 8 + 9 = 50$ days.—A 350 miles, B 400, and C 450, *Ans.*

32. Suppose A leaves Newburyport at 6 o'clock on Monday morning, and travels towards Providence, at the rate of 4 miles per hour without intermission ; and that, at 3 in the afternoon, B sets out from Providence for Newburyport, and travels constantly at the rate of 7 miles an hour ; Now, suppose the distance between the two towns to be 90 miles ; whereabouts on the road will they meet ?

$6 + 3 = 9$ hours, and $9 \times 4 = 36$ miles, the time and distance A had travelled before B started. Then $90 - 36 = 54$ miles remain to be travelled by both ; now, as both together lessen the distance $7 + 4 = 11$ miles an hour, therefore $\frac{4}{11}$ of $54 + 36 = 55\frac{7}{11}$ miles from Newburyport ; which is near Ames's at Dedham.

33. If, during ebb tide, a wherry should set out from Haverhill to come down the river, and, at the same time, another should set out from Newburyport, to go up the river, allowing the distance to be 18 miles ; suppose the current forwards one and retards the other $1\frac{1}{2}$ mile per hour ; the boats are equally laden, the rowers equally good, and, in the common way of working in still water, would proceed at the rate of 4 miles per hour ; Where, in the river, will the two boats meet ?

M. M. M. M. M. M. M.
 $4 + 1\frac{1}{2} = 5\frac{1}{2}$, and $4 - 1\frac{1}{2} = 2\frac{1}{2}$, then, $5\frac{1}{2} + 2\frac{1}{2} = 8$ in one hour
M. H. M. H. M. H. M.
by both. As $8 : 1 :: 18 : 2\frac{1}{4}$, then $5\frac{1}{2} \times 2\frac{1}{4} = 12\frac{3}{8}$ from Haverhill, and $2\frac{1}{2} \times 2\frac{1}{4} = 5\frac{5}{8}$ from Newburyport.

34. A gentleman making his addresses in a lady's family, who had 5 daughters ; she told him that their father had made a will, which imported that the first four of the girls' fortunes were, together, to make 50000*l.* ; the last four 60000*l.* ; the three last with the first 60000*l.* the three first with the last 50000*l.* and the two first with the two last 64000*l.* which, if he would unravel, and make it appear what each was to have, as he appeared to have a partiality for Harriet, her third daughter, he should be welcome to her ; Pray, what was Miss Harriet's fortune ?

358 MISCELLANEOUS QUESTIONS.

$$\begin{array}{rcl}
 A+B+C+D & = & 50000 \\
 B+C+D+E & = & 66000 \\
 A+C+D+E & = & 60000 \\
 A+B+C+E & = & 56000 \\
 A+B+D+E & = & 64000 \\
 \hline
 & = & 296000
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Then, } 296000 \div 4 \text{ the number of} \\ \text{combinations} = 74000 \text{ the sum of} \\ \text{their fortunes.} \\ \text{Then, } A+B+C+D+E = 74000 \\ \text{And } A+B+D+E = 64000 \end{array}$$

Ans. Harriet's fortune = £10000

35. Three persons purchase a vessel in company, towards the payment whereof A advanced $\frac{2}{5}$, B $\frac{3}{7}$, and C, 256*l.*; What did A and B pay, each, and what part of the vessel had C?

$$\frac{2}{5} + \frac{3}{7} = \frac{14+15}{35} = \frac{29}{35}, \text{ and } \frac{35}{35} - \frac{29}{35} = \frac{6}{35} \text{ C's part of the vessel.}$$

$$\text{As } \frac{6}{35} : 256 :: \left\{ \begin{array}{l} \frac{14}{35} : £597 \text{ 6s. 8d. A advanced.} \\ \frac{15}{35} : £640 \text{ B advanced.} \end{array} \right.$$

36. A and B cleared, by an adventure at sea, 45 guineas, which was 35*l.* per cent. upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; What money did each adventure?

As £35 : 100 :: 45 *Guin.* : £180 = the whole adventure.

$$\text{As } 11+8 : 180 :: \left\{ \begin{array}{l} 11 : £104 \text{ 4s. } 2\frac{10}{19}\text{d. A's.} \\ 8 : £75 \text{ 15s. } 9\frac{10}{19}\text{d. B's.} \end{array} \right.$$

37. A, B and C are to share 100*l.* in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively; but C dying, it is required to divide the whole sum properly between the other two?

$$\text{As } \frac{1}{3} + \frac{1}{4} + \frac{1}{5} : 100 :: \left\{ \begin{array}{l} \frac{1}{3} : 42\frac{26}{47} \text{ A's.} \\ \frac{1}{4} : 31\frac{23}{47} \text{ B's.} \\ \frac{1}{5} : 25\frac{25}{47} \text{ C's.} \end{array} \right.$$

$$\text{Again, As } \frac{1}{3} + \frac{1}{4} : 25\frac{25}{47} :: \frac{1}{3} : 14\frac{582}{947}$$

$$\text{Then, } 42\frac{26}{47} + 14\frac{582}{947} = £57 \text{ 2s. } 10\frac{1}{4}\text{d. A's share.}$$

$$\text{And } £100 - £57 \text{ 2s. } 10\frac{1}{4}\text{d.} = £42 \text{ 17s. } 1\frac{3}{4}\text{d. B's share.}$$

38. A, B and C have among them 135 guineas; A's+B's are to B's+C's, as 5 to 7, and C's—B's to C's+B's as 1 to 7; How many had each?

$$\text{--- } A+B. \quad A+C.$$

Suppose A's+B's = 50; then, as 5 to 7 :: 50 : 70; As 7 : 1 :: 70 : 10 = C's—B's; then, 70—10 = 60, and 60 ÷ 2 = 30 = B's, 30—10 = 20 = A's, and 30+10 = 40 = C's, by the supposition: Now 20+30+40 = 90, which should have been 135, therefore,

$$\text{As } 90 : 135 :: \left\{ \begin{array}{l} 20 : 30 = \text{A's.} \\ 30 : 45 = \text{B's.} \\ 40 : 60 = \text{C's.} \end{array} \right.$$

Sum = 135 proof. 39. There

39. There are three horses, belonging to different men, employed as a team to draw a load of salt from Newburyport to Boston for 2*l.* 10*s.* A and B are supposed to do $\frac{3}{11}$ of the work ; A and C $\frac{5}{13}$, and B and C $\frac{4}{14}$ of it ; they are to be paid proportionally ; Can you divide it as it should be ?

$$\left. \begin{array}{l} A+B = \frac{3}{11} = ,2727 \\ A+C = \frac{5}{13} = ,3846 \\ B+C = \frac{4}{14} = ,2857 \end{array} \right\}$$

$$\text{Sum} = ,943$$

$$\text{And } ,943 \div 2, \text{ the number combined} = ,4715 = A+B+C$$

$$\quad \quad \quad - ,2727 = A+B$$

Then, As $,4715 : 50 :: ,1988 : 51 \text{ } 1*s.* 0\frac{1}{2}d. = ,1988 = C.$

And in the same manner proceed for the rest.

40. I would put 20 hogheads of London beer into 10 wine pipes, and desire to know what the cask must contain, which will receive the difference, 231 solid inches being the wine gallon, and 282 that of beer ?

$$\begin{array}{l} \text{Beer hhd.} = 54 \text{ gal. and } 54 \times 282 \times 20 = 304560 \text{ solid inches.} \\ \text{Wine pipe.} = 126 \text{ gal. and } 126 \times 231 \times 10 = 291060 \text{ solid inches,} \\ \text{and } \frac{304560 - 291060}{282} = 47\frac{4}{7} \text{ beer gallons, } \textit{Ans.} \end{array}$$

41. Being about to plant 5292 trees equally distant in rows, the length of the grove is to be 3 times the breadth ; How many of the shorter rows will there be ?

$$\sqrt{\frac{5292}{3}} \times 3 = 126 \text{ rows, } \textit{Ans.} \text{ viz. } \frac{1}{3} \text{ of the trees are to form an exact square, the side whereof being 42, shews how many come into a short row.}$$

42. A general, disposing his army into a square battalion, found he had 231 over and above ; but increasing each side with one soldier, he wanted 44 to fill up the square ; How many men did his army consist of ?

$$231 + 44 = 275, \text{ and } \sqrt{275 - 1} \div 2 = 137, \text{ then } 137 \times 137 + 231 = 19000 \textit{ Ans.}$$

43. I want the length of a shoar, the bottom of which, being set 9 feet from the perpendicular side of a house, will support a weak place in the wall $22\frac{1}{2}$ feet from the ground ?

$$\sqrt{22,5 \times 22,5 + 9 \times 9} = 24 \text{ feet, } 2\frac{3}{4} \text{ inches, } \textit{Ans.}$$

44. A line 35 yards long will exactly reach from the top of a fort, standing on the brink of a river, known to be 27 yards broad, to the opposite bank ; What is the height of the wall ?

$$\sqrt{35 \times 35 - 27 \times 27} = 22 \text{ yards, } 9\frac{1}{2} \text{ inches, nearly.}$$

45. Suppose a light house built on the top of a rock ; the distance between the place of observation and that part of the rock level

level with the eye 620 yards; the distance from the top of the rock to the place of observation 846 yards, and from the top of the light house 900 yards: The height of the light house is required?

$$\sqrt{900 \times 900 - 620 \times 620} - \sqrt{846 \times 846 - 620 \times 620} = 76,77 \text{ yds. } \text{Ans.}$$

46. *The sum and difference of the squares of two numbers given, to find those numbers.*

RULE.—From the sum take the difference, and half the remainder is the square of the less, which, taken from the sum of the squares, will give the square of the greater.

A and B have between them a number of guineas, which are to be so divided, that the sum of their squares may be 208, and the difference of their squares 80; supposing A's the greater number, How many has he more than B?

$$208 - 80 \div 2 = 64 \text{ the square of B's, and } 208 - 64 = 144 \text{ the square of A's; therefore } \sqrt{144} - \sqrt{64} = 4 \text{ Ans.}$$

47. *Having the sum of two numbers, and the sum of their squares given, to find those numbers.*

RULE.—From the square of their sum take the sum of their squares: Then from the sum of their squares take this remainder, and the square root of the difference will be the difference of the two numbers. To half their sum add half their difference, and the sum will be the greater. From half the sum take half their difference, and the remainder will be the less.

A and B have 50 guineas between them, which are to be so divided, as that the sum of the squares of the two numbers shall be 1300; How many had each, supposing A to have the greater number?

$$50 \times 50 - 1300 = 1200; \text{ Then } \sqrt{1300 - 1200} = 10 \text{ difference.}$$

$$\text{Now } 50 \div 2 + 10 \div 2 = 30 = \text{A's. And } 50 \div 2 - 10 \div 2 = 20 = \text{B's, } \text{Ans.}$$

48. *Having the difference of two numbers, and the sum of their squares given, to find those numbers.*

RULE.—From the sum of their squares take the square of their difference: To the sum of the squares add the remainder, and the square root of this sum will be the sum of the required numbers; then, with the half sum and half difference proceed as in the last question.

A number of guineas are to be divided between A and B, in such a manner that A may have 50 more than B, and that the sum of the squares of the respective shares may be 12500; What number had each?

$$12500 - 50 \times 50 = 10000, \text{ and } \sqrt{12500 + 10000} = 150 = \text{sum of their shares.}$$

$$\text{Then, } 150 \div 2 + 50 \div 2 = 100 \text{ A's; and } 150 \div 2 - 50 \div 2 = 50 \text{ B's, } \text{Ans.}$$

49. *Having the sum of the squares of two numbers, and the square of their half sum given, to find those numbers.*

RULE.

RULE.—From the sum of the squares take *twice* the square of the half sum, and the square root of half the remainder will be their half difference, with which, and the half sum proceed as before directed.

Let the sum of the squares of two numbers, be 3161, and the square of their half sum 1560,25 ; Required those numbers ?

$3161 - 1560,25 \times 2 = 40,5$ $40,5 \div 2 = 20,25$, and $\sqrt{20,25} = 4,5 = \frac{1}{2}$ difference, and $\sqrt{1560,25} = 39,5 = \frac{1}{2}$ sum ; then, $39,5 + 4,5 = 44$ the greater, and $39,5 - 4,5 = 35$ the less, *Ans*.

50. 1. *If the quantity of matter, (or weights) of any two bodies, put in motion, be equal, the force by which they are moved will be in proportion to their velocities, or swiftness of motion.*

2. *If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them, that is, as their weights.*

3. *If both the quantities of matter and the velocities be unequal, the forces, with which the bodies are moved, will be in a proportion compounded of their quantities of matter and velocities.*

Suppose the battering ram of Vespasian weighed 60000℔ ; that it was moved at the rate of 24 feet in one second, and that this was sufficient to demolish the walls of Jerusalem ; With what velocity must a cannon ball, which weighs 42 ℔, be moved, to do the same execution ?

The velocity of the ram being 24, and the weight of the ball 42, compounded, will make a fraction $= \frac{24}{42} = \frac{4}{7}$, and $\frac{4}{7} \times 60000 = 34285\frac{5}{7}$ feet in a second, *Ans*.

51. A body weighing 30 ℔ is impelled by such a force as to send it 20 rods in a second ; With what velocity would a body weighing 12 ℔ move, if it were impelled by the same force ?

$$\frac{30 \times 20}{12} = 50 \text{ rods in a second, } \textit{Ans}.$$

Of GRAVITY.

52. *The gravity of bodies above the surface of the earth decreases in a duplicate ratio (or as the squares of their distances) in semidiameters of the earth, from the earth's centre.*

Supposing a body to weigh 400 ℔ at 2000 miles above the earth's surface ; What would it weigh at the surface, estimating the earth's semidiameter at 4000 miles ?

From the centre to the given height being $1\frac{1}{2}$ semidiameter ; multiply the square of $1\frac{1}{2}$ by the weight, and the product will be the answer.

$$1,5 \times 1,5 \times 400 = 900 \text{ ℔, } \textit{Ans}.$$

53. If a body weigh 900 ℔ at the surface of the earth ; What will it weigh at 2000 miles above the surface ?

This being the reverse of the last, therefore, $1 \div 1,5 = 1,5$ and $900 \div 1,5 \times 1,5 = 400 \text{ ℔, } \textit{Ans}.$

362 MISCELLANEOUS QUESTIONS.

54. A certain body on the surface of the earth, weighs 180 lb ; How high must it be carried, to weigh but 20 lb ?

$\sqrt{180 \div 20} = 3$, *Ans.* 3 semidiameters from the earth's centre, that is 8000 above its surface.

55. How high must a ball be raised, to lose *half* its weight ?

As $1 : 4000 \times 4000 :: 2 : 32000000$, and $\sqrt{32000000} = 5656,85$, and $5656,85 - 4000 = 1656,85$ miles, *Ans.*

56. If the attraction of the moon raise a tide on the earth 5 feet high ; What will be the height of a tide, raised by the earth on the surface of the moon under similar circumstances ?

The attraction of one of those bodies to the other's surface is directly as its quantity of matter, and inversely as its diameter ; therefore, As $2182 \times 2182 \times 2182 \times 494 : 5 :: 8000 \times 8000 \times 8000 \times 400 : 199,3$, And as $2182 : 199,3 :: 8000 : 54,8$ feet, inversely, *Ans.*

57. 1. If the diameters of two globes be equal, and their densities, (compactness, or closeness) different ; the weight of a body on their surfaces will be as their densities.

2. If their densities be equal, and their diameters different ; the weight of a body will be as $\frac{1}{3}$ of their circumferences.

3. If their diameters and densities be both different, the weight will be as $\frac{2}{3}$ of their semidiameters multiplied by their densities.

If a stone weigh 100 lb at the surface of the earth ; What will it weigh at the surfaces of the sun, and the several planets, whose densities are known, respectively ?

	Sun.	Jupiter.	Saturn.	Earth.	Moon.
Their densities	100.	94,5.	67.	400.	494
Diam.in geog.miles	776970.	135079.	98566.	6875.	1869,25
Semidiam.	388485.	67539,5.	49283.	3437,5.	934,67
$\frac{2}{3}$ Sem. Diam.	258990.	44693.	32855,33.	2295.	623,11
Then, As $2295 \times 400 : 100 ::$	$\left\{ \begin{array}{l} 258990 \times 100 : 2821,24 \text{ lb at the Sun.} \\ 44693 \times 94,5 : 460 \text{ lb at Jupiter.} \\ 32855,33 \times 67 : 239,8 \text{ lb at Saturn.} \\ 623,11 \times 494 : 33,5 \text{ at the Moon.} \end{array} \right.$				

OF THE FALL OF BODIES.

58. Heavy bodies near the surface of the earth, fall one foot the first quarter of a second ; three feet the second quarter ; five feet in the third, and seven feet in the fourth quarter ; that is, 16 feet in the first second.*

The velocities, acquired by bodies in falling, are in proportion to the squares of the times in which they fall ; for instance,
Let

* The exact velocity in *vacuo* is 16,1 in the second ; but in the air it will be scarcely 16 feet.

Let go three bullets together; stop the first at one second, and it will have fallen 16 feet. Stop the next at the end of the second second, and it will have fallen ($2 \times 2 = 4$) four times 16, or 64 feet; and stop the last at the end of the third second, and the distance fallen will be ($3 \times 3 = 9$) nine times 16, or 144 feet, and so on.

Or, which is the same, the space fallen through (in feet) is always equal to the square of the time in 4ths of a second.

Or, By multiplying 16 feet by so many of the odd numbers, beginning at unity, as there are seconds in any given time; viz. by 1 for the first second, by 3 for the second, by 5 for the third, and so on, these several products will give the spaces fallen through, in each of the several seconds, and their sum will be the whole distance fallen.

The velocity given, to find the space fallen through.

RULE 1.—The square root of the feet, in the space fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall; therefore,

2. Divide the velocity by 8, and the square of the quotient will be the distance fallen through, to acquire that velocity.

Suppose the velocity of a cannon ball to be about $\frac{1}{8}$ of a mile, or 660 feet per second; From what height must a body fall, to acquire the same velocity per second?

$$660 \div 8 = 82,5 \text{ and } 82,5 \times 82,5 = 6806\frac{1}{4} \text{ feet, } = 1\frac{37}{128} \text{ mile, } \text{Ans.}$$

59. *The time given, to find the space fallen through.*

RULE 1.—The square root of the feet, in the space fallen through, will ever be equal to four times the number of seconds the body has been falling: Therefore,

2. Multiply the time by 4, and the square of the products will be the space fallen through in the given time.

How many feet will a body fall in 5 seconds?

$$5 \times 4 = 20, \text{ and } 20 \times 20 = 400 \text{ feet, } \text{Ans.}$$

60. A bullet is dropped from the top of a building, and found to reach the ground in $1\frac{1}{4}$ second; Required its height?

$$1,75 \times 4 = 7, \text{ and } 7 \times 7 = 49 \text{ feet, } \text{Ans.} \quad \text{Or, } 1\frac{1}{4} = 7 \text{ qrs. and } 7 \times 7 = 49. \quad \text{Or, } 1,75 \times 1,75 \times 16 = 49 \text{ feet, } \text{Ans.}$$

61. What is the difference between the depth of two wells, into each of which should a stone be dropped in the same instant, one would reach the bottom in 5 seconds, and the other in 3?

$$5 \times 4 = 20, \text{ and } 20 \times 20 = 400 \text{ feet.}$$

$$3 \times 4 = 12, \text{ and } 12 \times 12 = 144 \text{ feet.}$$

Ans. 256 feet.

62. Ascending bodies are retarded in the same ratio that descending bodies are accelerated; therefore, if a ball, discharged from

from a gun, returned to the earth in 12 seconds ; How high did it ascend ?

The ball being half of the time, or 6 seconds, in its ascent, therefore, $6 \times 4 = 24$, and $24 \times 24 = 576$ feet, *Anf.*

63. *The velocity per second given, to find the time.*

RULE 1.—Four times the number of seconds, in which a body has been falling, is equal to one eighth of the velocity, in feet, per second, acquired at the end of the fall : Therefore,

2. Divide the given velocity by 8, and one fourth part of the quotient will be the answer.

How long must a bullet be falling to acquire a velocity of 160 feet per second ?

$$160 \div 8 = 20, \text{ and } 20 \div 4 = 5 \text{ seconds, } \textit{Ans.}$$

64. *The space, through which a body has fallen, given, to find the time it has been falling.*

RULE 1.—Four times the number of seconds, in which the body has been falling, will ever be equal to the square root of the space, in feet, through which it has fallen : Therefore,

2. Divide the square root of the space fallen through by 4, and the quotient will be the time, in which it was falling.

In how many seconds will a bullet fall through a space of 10125 feet ?

$$\sqrt{10125} = 100,6, \text{ and } 100,6 \div 4 = 25,15 \text{ seconds, } = 25'' 9''' \textit{ Ans.}$$

65. In what time will a musket ball, dropped from the top of a steeple 484 feet high, come to the ground ?

$$\sqrt{484} = 22, \text{ and } 22 \div 4 = 5\frac{1}{2} \text{ seconds, } \textit{Ans.}$$

66. *To find the velocity per second, with which a heavy body will begin to descend, at any distance from the earth's surface.*

RULE.—As the square of the earth's femidiameter is to 16 feet : So is the square of any other distance from the earth's centre inversely, to the velocity with which it begins to descend per second.

With what velocity per second will an iron ball begin to descend, if raised 3000 miles above the earth's surface ?

$$\text{As } 4000 \times 4000 : 16 :: 4000 + 3000 \times 4000 + 3000 : 5,2249 \text{ feet, } \textit{Ans.}$$

67. How high must a ball be raised above the earth's surface, to begin to descend with a velocity of 5,2249 feet per second ?

$$\text{As } 16 : 4000 \times 4000 :: 5,2249 : 49000000, \text{ and } \sqrt{49000000} = 7000, \\ \text{Wherefore, } 7000 - 4000 = 3000 \text{ miles, } \textit{Ans.}$$

68. *To find the mean velocity of a falling body.*

RULE.—Divide the space fallen through by the number of seconds it was falling, and the quotient will be the mean velocity.

A musket ball dropped from the top of a steeple 484 feet high in $5\frac{1}{2}$ seconds ; Required its mean velocity ?

$$484 \div 5,5 = 88 \text{ feet per second, } \textit{Ans.}$$

69. *To*

69. To find the velocity acquired by a falling body per second (or by a stream of water, having the perpendicular descent given) at the end of any given period of time.

RULE 1.—The velocity acquired at the end of any period is equal to twice the mean velocity, with which it passed during that period.

Or, 2. Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity required.

If a ball fall through a space of 484 feet in $5\frac{1}{2}$ seconds; with what velocity will it strike?

By the former part of the rule.

$$484 \div 5.5 = 88, \text{ and}$$

$$88 \times 2 = 176, \text{ Ans.}$$

By the latter part, without regarding the time.

$$\sqrt{484 \times 64} = 176, \text{ Ans.}$$

70. There is a sluice, (or flume) one end of which is $2\frac{1}{2}$ feet lower than the other; What is the velocity of the stream per second?

$$2.5 \times 64 = 160, \text{ and } \sqrt{160} = 12.649 \text{ feet, Ans.}$$

71. The velocity with which a falling body strikes, given, to find the space fallen through.

RULE.—Divide the square of the velocity by 64, and the quotient will be the height required.

If a ball strike the ground with a velocity of 56 feet per second; from what height did it fall?

$$56 \times 56 \div 64 = 49 \text{ feet, Ans.}$$

72. The mean velocity of a fluid, or stream, is 12,649 feet per second; What is the perpendicular fall of the stream?

$$12,649 \times 12,649 \div 64 = 2\frac{1}{2} \text{ feet, Ans.}$$

73. The weight of a body, and the space fallen through, given, to find the force with which it will strike.

RULE.—The momentum, or force, with which a falling body strikes, is equal to its weight multiplied by its velocity; therefore, find the velocity by Problem 69th, and multiply it by the weight, which will produce the force required.

If the rammer, used for driving the piles of Charlestown bridge, weighed $2\frac{1}{4}$ tons, or 4500 lb, and fell through a space of 10 feet, with what force did it strike the pile?

$$\sqrt{10 \times 64} = 25.3 = \text{velocity, and } 25.3 \times 4500 = 113850 \text{ lb, momentum, Ans.}$$

74. The weight and momentum, or striking force, given, to find the space fallen through.

RULE.—Divide the momentum by the weight, and the quotient will be the velocity; then divide the square of the velocity by 64, and the quotient will be the space fallen through.

If the aforementioned rammer weighed 4500 lb, and struck with a force of 113850 lb; From what height did it fall?

$$113850 \div 4500 = 25.3, \text{ and } 25.3 \times 25.3 \div 64 = 10 \text{ feet, Ans.}$$

75. If

75. If it were required to know with what quantity of motion, momentum, or force, a fluid, moving with a given velocity, strikes upon a fixed obstacle.

RULE.—By Problem 71st, find the fall, which will produce the given velocity: Multiply that height by 62,5lb, Avoird. for clean river water, by 63lb, for dirty water, and by 64 for sea water.

Suppose a stream of clear water to move at the rate of 5 feet per second, and to meet with a fixed obstacle (or bulk head) 15 feet wide and 4 feet high; What is the momentary, instantaneous pressure of the stream?

$5 \times 5 \div 64 = \frac{25}{64}$ and $25 \div 64 = ,39$ of a foot, for the perpendicular fall of the water. Now $62,5 \times ,39 = 24,375\text{lb}$, the pressure upon each square foot, which, multiplied by 60 (the number of square feet in the obstacle) gives 1462,5lb, going with the given velocity of 5 feet per second; therefore, $1462,5 \times 5 = 7312,5\text{lb}$, *Ans*.*

76. The velocity of water, spouting through a sluice, or aperture in a reservoir, or a bulk head, is the same that a body would acquire by falling through a perpendicular space equal to that between the top of the water in the reservoir, and the aperture.

What is the velocity of water issuing from a head of 5 feet deep?

By Problem 69th. $64 \times 5 = 320$, and $\sqrt{320} = 18$ feet, nearly, *Ans*.

77. If the velocity of a stream issuing through the bulk head of a mill, be 16 feet per second; What head of water is there?

$16 \times 16 \div 64 = 4$ feet, *Ans*.

78. The quantity of water discharged from a hole in a vessel, is as the square root of the height of water above the aperture.

A miller has a head of water 4 feet above the sluice; How high must the water be raised above the opening, so that half as much again water may be discharged from the sluice in the same time?

$\sqrt{4} = 2$, and half as much again as 2, is $2 + 1 = 3$, for the square root of the required depth; therefore, $3 \times 3 = 9$ feet high, *Answer*.

Of PENDULUMS.

79. The time of a vibration, in the cycloid, is to the time of a heavy body's descent through half its length, as the circumference of a circle to its diameter, that is, As 3,1416 to 1: Therefore, (as a body descends freely, by gravity, through about 193,5 inches in the first second) To find the length of a pendulum, vibrating seconds.

RULE.

* Water being a yielding substance, loses two thirds of its power in producing effects.

RULE.—As $3,1416 \times 3,1416 : 1 \times 1 :: 193,5 : 19,6$ inches, the half length, and $19,6 \times 2 = 39,2$ inches, the length.

80. To find the length of a pendulum, that will swing any given time.

RULE.—Multiply the square of the seconds in any given time by 39,2, and the product will be the length required, in inches.

Required the lengths of several pendulums, which will respectively swing $\frac{1}{4}$ seconds, $\frac{1}{2}$ seconds, seconds, minutes, and hours?

$,25 \times ,25 \times 39,2 = 2,45$ inches for $\frac{1}{4}$ seconds. $,5 \times ,5 \times 39,2 = 9,8$ inches for $\frac{1}{2}$ seconds. $1 \times 1 \times 39,2 = 39,2$ inches for seconds, as above; $60 \times 60 \times 39,2 =$ the inches in 2 miles and 1200 feet, for minutes; and 1 hour = 3600 seconds, therefore $3600 \times 3600 \times 39,2 =$ the inches in 8018 miles and 96 feet, for hours, *Ans*.

81. What is the difference between the length of a pendulum, which vibrates half seconds, and one which swings 3 seconds?

$3 \times 3 \times 39,2 - ,5 \times ,5 \times 39,2 = 28\frac{7}{12}$ feet, *Ans*.

82. To find the time which a pendulum of any given length will swing.

RULE.—Divide the given length by 39,2, and the quotient will be the square of the time in seconds.

Or, As 6,261 (the square root of 39,2) is to the square root of the given length: So is 1 second, to the time of one oscillation: That is, divide the square root of the given length by 6,261, and the quotient will be the time of one vibration of that pendulum.

How often will a pendulum of 9,8 inches vibrate in a second?

By the former part of the rule, $9,8 \div 39,2 = ,25$ of a second, and $\sqrt{,25} = ,5$ of a second, the time of one vibration, that is, it vibrates half seconds, or $60 \div ,5 = 120$ times in a minute.

By the latter part. $\sqrt{9,8} = 3,13$, and $\sqrt{39,2} = 6,261$, therefore, $3,13 \div 6,261 = ,5$ of a second.

83. I observed, that while a stone was falling from a precipice, a string, (with a bullet at the end) which measured 25 inches, (to the middle of the ball) had made 5 vibrations; What was the height of the precipice?

$25 \div 39,2 = ,6377$, and $\sqrt{,6377} = ,7985$ of a second, the time of one vibration, and $,7985 \times 5 = 4$ seconds. nearly, the time of the stone's descent; then $4 \times 4 = 16$, and $16 \times 16 = 256$ feet, *Ans*.

84. To find the true depth of a well, by dropping a stone into it, also the time of the stone's descent, and of the sound's ascent.

RULE 1.—Take a line of any length, and by the last Problem find the time from the dropping of the stone till you hear it strike the bottom.

2. Multiply 73088 ($= 16 \times 4 \times 1142$; 1142 feet being the distance, which sound moves in a second) by the number of seconds till you hear the stone strike the bottom.

3. To

368 MISCELLANEOUS QUESTIONS.

3. To this product add 1304164 (= the square of 1142) and from the square root of the sum take 1142.

4. Divide the square of the remainder by 64 (= 16×4) and the quotient will be the depth of the well in feet.

5. Divide the depth by 1142, and the quotient will be the time of the sound's ascent, which, being taken from the whole time, will leave the time of the stone's descent in seconds.

Suppose I drop a stone into a well, and a string with a plummet, which measured to the middle of the ball 25 inches, made 5 vibrations before I heard the stone strike the bottom; Required the depth, time of the stone's descent, and of the sound's ascent?

$25 \div 39,2 = ,6377$, and $\sqrt{,6377} = ,7985$, and $,7985 \times 5 = 4$ seconds to the hearing of it strike; then $\sqrt{73088 \times 4 + 1304164} - 1142 = 121,53$ and $121,53 \times 121,53 \div 64 = 230,77$ feet, the depth, and $230,77 \div 1142 = ,2$ of a second, the time of the sound's ascent, and $4 - ,2 = 3,8$ seconds, the time of the stone's descent.

Of the LEVER or STEELYARD.

85. It is a principle in mechanics, that the power is to the weight, as the velocity of the weight, to the velocity of the power. Therefore, to find what weight may be raised or balanced by any given power, say;

As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied: So is the power to the weight which it will balance.

If a man, weighing 160 lb, rest on the end of a lever 10 feet long; What weight will he balance on the other end, supposing the prop one foot from the weight?

The distance between the weight and prop being 1 foot, the distance from the prop to the power is $10 - 1 = 9$ feet; therefore, As 1 foot : 9 feet :: 160 lb : 1440 lb, *Ans.*

86. If a weight of 1440 lb were to be raised with a lever 10 feet long, and the prop fixed 1 foot from the weight; What power, or weight, applied to the other end of the lever, would balance it?

As 9 : 1 :: 1440 : 160 lb, *Ans.*

87. If a weight of 1440 lb be placed 1 foot from the prop; at what distance from the prop must a power of 160 lb be applied, to balance it?

As 160 : 1440 :: 1 : 9 feet, *Ans.*

88. At what distance from a weight of 1440 lb must a prop be placed, so as that a power of 160 lb applied 9 feet from the prop, may balance it?

As 1440 : 160 :: 9 : 1 foot, *Ans.*

89. In giving directions for making a chaise, the length of the shafts, between the axletree and backband, being settled at 9 feet, a dispute arose whereabouts on the shafts the centre of the body should

should be fixed. The chaisemaker advised to place it 30 inches before the axletree ; others supposed 20 inches would be a sufficient incumbrance for the horse : Now, supposing two passengers to weigh 3 *Cwt.* and the body of the chaise $\frac{1}{4}$ *Cwt.* more ; What will the beast in both these cases bear, more than his harness ?

Weight of the chaise and passengers $3\frac{1}{4}$ *Cwt.* = 420 *lb.* and 9 feet = 108 inches.

$$\text{Then, As } 108 : 420 :: \left\{ \begin{array}{l} 30 : 116\frac{2}{3} \\ 20 : 77\frac{2}{3} \end{array} \right\} \text{ Ans.}$$

Of the W H E E L and A X L E.

90. The proportion for the wheel and axle (in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, which coils about the axle as the wheel turns round) is, As the diameter of the axle is to the diameter of the wheel ; so is the power applied to the wheel, to the weight suspended by the axle.

A mechanic would make a windlass in such a manner, as that 1 *lb.* applied to the wheel, should be equal to 10 *lb.* suspended from the axle ; now, supposing the axle to be 6 inches diameter ; Required the diameter of the wheel ?

lb. in. lb. in.

As 10 : 6 :: 1 : 60 inversely, the diameter required.

91. Suppose the diameter of the wheel to be 60 inches ; Required the diameter of the axle, so as that 1 *lb.* on the wheel may balance 10 *lb.* on the axle ?

lb. in. lb. in.

Inversely, As 1 : 60 :: 10 : 6 diameter required.

92. Suppose the diameter of the axle 6 inches, and that of the wheel 60 inches ; What power at the wheel will balance 10 *lb.* at the axle ?

in. lb. in. lb.

Inversely, As 6 : 10 :: 60 : 1 *Ans.*

93. Suppose the diameter of the wheel 60 inches, and that of the axle 6 inches ; What weight at the axle will balance 1 *lb.* at the wheel ?

in. lb. in. lb.

Inversely, As 60 : 1 :: 6 : 10 *Ans.*

Of the S C R E W.

94. The power is to the weight, which is to be raised, as the distance between two threads of the screw, is to the circumference of a circle described by the power applied at the end of the lever.

RULE.—Find the circumference of the circle described by the end of the lever ; then, As that circumference is to the distance between the spiral threads of the screw ; so is the weight to be raised, to the power which will raise it, abating the friction, which

370 MISCELLANEOUS QUESTIONS.

is not proportional to the quantity of surface ; but to the weight of the incumbent part ; and, at a medium, $\frac{1}{3}$ part of the effect of the machine is destroyed by it, sometimes more and sometimes less.

There is a screw, whose threads are an inch asunder ; the lever by which it is turned, 30 inches long, and the weight to be raised a ton, or 2240 lb ; What power or force must be applied to the end of the lever, sufficient to turn the screw—that is, to raise the weight ?

The lever being the semidiameter of the circle, the diameter is 60 inches ; then, $3,1416 \times 60 = 188,496$ inches, the circumference :

Therefore, As 188,496 : 1 :: 2240 : 11,88, *Ans.*

95. Let the lever be 30 inches, (the circumference of which is found to be 188,496) the threads 1 inch asunder, and the power 11,88 lb ; Required the weight to be raised ?

As 1 : 188,496 :: 11,88 : 2240 nearly, *Ans.*

96. Let the weight be 2240 lb, the power 11,88 lb, and the lever 30 inches ; Required the distance between the threads ?

As 2240 : 11,88 :: 188,496 : 1 nearly, *Ans.*

97. Let the power be 11,88 lb, the weight 2240 lb, and the threads an inch asunder, to find the length of the lever ?

As 11,88 : 2240 :: 1 : 188,5 ; Then, As 355 : 113 :: 188,5 : 60 inches nearly, the diameter, and $60 \div 2 = 30$ inches, *Ans.*

98. Suppose one of those meteors, called fire balls, to move parallel to the earth's surface, and 50 miles from it, at the rate of 20 miles per second ; In what time would it move round the earth ?

Suppose the earth's diameter 8000 miles, then $8000 + 50 \times 2 = 8100$ the diameter of the circle described by the ball : Then, As 113 : 355 :: 8100 : 25446,9 miles nearly, its circumference, and $25446,9 \div 20 = 1272,345$ seconds, $= 21' 12'' 19'''$ *Ans.*

99. Sound, uninterrupted, moves about 1142 feet in a second ; How long, then, after firing of a cannon at Newburyport, before it will be heard at Ipswich, estimating the distance at 10 miles in a right line ?

10 miles $= 52800$ feet, and $52800 \div 1142 = 46\frac{134}{571}$ seconds, *Ans.*

100. In a thunder storm I observed by my clock that it was 6 seconds between the lightning and thunder ; at what distance was the explosion ? $1142 \times 6 = 6852$ feet $= 1\frac{13}{20}$ mile, *Ans.*

101. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot ; What would be the weight of such a tube, which would extend across the Atlantic, from Boston to London, estimating the distance at 1000 leagues ?

$1000 \times 3 = 3000$ miles, and $3000 \times 5280 = 15840000$ feet, and $15840000 \times \frac{1}{1625} = 9747\frac{69}{1000}$ gr. or rather, 1 lb 8 oz. 6 pwt. 3 $\frac{62}{1000}$ gr. *Ans.*
102. The

102. The mean distances of the planets from the Sun in English miles are as follows; viz. Mercury 36841468; Venus 68891486; the Earth 95173000; Mars 145014148; Jupiter 494990976; Saturn 907956130, and the Moon 240000 from the Earth: Now, as a cannon ball, at its first discharge, flies about a mile in 8 seconds, and sound, 1142 feet per second; How long would a bullet, at the aforementioned rate, be in passing from the Earth to the Sun, and sound in moving from the Sun to Saturn?

$95173000 \times 8'' = 24$ years, 52 days, 7 hours, 33 minutes, 20 seconds, for the passage of the ball; and $907956130 \times 5280 = 4794008366400$ feet, and $4794008366400 \div 1142 = 133$ yrs. 41 d. 20 h. 55 m. $49\frac{5}{771}$ s. So long would sound be in passing from the Sun to Saturn.

103. Light passes from the Sun to the Earth in about 8 minutes; How long would it be in passing from the Sun to *Herschell's* Planet, or the *Georgium Sidus*, supposing it to be 5000000000 miles?

As $95173000 : 8' :: 5000000000 : 7$ h. 0 m. 17 s. 7'', *Ans.*

104. The diameter of the Sun is 890000 miles; Mercury's diameter 3000; Venus's 7924; the Earth's 7970; Mars's 7338; Jupiter's 94000; Saturn's 78000, and the Moon's 2182; What is the comparative magnitude between the Sun and the Earth, and between the Earth and all the others?

The Sun $= 890000 \times 890000 \times 890000 \div 7970 \times 7970 \times 7970 = 1392499,52$ times larger than the Earth. The Earth $= 7970 \times 7970 \times 7970 \div 3000 \times 3000 \times 3000 = 18,75$ larger than Mercury. $= 7970 \times 7970 \times 7970 \div 7924 \times 7924 \times 7924 = 1,0175$ times larger than Venus. $= 7970 \times 7970 \times 7970 \div 5400 \times 5400 \times 5400 = 3,21$ times larger than Mars. $= 7970 \times 7970 \times 7970 \div 2182 \times 2182 \times 2182 = 48,82$ times larger than the Moon. Jupiter $= 94000 \times 94000 \times 94000 \div 7970 \times 7970 \times 7970 = 1640,62$ times larger than the Earth. Saturn $= 78000 \times 78000 \times 78000 \div 7970 \times 7970 \times 7970 = 937,36$ times larger than the Earth.

105. The density of the Moon is to that of the Earth, as 123,5 to 100: What is the proportion between the quantity of matter in the Earth and that of the Moon, allowing the Earth's diameter to be 7970, and the Moon's 2182 miles, and supposing the Earth to be a complete sphere, which however it is not?

There is $\frac{7970 \times 7970 \times 7970 \times 100}{2182 \times 2182 \times 2182 \times 123,5} = 39,534$ times the quantity of matter in the Earth more than is in the Moon. In other words, the Earth weighs so much more than the Moon.

106. The mean diameter of the Earth's orbit (or annual path round the Sun) is 191263000 miles; Required its mean motion, (or the space through which it moves,) per minute?

$191263000 \times 3,1416 = 600871840,8$ miles circumference; then,
Days. Miles. Miles.

As 365,25 : 600871840,8 :: 1' : 1142,44 *Ans.*

N. B. The Earth's diurnal motion round its axis is $17\frac{1}{4}$ miles per minute, at the equator.

Of the SPECIFIC GRAVITIES of BODIES.

The specific gravities of bodies are as their densities, or weights, bulk for bulk ; thus, a body is said to have two or three times the specific gravity of another, when it contains two or three times as much matter in the same space.

A body, immersed in a fluid, will sink, if it be heavier than its bulk of the fluid. If it be suspended therein, it will lose so much of what it weighed in the air, as its bulk of the fluid weighs. Hence, all bodies of equal bulks, which will sink in fluids, lose equal weights when suspended therein, and unequal bodies lose in proportion to their bulks.

The *hydrostatic balance* differs very little from a common balance that is nicely made ; only it has a hook at the bottom of each scale, on which small weights may be hung by horse hairs, so that a body suspended by the hair, may be immersed in water without wetting the scales.

How to find the Specific Gravities of Bodies.

If the body thus suspended under the scale, at one end of the balance, be first counterpoised in air by weights in the opposite scale, and then immersed in water, the equilibrium will be immediately destroyed ; then, if as much weight be put into the scale, to which the body is suspended, as will restore the equilibrium, (without altering the weights in the opposite scale) that weight, which restores the equilibrium, will be equal to a quantity of water as big as the immersed body ; and if the weight of the body in air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus, if a guinea, suspended in air, be counterbalanced by 129 grains in the opposite scale, and then, upon being immersed in water, it becomes so much lighter, as to require $7\frac{1}{4}$ grains to be put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs 7,25 grains ; by which divide 129 (the weight of the guinea in air) and the quotient will be 17,793 ; which shews that the guinea is 17,793 times as heavy as its bulk of water.

Thus may any piece of gold be tried, by weighing it first in air, and then in water ; and if, upon dividing the weight in air by the loss in water, the quotient comes out 17,793, the gold is good : If the quotient be 18, or between 18 and 19, the gold is very fine ; but, if it be less than 17, the gold is too much alloyed, by being mixed with some other metal.

If silver be tried in this manner and found to be 11 times as heavy as water, it is very fine : If it be $10\frac{1}{2}$ times as heavy, it is standard ; but if it be of any less weight, compared with water, it is mixed with some lighter metal, such as tin, &c.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, the different losses of weight therein will shew how much heavier it is than its bulk of the fluid ;

fluid ; that fluid being lightest, in which the immersed body loses least of its aerial weight.

Common clear water, for common uses, is generally made a standard for comparing bodies by, whose gravity may be represented by unity, or 1, or, in case great accuracy be required, by 1,000, where 3 cyphers are annexed to give room to express the ratios of other gravities in larger numbers in the table. In doing this there is a twofold advantage ; the first is, that, by this mean, the specific gravities of bodies may be expressed to a much greater degree of accuracy. The second is, that the numbers of the Table, considered as whole numbers, do also express the ounces Avoirdupois contained in a cubic foot of every sort of matter therein specified ; because a cubic foot of common water is found by experiment to weigh very nearly 1000 ounces Avoirdupois, or $62\frac{1}{2}$ pounds.

A TABLE of the Specific Gravities of several solid and fluid Bodies ; where the second Column contains their Absolute Weight, and the third, their Relative Weight, in Avoirdupois Ounces.

<i>A Cubic Foot of</i>	<i>Abso. wt.</i>	<i>Rela. wt.</i>	<i>A Cubic Foot of</i>	<i>Abso. wt.</i>	<i>Rela. wt.</i>
Platina rendered malleable and hammered }	20170	20,170	Brick - -	2000	2,000
Very fine Gold - -	19637	19,637	Live Sulphur - -	2000	2,000
Standard Gold - -	18888	18,888	Nitre - - -	1900	1,900
Guinea Gold - -	17793	17,793	Alabaster - -	1875	1,875
Moidore Gold - -	17140	17,140	Dry Ivory - -	1825	1,825
Quicksilver - -	13600	13,600	Brimstone - -	1800	1,800
Lead - - -	11325	11,325	Solid subf. of Gun Pow.	1745	1,745
Fine Silver - -	11087	11,087	Alum - - -	1714	1,714
Standard Silver - -	10535	10,535	Ebony - - -	1117	1,117
Rose Copper - -	9000	9,000	Human Blood - -	1054	1,054
Copper - - -	8843	8,843	Amber - - -	1030	1,030
Plate Brass - -	8000	8,000	Cows' Milk - -	1030	1,030
Steel - - -	7852	7,852	Sea Water - -	1030	1,030
Cast Brass - -	7850	7,850	Pure Water - -	1000	1,000
Iron - - -	7645	7,645	Red Wine - -	993	0,993
Block Tin - -	7321	7,321	Oil of Amber - -	978	0,978
Cast Iron - -	7135	7,135	Proof Spirits* - -	925	0,925
Lead Ore - -	6800	6,800	Dry Oak - - -	925	0,925
Copper Ore - -	3775	3,775	Olive Oil - - -	913	0,913
Diamond - -	3400	3,400	Loose Gun Powder - -	872	0,872
Chrysal Glafs - -	3150	3,150	Spirit of Turpentine -	864	0,864
White Marble - -	2707	2,707	Alcohol or pure Spirit	850	0,850
Black ditto - -	2704	2,704	Elm and Ash - -	800	0,800
Rock Chrysal - -	2658	2,658	Oil of Turpentine - -	772	0,772
Green Glafs - -	2620	2,620	Dry Crab Tree - -	765	0,765
Clear Glafs - -	2600	2,600	Æther - - -	732	0,732
Stone {	Flint - -	2582	White Pine - - -	569	0,569
	Paving - -	2570	Sassafras Wood - -	482	0,482
	Cornelian - -	2568	Cork - - -	240	0,240
	Free - -	2352	Common Air - -	$1\frac{25}{100}$	0,00125
			Inflammable Air - -	$0\frac{12}{100}$	0,00012

Specific

* Although the specific gravity be 925 according to theory, yet, in fact, it will amount to about 927.

*Specific Gravities of the
Solar System.*

<i>A cubic Foot of</i>	<i>Abso. wt.</i>	<i>Relat. wt.</i>
The Sun	11333	11,333
Mercury	9166	9,166
Venus	5733	5,733
Earth	4500	4,500
Mars	3286	3,286
Moon	3092	3,092
Jupiter	1042	1,042
Saturn	406	0,406

The use of the Table of Specific Gravities will best appear by several Examples.

*How to discover the quantity of adulteration
in metals.*

Suppose a body be compounded of gold and silver, and it be required to find the quantity of each metal in the compound.

First find the specific gravity of the compound, by weighing it in air and in water, and dividing its ærial weight by

what it loses thereof in water, and the quotient will shew its specific gravity, or how many times it is heavier than its bulk of water. Then, subtract the specific gravity of silver (found in the Table) from that of the compound, and the specific gravity of the compound from that of the gold: The first remainder will shew the bulk of gold, and the latter, the bulk of silver in the whole compound; and if these remainders be multiplied by the respective specific gravities, the products will shew the proportional weights of each metal in the body.

Suppose the specific gravity of the compounded body be 14; that of standard silver (by the Table) is 10,535, and that of standard gold 18,888; therefore, 10,535 from 14, remains 3,465, the proportional *bulk* of the gold in the compound; and 14 from 18,888, remains 4,888, the proportional *bulk* of silver in the compound: Then 18,888, the specific gravity of gold, multiplied by the first remainder 3,465, produces 65,447 for the proportional *weight* of gold; and 10,535, the specific gravity of silver, multiplied by the last remainder, produces 51,495 for the proportional weight of silver in the whole body: So that for every 65,447 ounces or pounds of gold, there are 51,495 ounces or pounds of silver in the body.

Hence it is easy to know whether any suspected metal be genuine, or alloyed or counterfeit, by finding how much heavier it is than its bulk of water, and comparing the same with the table; if they agree, the metal is good; if they differ, it is alloyed or counterfeited.

How to try Spirituous Liquors.

A cubic inch of good brandy, rum, or other proof spirits, weighs 234 grains; therefore, if a true inch cube of any metal weighs 234 grains less in spirits than in air, it shews the spirits are proof: if it lose less of its ærial weight in spirits, they are above proof; if it lose more, they are under proof; for the better the spirits are, the lighter they are, and the worse, the heavier. Or, let any solid, of sufficient specific gravity, be weighed first in air, then in water, and then in another liquid; from its weight

weight in the air take its weight in water, and the remainder is the weight of its bulk of water. From its weight in air take its weight in the other liquid, and the remainder is the weight of the same quantity of that liquid. Divide the weight of this quantity of liquid by the weight of the same quantity of water, and the quotient will be the specific gravity of the liquid.

All bodies expand with heat, and contract with cold; but some more, and some less than others; therefore, the specific gravities of bodies are not precisely the same in summer as in winter.

The four following Problems, relating to Spirituous Liquors, are wrought by Alligation.

107. What proportion of rectified spirits of wine must be mixed with water, to make proof spirit; the specific gravity of the rectified spirits being 850, that of proof spirit 925, and of water 1000?

$$925 \left\{ \begin{array}{l} 1000 \\ 850 \end{array} \right\} \begin{array}{l} 75 \\ 75 \end{array} \left. \vphantom{\begin{array}{l} 1000 \\ 850 \end{array}} \right\} \text{ Or equal measures.}$$

108. What proportional weight of rectified spirits of wine and water must be mixed, to make proof spirit, the specific gravities as before?

$$\text{Ans. } \frac{1000}{850} = \frac{20}{17}, \text{ or as 20 to 17.}$$

109. What is the specific gravity of best French brandy, consisting of 5 parts, measure, of rectified spirits of wine, and 3 parts water?

$$\begin{array}{r} 850 \times 5 = 4250 \\ 1000 \times 3 = 3000 \end{array}$$

$$5 + 3 = 8 \quad) \quad 7250$$

906,25 = specific gravity.

110. A retailer has 30 gallons of rum, whose specific gravity is 900; How much water must he add, to reduce it to standard proof?

$$925 \left\{ \begin{array}{l} 1000 \\ 900 \end{array} \right\} \begin{array}{l} 25 \\ 75 \end{array} \left. \vphantom{\begin{array}{l} 1000 \\ 900 \end{array}} \right\} \begin{array}{l} G. \text{ rum. } G. \text{ wat. } G. \text{ rum. } G. \text{ wat.} \\ \text{As } 75 : 25 :: 30 : 10 \text{ to be added.} \end{array}$$

111. The cubic inch of common glass, weighs about 1,36oz. Troy; ditto of salt water, 5427oz. ditto of brandy, 48927oz. Suppose then, a seaman has a gallon of brandy in a bottle, which weighs $4\frac{1}{2}$ lb Troy, out of water, and, to conceal it, throws it overboard into salt water; Pray, will it sink or swim, and by how much is it heavier or lighter than the same bulk of salt water?

$$4\frac{1}{2} \text{ lb} = 54 \text{ oz.} = \text{weight of bot. } \frac{54}{1,36} = 39,7059 \text{ cub. in. in the bot.}$$

$$\text{Add } 231 = \text{do. in the brandy.}$$

$$270,7059 = \text{ditto in both.}$$

Then, $270,7059 \times 5427 = 146,912 \text{ oz.} = \text{weight of salt water occupied by the bottle and brandy. And, } 48927 (= \text{weight of a cubic$

376 MISCELLANEOUS QUESTIONS.

cubic inch of brandy) $\times 231 = 113,020z.$ and $113,02 + 54 = 167,020z. =$ weight of the bottle and brandy. From this take the weight of the salt water, viz. $146,9120z.$ and it leaves $20,110z.$ *Ans.* Supposing the bottle full, it is $20,110z.$ heavier than the same bulk of salt water, and therefore will sink.

Given the weight to be raised by a balloon, to find its diameter.

RULE 1.—As the specific difference between common and inflammable air, is to one cubic foot; so is any weight to be raised, to the cubic feet contained in the balloon.

2. Divide the cubic feet by $,5236$, and the cube root of the quotient will be the diameter required, to balance it with common air; but, to raise it, the diameter must be somewhat greater, or the weight somewhat less.

112. I would construct a spherical balloon of sufficient capacity to ascend with 4 persons weighing one with another 160 lb, and the balloon and a bag of sand weighing 60 lb; Required the diameter of the balloon?

By the Table of Specific Gravities, page 373d, I find a cubic foot of common air weighs $1,25$ ounce Avoirdupois, and a cubic foot of inflammable air $,12$ of an ounce Avoirdupois; therefore,

$1,25 - ,12 = 1,130z.$ difference. And $160 \times 4 + 60 = 700 = 11200.$

$oz.$ Cub. foot. $oz.$ Cub. feet.
As $1,13 : 1 :: 11200 : 9911,5044.$ And $\sqrt[3]{\frac{9911,5044}{,5236}} = 26,65$ [feet, *Ans.*]

Given the diameter of a balloon, to find what weight it is capable of raising.

RULE 1.—Multiply the cube of the diameter by $,5236$, and the product will be the content in cubic feet.

2. As one cubic foot is to the specific difference between common and inflammable air; so is the content of the balloon to the weight it will raise.

113. The diameter of a balloon is $26,65$ feet; What weight is it capable of raising?

$26,65 \times 26,65 \times 26,65 \times ,5236 = 9911,4 +$ cubic feet. And

$Cub. foot. oz. Cub. feet. oz.$
As $1 : 1,13 :: 9911,4 + : 11199,882 = 700$ lb, nearly.

If the magnitude of any body be multiplied by its specific gravity, the product will be its absolute weight.

114. What weight of lead will cover a house, the area of whose roof is 6000 feet, and the thickness of the lead $\frac{1}{120}$ of a foot?

$6000 \times \frac{1}{120} = 50$ cub. feet, and its specif. grav. $11325 \times 50 = 566250$

$Tons. Cwt. qrs. lb. oz.$
 $ounces = 15 \ 15 \ 3 \ 26 \ 10$ *Ans.*

To find the magnitude of any thing, when the weight is known.

Divide the weight by the specific gravity in the Table, and the quotient will be the magnitude sought.

116. What is the magnitude of several fragments of clear glass, whose weight is 13 ounces ?

$13 \div 2600 = ,005$ of a cubic foot, and $,005 \times 1728 = 8,640$ cubic inches, *Ans.*

Having the magnitude and weight of any body given, to find its specific gravity.

Divide the weight by the magnitude, and the quotient will be the specific gravity.

117. Suppose a piece of marble contains 8 cubic feet, and weighs $1353\frac{1}{2}$ lb, or 21656 ounces ; What is the specific gravity ?

$21656 \div 8 = 2707$. the specific gravity required, as by the Table.

To find the quantity of pressure against the sluice or bank, which pens water.

Multiply the area of the sluice, under water, by the depth of the centre of gravity, (which is equal to half the depth of the water) in feet, and that product again by $62\frac{1}{2}$ (the number of pounds Avoirdupois in a cubic foot of fresh water) or by 64,4 lb, (the Avoirdupois weight of a cubic foot of salt water) and the product will be the number of pounds required.

118. Suppose the length of a sluice or flume be 30 feet, and the depth of the water 4 feet ; What is the pressure against the side of the sluice ?

$30 \times 4 = 120$ feet, the area of the side, and 120×2 (the depth of the centre of gravity) gives 240 cubic feet, and $240 \times 62,5 = 15000$ lb = 6T. 13cwt. 3qrs. 20lb, *Ans.*

The perpendicular pressure of fluids on the bottoms of vessels is estimated by the area of the bottom multiplied by the altitude of the fluid.

119. Suppose a vessel 3 feet wide, 5 feet long, and 4 feet high ; What is the pressure on the bottom, it being filled with water to the brim ?

$3 \times 5 = 15$ square feet, the area of the bottom, and $15 \times 4 = 60$ cubic feet, and $60 \times 62,5 = 3750$ lb = 33cwt. 1qr. 26lb.

The Use of the BAROMETER.

The barometer is so formed, that a column of quicksilver is supported within it, to such a height as to counterbalance the weight of a column of air, of an equal diameter, extending from the barometer to the top of the atmosphere.

120. At the surface of the earth, the height of this column of quicksilver is, at an average, almost 30 inches ; when the barometer is at that height, What is the pressure of atmosphere on a square foot, and on the surface of a man's body, estimated at 14 square feet ?

378 MISCELLANEOUS QUESTIONS.

As the cubic foot of quicksilver is 13600 ounces, Avoirdupois, and as the height in the barometer is 2,5 feet, therefore $13600 \times 2,5 = 34000$ ounces, $= 2125$ pounds on a square foot; and $2125 \times 14 = 29750$ pounds on a man's body.

121. If the mercury in a barometer, at the bottom of a tower, be observed to stand at 30 inches, and, on being carried to the top of it, be observed at 29,9 inches; What is the height of the tower?

Divide 13600, the specific gravity of quicksilver, by 1,25, the specific gravity of air, and the quotient will be the height of the tower, in tenths of an inch.

$$\frac{13600}{1,25} = 10880 \text{ tenths, and } \frac{10880}{10} = 1088 \text{ inch.} = 90\frac{2}{3} \text{ ft. Ans.}$$

The number of feet, in height, of the atmosphere, corresponding with $\frac{1}{10}$ of an inch on the barometer is variable, depending on the temperature and density of the atmosphere.

The variation, depending on the temperature, is shewn in the following Table, calculated for every 5 degrees, from 32 to 80, Farenheit's Thermometer, from whence it may be easily calculated for the intermediate degrees, by allowing $\frac{21}{100}$ of a foot for each degree.

TABLE.
Thermo. feet.

32°	86,86	The altitude, thus found, will be to the altitude corrected for the density of the air, inversely, as the mean height of the barometer, at the two stations, is to 30 inches; therefore,
35	87,49	
40	88,54	
45	89,60	
50	90,66	
55	91,72	RULE.—Multiply the mean height corresponding to the mean temperature of the two barometers (found in the Table) by the tenths of an inch in the difference of the two barometers, and this product by 30; divide this last product by the mean height of the two barometers, and the quotient will be the answer, or height required, with the error of a few feet only, if the height be less than a mile.*
60	92,77	
65	93,82	
70	94,88	
75	95,93	
80	96,99	

122. At the 1st station, suppose the barometer to stand at 29, and the thermometer at 60; at the 2d station, the barometer at 28, and the thermometer at 40; What is the height of the 2d station, or the distance between the two places of observation?

Barometer.

* Let h = mean height of the barometer at its two stations, (or of two barometers, one at each station) in inches, d = difference of the two barometers in tenths of an inch; and n = number from the Table answering to the mean temperature of the two thermometers accompanying the barometer, then $\frac{30dn}{h}$ = the altitude required nearly.

Barometer.

Add { First station = 29
Second station = 28

$\frac{1}{2}$)57

$\frac{1}{2}$ sum = 28,5 = mean height of the two barometers.

29
28

- Difference = 1 = 10 tenths of an inch.

Thermometer.

First station = 60
Second station = 40

$\frac{1}{2}$)100

50 = mean height of the two thermometers, against which, in the Table, you will find 90,66, the mean temperature of the two barometers. Now according to the rule $90,66 \times 10 \times 30 \div 28,5 = 954,3$ feet, the Answer, nearly.

T A B L E S.

Value of English and Portuguese gold, in dollars, cents and mills, throughout the United States.

Gr.	Cts.	m.	Pwts.	Dols.	cts.
1	3	7	1	0	89
2	7	4	2	1	77 $\frac{1}{2}$
3	11	1	3	2	66 $\frac{1}{2}$
4	14	8	4	3	55 $\frac{1}{2}$
5	18 $\frac{1}{2}$	0	5	4	44 $\frac{1}{2}$
6	22	2	6	5	33 $\frac{1}{2}$
7	25	9	7	6	22
8	29	6	8	7	11
9	33 $\frac{1}{2}$	0	9	8	0
10	37	0	10	8	89
11	40	7	11	9	77 $\frac{1}{2}$
12	44	4	12	10	66 $\frac{1}{2}$
13	48	1	13	11	55 $\frac{1}{2}$
14	51	8	14	12	44 $\frac{1}{2}$
15	55	5	15	13	33 $\frac{1}{2}$
16	59 $\frac{1}{2}$	0	16	14	22
17	63	0	17	15	11
18	66 $\frac{1}{2}$	0	18	16	0
19	70	4	19	16	89
20	74	0	10Z.	17	77 $\frac{1}{2}$
21	77 $\frac{1}{2}$	0			
22	81 $\frac{1}{2}$	0			
23	85	2			

Note. 30 cents is the value of 1 penny-weight of English and Portuguese Gold.

Value of French and Spanish gold, in dollars, cents and mills, throughout the United States.

Gr.	Cts.	m.	Pwts.	Dols.	cts.	m.
1	3	6	1	0	87	6
2	7	3	2	1	75	2
3	11	0	3	2	62 $\frac{1}{2}$	0
4	14	6	4	3	50	3
5	18	2	5	4	38	0
6	21	9	6	5	25	5
7	25	5	7	6	13	1
8	29	2	8	7	0	7
9	32	8	9	7	88	3
10	36 $\frac{1}{2}$	0	10	8	76	0
11	40	1	11	9	63 $\frac{1}{2}$	0
12	43	8	12	10	51	1
13	47	4	13	11	38	7
14	51	1	14	12	26	3
15	54 $\frac{1}{2}$	0	15	13	13	9
16	58	4	16	14	1 $\frac{1}{2}$	0
17	62	0	17	14	89	0
18	65	7	18	15	76	6
19	69	3	19	16	64	2
20	73	0	10Z.	17	51	8
21	76	6				
22	80	3				
23	83	9				

Note. 87 cents 6 mills, the value of 1 penny-weight French and Spanish gold.

TABLES OF EXCHANGE.

<i>Federal Money.</i>	<i>N. Hamp- shire, Massachusetts, R. Island, Connecticut, & Virginia.</i>	<i>New York and N. Carolina.</i>	<i>N. Jersey, Pennsylvania, Delaware & Maryland.</i>	<i>S. Carolina & Georgia.</i>	<i>Canada and Nova Scotia.</i>	<i>French.</i>
<i>Dol. d. c.</i>	<i>£. s. d.</i>	<i>£. s. d.</i>	<i>£. s. d.</i>	<i>£. s. d.</i>	<i>£. s. d.</i>	<i>Liv. } Sous. Tour. }</i>
0,0 1	$\frac{18}{25}$	$\frac{24}{25}$	$\frac{9}{10}$	$\frac{14}{25}$	$\frac{3}{5}$	$1\frac{1}{20}$
0,0 2	$1\frac{11}{25}$	$1\frac{23}{25}$	$1\frac{8}{10}$	$1\frac{3}{25}$	$1\frac{1}{5}$	$2\frac{1}{10}$
0,0 3	$2\frac{4}{25}$	$2\frac{22}{25}$	$2\frac{7}{10}$	$2\frac{2}{25}$	$1\frac{2}{5}$	$3\frac{3}{10}$
0,0 4	$2\frac{22}{25}$	$3\frac{21}{25}$	$3\frac{6}{10}$	$2\frac{2}{25}$	$2\frac{2}{5}$	$4\frac{1}{5}$
0,0 5	$3\frac{3}{25}$	$4\frac{4}{25}$	$4\frac{5}{10}$	$3\frac{2}{25}$	3	$5\frac{1}{4}$
0,0 6	$4\frac{3}{25}$	$5\frac{19}{25}$	$5\frac{4}{10}$	$3\frac{9}{25}$	$3\frac{3}{5}$	$6\frac{3}{10}$
0,0 7	$5\frac{1}{25}$	$6\frac{18}{25}$	$6\frac{3}{10}$	$3\frac{23}{25}$	$4\frac{1}{5}$	$7\frac{7}{10}$
0,0 8	$5\frac{19}{25}$	$7\frac{17}{25}$	$7\frac{2}{10}$	$4\frac{12}{25}$	$4\frac{4}{5}$	$8\frac{3}{5}$
0,0 9	$6\frac{12}{25}$	$8\frac{16}{25}$	$8\frac{1}{10}$	$5\frac{1}{25}$	$5\frac{1}{5}$	$9\frac{9}{10}$
0,1 0	$7\frac{1}{25}$	$9\frac{3}{25}$	9	$5\frac{3}{25}$	6	$10\frac{1}{2}$
0,2 0	1 2	1 7 $\frac{1}{2}$	1 6	11 $\frac{1}{5}$	1 0	1 1
0,3 0	1 9	2 4 $\frac{4}{5}$	2 3	1 4	1 6	1 11 $\frac{1}{2}$
0,4 0	2 4	3 2 $\frac{2}{5}$	3 0	1 10 $\frac{2}{5}$	2 0	2 2
0,5 0	3 0	4 0	3 9	2 4	2 6	2 12 $\frac{1}{2}$
0,6 0	3 7 $\frac{1}{5}$	4 9 $\frac{1}{5}$	4 6	2 9 $\frac{3}{5}$	3 0	3 3
0,7 0	4 2	5 7 $\frac{2}{5}$	5 3	3 3 $\frac{1}{5}$	3 6	3 13 $\frac{1}{2}$
0,8 0	4 9	6 4 $\frac{3}{5}$	6 0	3 8 $\frac{4}{5}$	4 0	4 4
0,9 0	5 4 $\frac{4}{5}$	7 2 $\frac{4}{5}$	6 9	4 2 $\frac{2}{5}$	4 6	4 14 $\frac{1}{2}$
1,0 0	6 0	8 0	7 6	4 8	5 0	5 5
2,0 0	12 0	16 0	15 0	9 4	10 0	10 10
3,0 0	18 0	1 4 0	1 2 6	14 0	15 0	15 15
4,0 0	1 4 0	1 12 0	1 10 0	18 8	1 0 0	21 0
5,0 0	1 10 0	2 0 0	1 17 6	1 3 4	1 5 0	26 5
6,0 0	1 16 0	2 8 0	2 5 0	1 8 0	1 10 0	31 10
7,0 0	2 2 0	2 16 0	2 12 6	1 12 8	1 15 0	36 15
8,0 0	2 8 0	3 4 0	3 0 0	1 17 4	2 0 0	42 0
9,0 0	2 14 0	3 12 0	3 7 6	2 2 0	2 5 0	47 5
10,0 0	3 0 0	4 0 0	3 15 0	2 6 8	2 10 0	52 10
20,0 0	6 0 0	8 0 0	7 10 0	4 13 4	5 0 0	105 0
30,0 0	9 0 0	12 0 0	11 5 0	7 0 0	7 10 0	157 10
40,0 0	12 0 0	16 0 0	15 0 0	9 6 8	10 0 0	210 0
50,0 0	15 0 0	20 0 0	18 15 0	11 13 4	12 10 0	262 10
60,0 0	18 0 0	24 0 0	22 10 0	14 0 0	15 0 0	315 0
70,0 0	21 0 0	28 0 0	26 5 0	16 6 8	17 10 0	367 10
80,0 0	24 0 0	32 0 0	30 0 0	18 13 4	20 0 0	420 0
90,0 0	27 0 0	36 0 0	33 15 0	21 0 0	22 10 0	472 10
100,0 0	30 0 0	40 0 0	37 10 0	23 6 8	25 0 0	525 0
200,0 0	60 0 0	80 0 0	75 0 0	46 13 4	50 0 0	1050 0
300,0 0	90 0 0	120 0 0	112 10 0	70 0 0	75 0 0	1575 0
400,0 0	120 0 0	160 0 0	150 0 0	93 6 8	100 0 0	2100 0
500,0 0	150 0 0	200 0 0	187 10 0	116 13 4	125 0 0	2625 0
600,0 0	180 0 0	240 0 0	225 0 0	140 0 0	150 0 0	3150 0
700,0 0	210 0 0	280 0 0	262 10 0	163 6 8	175 0 0	3675 0
800,0 0	240 0 0	320 0 0	300 0 0	186 13 4	200 0 0	4200 0
900,0 0	270 0 0	360 0 0	337 10 0	210 0 0	225 0 0	4725 0
1000,0 0	300 0 0	400 0 0	375 0 0	233 6 8	250 0 0	5250 0

TABLES OF EXCHANGE.

381

V. Hamp. Massachu. R. Island, Connecticut, and Virgin.			Newyork and Northear- olina.			Newjersey, Pennsylvania, Delaware & Maryland.			S. Carolina and Georgia.			Eng. Money.			French Money.		
Federal Coin.															Liv. } Tour. } Sous.		
£.	s.	d.	Dol.	d.	c.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1			0,01	$\frac{7}{8}$		1	$\frac{1}{4}$				$\frac{7}{8}$			$\frac{1}{4}$			1 $\frac{1}{2}$
2			0,02	$\frac{7}{8}$		2	$\frac{1}{2}$				1 $\frac{5}{8}$			1 $\frac{1}{2}$			2 $\frac{1}{2}$
3			0,04	$\frac{1}{6}$		4					2 $\frac{3}{8}$			2 $\frac{1}{4}$			4 $\frac{3}{8}$
4			0,05	$\frac{5}{8}$		5	$\frac{1}{8}$				3 $\frac{1}{8}$			3			5 $\frac{5}{8}$
5			0,06	$\frac{3}{8}$		6	$\frac{3}{8}$				3 $\frac{3}{8}$			3 $\frac{1}{4}$			7 $\frac{7}{8}$
6			0,08	$\frac{1}{2}$		8					4 $\frac{1}{8}$			4 $\frac{1}{2}$			8 $\frac{1}{2}$
7			0,09	$\frac{3}{4}$		9	$\frac{1}{2}$				5 $\frac{1}{8}$			5 $\frac{1}{4}$			10 $\frac{5}{8}$
8			0,11	$\frac{1}{2}$		10	$\frac{3}{4}$				6 $\frac{1}{8}$			6			11 $\frac{3}{4}$
9			0,12	$\frac{1}{2}$		10	$\frac{3}{4}$				7 $\frac{1}{8}$			6 $\frac{1}{4}$			13 $\frac{1}{8}$
10			0,13	$\frac{1}{2}$		11	$\frac{1}{2}$				7 $\frac{7}{8}$			7 $\frac{1}{2}$			14 $\frac{7}{8}$
11			0,15	$\frac{1}{2}$		12	$\frac{1}{2}$				8 $\frac{5}{8}$			8 $\frac{1}{4}$			16 $\frac{1}{2}$
100			0,16	$\frac{2}{3}$		14					9 $\frac{3}{8}$			9			17 $\frac{1}{2}$
200			0,33	$\frac{1}{3}$		28					6 $\frac{6}{8}$			16			35
300			0,50			40					24			23			52
400			0,66	$\frac{2}{3}$		54					3 $\frac{1}{8}$			30			70
500			0,83	$\frac{1}{3}$		68					3 $\frac{10}{8}$			39			87
600			1,00			80					48			46			105
700			1,16	$\frac{2}{3}$		94					5 $\frac{4}{8}$			53			122
800			1,33	$\frac{1}{3}$		108					6 $\frac{2}{8}$			60			140
900			1,50			120					70			69			157
1000			1,66	$\frac{2}{3}$		134					7 $\frac{9}{8}$			76			175
10000			3,33	$\frac{1}{3}$		68					15 $\frac{6}{8}$			150			350
20000			6,66	$\frac{2}{3}$		134					11 $\frac{1}{8}$			110			525
30000			10,00			200					26			250			700
40000			13,33	$\frac{1}{3}$		268					3 $\frac{2}{8}$			300			875
50000			16,66	$\frac{2}{3}$		334					3 $\frac{17}{8}$			315			1050
60000			20,00			400					4 $\frac{13}{8}$			410			1225
70000			23,33	$\frac{1}{3}$		468					5 $\frac{8}{8}$			510			1400
80000			26,66	$\frac{2}{3}$		534					6 $\frac{4}{8}$			600			1575
90000			30,00			600					70			615			1750
100000			33,33	$\frac{1}{3}$		668					7 $\frac{15}{8}$			710			2250
200000			66,66	$\frac{2}{3}$		1334					15 $\frac{11}{8}$			1500			3500
300000			100,00			2000					23			2210			5250
400000			133,33	$\frac{1}{3}$		2668					31			3000			7000
500000			166,66	$\frac{2}{3}$		3334					38			3710			8750
600000			200,00			4000					46			4500			10500
700000			233,33	$\frac{1}{3}$		4668					54			5210			12250
800000			266,66	$\frac{2}{3}$		5334					62			6000			14000
900000			300,00			6000					70			6710			15750
1000000			333,33	$\frac{1}{3}$		6668					77			7500			17500
2000000			666,66	$\frac{2}{3}$		13334					155			15000			35000
3000000			1000,00			20000					233			22500			52500
4000000			1333,33	$\frac{1}{3}$		26668					311			30000			70000
5000000			1666,66	$\frac{2}{3}$		33334					388			37500			87360

TABLE of the Value of several Pieces of Coin, in the Federal Coin, and the several Currencies of the United States.

	Federal Coin.	N. Hampshire, Massachusetts, Rhode Island, Connecticut, Virginia.	New York and North Carolina.	N. Jersey, Pennsylvania, Delaware and Maryland.	South Carolina and Georgia.
	Cents.	£. s. d.	£. s. d.	£. s. d.	£. s. d.
$\frac{1}{10}$ of a Dollar	0,06 $\frac{1}{4}$	4 $\frac{1}{2}$	6	5 $\frac{5}{8}$	3 $\frac{1}{2}$
$\frac{1}{2}$ a Pistareen	0,10	7 $\frac{1}{5}$	9 $\frac{3}{5}$	9	5 $\frac{3}{5}$
		Vir. 8			
$\frac{1}{9}$ of a Dollar	0,11 $\frac{1}{9}$	8	10 $\frac{2}{3}$	10	6 $\frac{2}{3}$
$\frac{1}{8}$ of ditto	0,12 $\frac{1}{2}$	9	1 0	11 $\frac{1}{5}$	7
A Pistareen	0,20	1 2 $\frac{2}{5}$	1 4 $\frac{1}{2}$	1 6	11 $\frac{1}{5}$
		Vir. 1 4			
An Eng. Shill.	0,22 $\frac{2}{9}$	1 4	1 7 $\frac{2}{3}$	1 8	1 C $\frac{4}{5}$
$\frac{1}{4}$ of a Dollar	0,25	1 6	2 0	1 10 $\frac{1}{2}$	1 2
Half ditto	0,50	3 0	4 0	3 9	2 4
A Dollar	1,00	6 0	8 0	7 6	4 8
En. or Br. Crown	1,11 $\frac{1}{9}$	6 8	N. York 9 0 N. Caro. 8 9	8 4	5 2 $\frac{2}{5}$
	pwt. gr.				
Fr. Guin. 5 5	4,62 $\frac{26}{27}$	1 7 6	1 16 0	1 14 6	1 1 5
In Massa. 5 6	4,55 $\frac{1}{5}$	1 7 4			
En. Guin. 5 6	4,66 $\frac{2}{3}$	1 8 0	1 17 0	1 15 0	1 1 9
In S. Caro. 5 7					1 1 10
$\frac{1}{2}$ Johann. 9 0	4,00	2 8 0	3 4 0	3 0 0	1 17 4
Pistole 2 5	3,66 $\frac{2}{3}$	1 2 0	1 8 0	1 7 0	17 6
In Massa. 4 3					
Moidore 6 18	6,00	1 16 0	2 8 0	2 5 0	1 8 0
Doubloon 17 0	14,66 $\frac{2}{3}$	4 8 0	5 16 0	5 12 0	3 10 0

The standard weight of an eagle 11pwt. 4 $\frac{2}{3}$ gr. Half ditto 5pwt. 14 $\frac{1}{3}$ gr. A dollar 17pwt. 1 $\frac{1}{5}$ gr. Half ditto 8pwt. 12 $\frac{2}{3}$ gr. A double dime 3pwt. 9 $\frac{4}{5}$ gr. A dime 1pwt. 16 $\frac{1}{5}$ gr.

TABLE of Refner's Weight.

Blanks

24 = 1 Perrot.

480 = 20 = 1 Mitc.

9600 = 400 = 20 = 1 Grain.

Note, What they denominate a carat, is the $\frac{1}{4}$ of a lb, an oz. or any other weight.

DUTCH WEIGHTS for GOLD and SILVER.

Note, 32 aces = 1 engel, 20 engels = 1 ounce, 8 ounces = 1 mark, for gross gold. Also, 24 parts = 1 grain, 12 grains = 1 carat, 24 carats = 1 mark, for fine gold.

The mark weights are 1 per cent. lighter than our Troy weight.

T A B L E 3.

A TABLE of Commission or Brokage.

283

Goods or Stock sold				at $\frac{1}{2}$ per cent.	at 1 per cent.	at $1\frac{1}{2}$ per cent.	at 2 per cent.	at $2\frac{1}{2}$ per cent.	at 3 per cent.
Shill.	1	£.	s. d.	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	2	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	3	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	4	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	5	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	6	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	7	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	8	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	9	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	10	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	11	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	12	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	13	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	14	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	15	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	16	0	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	17	0	0 1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	18	0	0 1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	19	0	0 1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
Pounds	1	0	0 1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	2	0	0 2	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	3	0	0 3	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	4	0	0 5	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	5	0	0 6	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	6	0	0 7	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	7	0	0 8	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	8	0	0 9	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	9	0	0 10	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	10	0	1 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	20	0	2 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	30	0	3 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	40	0	4 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	50	0	5 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	60	0	6 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	70	0	7 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	80	0	8 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	90	0	9 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	100	0	10 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	200	1	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	300	1	10 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	400	2	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	500	2	10 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	600	3	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	700	3	10 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	800	4	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	900	4	10 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	1000	5	0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0

A TABLE of the Returns of the Neat Proceeds of an Account of Sales from a Factor to his Employer, reserving his Commissions for Remittance.

Neat Pro-ceeds.	Sum to be re-mitted, reserv-ing $2\frac{1}{2}$ pr cent. Commission.	Sum to be re-mitted, reserv-ing 5 pr cent. Commission.	Neat Pro-ceeds.	Sum to be remit-ted, reserving $2\frac{1}{2}$ per cent. Com-mission.	Sum to be remit-ted, reserving 5 per cent. Com-mission.
£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.
3	3	$2\frac{3}{4}$	6 0 0	5 17 $0\frac{3}{4}$	5 14 $5\frac{1}{2}$
4	4	$3\frac{3}{4}$	7 0 0	6 16 7	6 13 4
5	5	$4\frac{3}{4}$	8 0 0	7 16 $1\frac{1}{4}$	7 12 $4\frac{1}{2}$
6	$5\frac{3}{4}$	$5\frac{3}{4}$	9 0 0	8 15 $7\frac{1}{4}$	8 11 $5\frac{1}{4}$
7	$6\frac{3}{4}$	$6\frac{3}{4}$	10 0 0	9 15 $1\frac{1}{2}$	9 10 $5\frac{3}{4}$
8	$7\frac{3}{4}$	$7\frac{1}{2}$	20 0 0	19 10 3	19 0 $11\frac{1}{2}$
9	$8\frac{3}{4}$	$8\frac{1}{2}$	30 0 0	29 5 $4\frac{1}{4}$	28 11 $5\frac{1}{4}$
10	$9\frac{3}{4}$	$9\frac{1}{2}$	40 0 0	39 0 $5\frac{1}{4}$	38 1 $10\frac{1}{4}$
11	$10\frac{3}{4}$	$10\frac{1}{4}$	50 0 0	48 15 $7\frac{1}{4}$	47 12 $4\frac{1}{2}$
1 0	$11\frac{3}{4}$	$11\frac{1}{4}$	60 0 0	58 10 $8\frac{3}{4}$	57 2 $10\frac{1}{4}$
2 0	1 $11\frac{1}{2}$	1 $10\frac{3}{4}$	70 0 0	68 5 $10\frac{1}{4}$	66 13 4
3 0	2 $11\frac{1}{4}$	2 $10\frac{1}{4}$	80 0 0	78 0 $11\frac{1}{2}$	76 3 $9\frac{3}{4}$
4 0	3 $10\frac{3}{4}$	3 $9\frac{3}{4}$	90 0 0	87 16 1	85 14 $3\frac{1}{2}$
5 0	4 $10\frac{1}{2}$	4 $9\frac{1}{4}$	100 0 0	97 11 $2\frac{3}{4}$	95 4 9
6 0	5 $10\frac{1}{4}$	5 $8\frac{1}{2}$	200 0 0	195 2 $5\frac{1}{4}$	190 9 $6\frac{1}{4}$
7 0	6 10	6 8	300 0 0	292 13 8	285 14 $3\frac{1}{4}$
8 0	7 $9\frac{3}{4}$	7 $7\frac{1}{2}$	400 0 0	390 4 $10\frac{1}{2}$	380 19 $0\frac{1}{2}$
9 0	8 $9\frac{1}{4}$	8 $6\frac{3}{4}$	500 0 0	487 16 $1\frac{1}{4}$	476 2 $9\frac{1}{2}$
10 0	9 9	9 $6\frac{1}{4}$	600 0 0	585 7 $3\frac{3}{4}$	571 8 $6\frac{3}{4}$
1 0 0	19 $6\frac{1}{4}$	19 $0\frac{1}{2}$	700 0 0	682 18 $4\frac{1}{4}$	666 13 4
2 0 0	1 19 $0\frac{1}{4}$	1 18 $1\frac{1}{4}$	800 0 0	780 9 9	761 18 1
3 0 0	2 18 $6\frac{1}{2}$	2 17 $1\frac{3}{4}$	900 0 0	878 0 $11\frac{3}{4}$	857 2 10
4 0 0	3 18 $0\frac{1}{2}$	3 16 $2\frac{1}{4}$	1000 0 0	975 12 $2\frac{1}{4}$	952 7 $7\frac{1}{4}$
5 0 0	4 17 $6\frac{3}{4}$	4 15 $2\frac{1}{4}$			

Suppose I have the neat proceeds, or balance of an account of sales 325*l.* 17*s.* 9*d.* in my hands, and would make remittance to my employer, reserving my commission at $2\frac{1}{2}$ per cent. What sum must be remitted, so that my employer's account may be closed?

$$\text{Against } \left\{ \begin{array}{l} \text{£. s. d.} \\ 300 \text{ } 0 \text{ } 0 \\ 20 \text{ } 0 \text{ } 0 \\ 5 \text{ } 0 \text{ } 0 \\ 10 \text{ } 0 \text{ } 0 \\ 7 \text{ } 0 \text{ } 0 \\ 9 \end{array} \right\} \text{ stands } \left\{ \begin{array}{l} \text{£. s. d.} \\ 292 \text{ } 13 \text{ } 8 \\ 19 \text{ } 10 \text{ } 3 \\ 4 \text{ } 17 \text{ } 6\frac{3}{4} \\ 9 \text{ } 9 \\ 6 \text{ } 10 \text{ } 8\frac{3}{4} \end{array} \right.$$

To be remitted £317 18 $9\frac{1}{2}$ Answer.

A TABLE, shewing the number of Days from any Day in any Month to the same Day in any other Month through the Year.

From January	February	March	April	May	June	July	August	September	October	November	December
Feb. 3	Mar. 28	April 3	May 3	Jun. 31	July 30	Aug. 31	Sept. 31	Oct. 30	Nov. 31	Dec. 30	Jan. 31
Mar. 59	April 59	May 61	June 61	July 61	Aug. 61	Sept. 62	Oct. 62	Nov. 61	Dec. 61	Jan. 61	Feb. 62
April 90	May 89	June 92	July 91	Aug. 92	Sept. 92	Oct. 92	Nov. 92	Dec. 91	Jan. 92	Feb. 92	Mar. 90
May 120	June 120	July 122	Aug. 122	Sept. 123	Oct. 122	Nov. 123	Dec. 122	Jan. 122	Feb. 123	Mar. 120	April 121
June 151	July 150	Aug. 153	Sept. 153	Oct. 153	Nov. 153	Dec. 153	Jan. 153	Feb. 153	Mar. 151	April 151	May 151
July 181	Aug. 181	Sept. 181	Oct. 183	Nov. 184	Dec. 183	Jan. 181	Feb. 184	Mar. 181	April 182	May 181	June 182
Aug. 212	Sept. 212	Oct. 214	Nov. 214	Dec. 214	Jan. 214	Feb. 215	Mar. 212	April 21	May 212	June 212	July 212
Sept. 243	Oct. 242	Nov. 245	Dec. 244	Jan. 245	Feb. 245	Mar. 243	April 243	May 242	June 243	July 242	Aug. 243
Oct. 273	Nov. 273	Dec. 275	Jan. 275	Feb. 276	Mar. 273	April 274	May 273	June 273	July 273	Aug. 273	Sept. 274
Nov. 304	Dec. 303	Jan. 306	Feb. 306	Mar. 304	April 304	May 304	June 304	July 303	Aug. 304	Sept. 304	Oct. 304
Dec. 334	Jan. 334	Feb. 337	Mar. 334	April 335	May 331	June 325	July 334	Aug. 334	Sept. 335	Oct. 334	Nov. 335
Jan. 365	Feb. 365	Mar. 365	April 365	May 365	June 365	July 365	Aug. 365	Sept. 365	Oct. 365	Nov. 365	Dec. 365
To											

The use of the preceding Table of Number of Days, will easily appear from the following Examples.

Suppose the number of days between the 1st, or 10th, or 30th, &c. of January, and the 1st, or 10th, or 30th &c. of October, were required? Look in the column under January for October, and against that month you will find 273, which is the number of days between the said times; and so for the days between any other two months.

If the given days be different, it is only adding or subtracting their inequality to or from the tabular number.

How many days from the 6th of April to the 12th of January? From the 6th of April to the 6th of January is 275, and adding the 6 overplus days, it makes 281 days. And from the 5th of June to the 1st of February is 240 days.

Note, After February 31, (in leap years) increase each number with an unit or 1.

A TABLE of the Measure of Length of the principal places in Europe, compared with the American yard.

100 Aunes or Ells of England,	-	-	= 125
100 — of Holland or Amsterdam, Hærlém, Ley-	}	= 75	
den, the Hague, Rotterdam, Nuremberg,			
and other cities of Holland,			
100 — of Brabant or Antwerp,	-	-	= 76
100 — of France and Oznaburg,	-	-	= $128\frac{1}{2}$
100 — of Hamburg, Francfort, Leipzig, Bern and Basil,	-	-	= $62\frac{1}{2}$
100 — of Breslau,	-	-	= 60
100 — of Dantzick,	-	-	= $61\frac{3}{4}$
100 — of Bergen and Drontheim,	-	-	= $68\frac{1}{4}$
100 — of Sweden and Stockholm,	-	-	= $65\frac{3}{4}$
100 — of St. Gall. for Linens,	-	-	= $87\frac{1}{2}$
100 — of ditto for Cloths,	-	-	= 67
100 — of Geneva,	-	-	= $124\frac{3}{4}$
100 Canes of Marfeilles and Montpelier,	-	-	= $214\frac{1}{2}$
100 — of Thoulouse and High Languedoc,	-	-	= 200
100 — of Genoa, of 9 palms,	-	-	= $245\frac{1}{4}$
100 — of Rome,	-	-	= $227\frac{1}{4}$
100 Varas of Spain,	-	-	= $34\frac{3}{4}$
100 — of Portugal,	-	-	= 123
100 Cavidos of Portugal,	-	-	= 75
100 Brasses of Venice,	-	-	= $78\frac{1}{2}$
100 — of Bergamo,	-	-	= $71\frac{1}{4}$
100 — of Florence and Leghorn,	-	-	= 64
100 — of Milan,	-	-	= $58\frac{1}{2}$

The use of the following TABLE, directing how to buy and sell by the hundred.

If you buy or sell any thing by the great hundred (112 lb.) and desire to know, by the pound, what the hundred is valued at, observe the following Examples.

1. If you buy sugar at $6\frac{3}{4}d.$ per lb, look for $6\frac{3}{4}d.$ in the left hand column of the Table, and against it, in the second column, you will find £3 3s. which is the value of 1 *cwt.* at that rate.

2. If 1 *cwt.* (112 lb.) cost £9 4s. 4d. to know how much it is per lb, look £9 4s. 4d. in the fourth column, and against it, in the next left hand column, you will find 1s. $7\frac{3}{4}d.$ which is the price per lb.

Again, if you buy one hundred weight of goods for 9l. 4s. 4d. and retail it at 1s. $9\frac{3}{4}d.$ per lb, it comes at that rate, to 10l. 3s. ; then take 9l. 4s. 4d. from 10l. 3s. and by the remainder, you will find that you have gained 18s. 8d.

And in this manner, you may, with ease, calculate any quantity by the following Table.

A TABLE directing how to buy and sell by the hundred.

d.	l.	s.	d.	l.	s.	d.	l.	s.	d.	l.	s.	d.	l.	s.	d.
$\frac{1}{4}$	0	2	4	1	0	$\frac{1}{4}$	5	14	4	2	0	$\frac{1}{4}$	11	6	4
$\frac{1}{2}$	0	4	8	1	0	$\frac{1}{2}$	5	16	8	2	0	$\frac{1}{2}$	11	8	8
$\frac{3}{4}$	0	7	0	1	0	$\frac{3}{4}$	5	19	0	2	0	$\frac{3}{4}$	11	11	0
1	0	9	4	1	1	1	6	1	4	2	1	1	11	13	4
$1\frac{1}{4}$	0	11	8	1	$1\frac{1}{4}$	$\frac{1}{4}$	6	3	8	2	$1\frac{1}{4}$	$\frac{1}{4}$	11	15	8
$1\frac{1}{2}$	0	14	0	1	$1\frac{1}{2}$	$\frac{1}{2}$	6	6	0	2	$1\frac{1}{2}$	$\frac{1}{2}$	11	18	0
$1\frac{3}{4}$	0	16	4	1	$1\frac{3}{4}$	$\frac{3}{4}$	6	8	4	2	$1\frac{3}{4}$	$\frac{3}{4}$	12	0	4
2	0	18	8	1	2	1	6	10	8	2	2	1	12	2	8
$2\frac{1}{4}$	1	1	0	1	$2\frac{1}{4}$	$\frac{1}{4}$	6	13	0	2	$2\frac{1}{4}$	$\frac{1}{4}$	12	5	0
$2\frac{1}{2}$	1	3	4	1	$2\frac{1}{2}$	$\frac{1}{2}$	6	15	4	2	$2\frac{1}{2}$	$\frac{1}{2}$	12	7	4
$2\frac{3}{4}$	1	5	8	1	$2\frac{3}{4}$	$\frac{3}{4}$	6	17	8	2	$2\frac{3}{4}$	$\frac{3}{4}$	12	9	8
3	1	8	0	1	3	1	7	0	0	2	3	1	12	12	0
$3\frac{1}{4}$	1	10	4	1	$3\frac{1}{4}$	$\frac{1}{4}$	7	2	4	2	$3\frac{1}{4}$	$\frac{1}{4}$	12	14	4
$3\frac{1}{2}$	1	12	8	1	$3\frac{1}{2}$	$\frac{1}{2}$	7	4	8	2	$3\frac{1}{2}$	$\frac{1}{2}$	12	16	8
$3\frac{3}{4}$	1	15	0	1	$3\frac{3}{4}$	$\frac{3}{4}$	7	7	0	2	$3\frac{3}{4}$	$\frac{3}{4}$	12	19	0
4	1	17	4	1	4	1	7	9	4	2	4	1	13	1	4
$4\frac{1}{4}$	1	19	8	1	$4\frac{1}{4}$	$\frac{1}{4}$	7	11	8	2	$4\frac{1}{4}$	$\frac{1}{4}$	13	3	8
$4\frac{1}{2}$	2	2	0	1	$4\frac{1}{2}$	$\frac{1}{2}$	7	14	0	2	$4\frac{1}{2}$	$\frac{1}{2}$	13	6	0
$4\frac{3}{4}$	2	4	4	1	$4\frac{3}{4}$	$\frac{3}{4}$	7	16	4	2	$4\frac{3}{4}$	$\frac{3}{4}$	13	8	4
5	2	6	8	1	5	1	7	18	8	2	5	1	13	10	8
$5\frac{1}{4}$	2	9	0	1	$5\frac{1}{4}$	$\frac{1}{4}$	8	1	0	2	$5\frac{1}{4}$	$\frac{1}{4}$	13	13	0
$5\frac{1}{2}$	2	11	4	1	$5\frac{1}{2}$	$\frac{1}{2}$	8	3	4	2	$5\frac{1}{2}$	$\frac{1}{2}$	13	15	4
$5\frac{3}{4}$	2	13	8	1	$5\frac{3}{4}$	$\frac{3}{4}$	8	5	8	2	$5\frac{3}{4}$	$\frac{3}{4}$	13	17	8
6	2	16	0	1	6	1	8	8	0	2	6	1	14	0	0
$6\frac{1}{4}$	2	18	4	1	$6\frac{1}{4}$	$\frac{1}{4}$	8	10	4	2	$6\frac{1}{4}$	$\frac{1}{4}$	14	2	4
$6\frac{1}{2}$	3	0	8	1	$6\frac{1}{2}$	$\frac{1}{2}$	8	12	8	2	$6\frac{1}{2}$	$\frac{1}{2}$	14	4	8
$6\frac{3}{4}$	3	3	0	1	$6\frac{3}{4}$	$\frac{3}{4}$	8	15	0	2	$6\frac{3}{4}$	$\frac{3}{4}$	14	7	0
7	3	5	4	1	7	1	8	17	4	2	7	1	14	9	4
$7\frac{1}{4}$	3	7	8	1	$7\frac{1}{4}$	$\frac{1}{4}$	8	19	8	2	$7\frac{1}{4}$	$\frac{1}{4}$	14	11	8
$7\frac{1}{2}$	3	10	0	1	$7\frac{1}{2}$	$\frac{1}{2}$	9	2	0	2	$7\frac{1}{2}$	$\frac{1}{2}$	14	14	0
$7\frac{3}{4}$	3	12	4	1	$7\frac{3}{4}$	$\frac{3}{4}$	9	4	4	2	$7\frac{3}{4}$	$\frac{3}{4}$	14	16	4
8	3	14	8	1	8	1	9	6	8	2	8	1	14	18	8
$8\frac{1}{4}$	3	17	0	1	$8\frac{1}{4}$	$\frac{1}{4}$	9	9	0	2	$8\frac{1}{4}$	$\frac{1}{4}$	15	1	0
$8\frac{1}{2}$	3	19	4	1	$8\frac{1}{2}$	$\frac{1}{2}$	9	11	4	2	$8\frac{1}{2}$	$\frac{1}{2}$	15	3	4
$8\frac{3}{4}$	4	1	8	1	$8\frac{3}{4}$	$\frac{3}{4}$	9	13	8	2	$8\frac{3}{4}$	$\frac{3}{4}$	15	5	8
9	4	4	0	1	9	1	9	16	0	2	9	1	15	8	0
$9\frac{1}{4}$	4	6	4	1	$9\frac{1}{4}$	$\frac{1}{4}$	9	18	4	2	$9\frac{1}{4}$	$\frac{1}{4}$	15	10	4
$9\frac{1}{2}$	4	8	8	1	$9\frac{1}{2}$	$\frac{1}{2}$	10	0	8	2	$9\frac{1}{2}$	$\frac{1}{2}$	15	12	8
$9\frac{3}{4}$	4	11	0	1	$9\frac{3}{4}$	$\frac{3}{4}$	10	3	0	2	$9\frac{3}{4}$	$\frac{3}{4}$	15	15	0
10	4	13	4	1	10	1	10	5	4	2	10	1	15	17	4
$10\frac{1}{4}$	4	15	8	1	$10\frac{1}{4}$	$\frac{1}{4}$	10	7	8	2	$10\frac{1}{4}$	$\frac{1}{4}$	15	19	8
$10\frac{1}{2}$	4	18	0	1	$10\frac{1}{2}$	$\frac{1}{2}$	10	10	0	2	$10\frac{1}{2}$	$\frac{1}{2}$	16	2	0
$10\frac{3}{4}$	5	0	4	1	$10\frac{3}{4}$	$\frac{3}{4}$	10	12	4	2	$10\frac{3}{4}$	$\frac{3}{4}$	16	4	4
11	5	2	8	1	11	1	10	14	8	2	11	1	16	6	8
$11\frac{1}{4}$	5	5	0	1	$11\frac{1}{4}$	$\frac{1}{4}$	10	17	0	2	$11\frac{1}{4}$	$\frac{1}{4}$	16	9	0
$11\frac{1}{2}$	5	7	4	1	$11\frac{1}{2}$	$\frac{1}{2}$	10	19	4	2	$11\frac{1}{2}$	$\frac{1}{2}$	16	11	4
$11\frac{3}{4}$	5	9	8	1	$11\frac{3}{4}$	$\frac{3}{4}$	11	1	8	2	$11\frac{3}{4}$	$\frac{3}{4}$	16	13	8
12	5	2	0	2	0	1	11	4	0	3	0	1	16	16	0

A Comparison of the American Foot with the Feet of other Countries.

The American foot being divided into 1000 parts, or into 12 inches, the feet of several other countries will be as follow.

	Parts.		Inch.	lin.	points.
America, - - -	1000	- - -	12	0	0 dec.
London, - - -	1000	- - -	12	0	0
Antwerp, - - -	946	- - -	11	4	1,32
Bologna, - - -	1204	- - -	14	5	2,25
Bremen, - - -	964	- - -	11	6	4,89
Cologne, - - -	954	- - -	11	5	2,25
Copenhagen, - -	965	- - -	11	6	5,76
Amsterdam, - -	942	- - -	11	3	3,88
Dantzick, - - -	944	- - -	11	3	5,61
Dort, - - -	1184	- - -	14	2	2,97
Frankfort on the Main, 9 8		- - -	11	4	3,07
The Greek, - - -	1007	- - -	12	1	0,04
Lorrain, - - -	958	- - -	11	5	5,71
Mantua, - - -	1569	- - -	18	9	5,61
Mecklin, - - -	919	- - -	11	0	2,01
Middleburg, - -	991	- - -	11	10	4,22
France, - - -	938	- - -	11	3	0,43
Prague, - - -	1026	- - -	12	3	4,46
Rhyneland or Leyden, 1033		- - -	12	4	4,51
Riga, - - -	1831	- - -	21	11	3,98
Roman, - - -	967	- - -	11	7	1,48
Old Roman, - - -	970	- - -	11	8	0
Scotch, - - -	1005	- - -	12	0	4,32
Straßburgh, - -	920	- - -	11	0	2,88
Toledo, - - -	899	- - -	10	9	2,73
Turin, - - -	1062	- - -	12	8	5,66
Venice, - - -	1162	- - -	13	11	1,96

A TABLE representing the Conformity of the Weights of the principal trading Cities of Europe with those of America.

lb	of America.
100 of England, Scotland and Ireland,	Equal 100lb 0oz.
100 of Amsterdam, Paris, Bourdeaux, &c.	— 109 8
100 of Antwerp, or Brabant, - - -	— 103 12
100 of Rouën, the Viscounty, - - -	— 113 14
100 of Lyons, the city, - - -	— 94 3
100 of Rochelle, - - -	— 110 9
100 of Thouloufe, and Upper Languedoc, -	— 92 6
100 of Marseilles and Provence, - -	— 88 11
100 of Geneva, - - -	— 123
100 of Hamburg, - - -	— 107 5
100 of Francfort, - - -	— 111 11
100 of Leipzig, - - -	— 104 5

A TABLE representing the Conformity of the Weights of the principal trading Cities of Europe with those of America.

£	of America.
100 of Genoa,	Equal 73
100 of Leghorn,	75 8
100 of Milan,	65 3
100 of Venice,	65 11
100 of Naples,	64 10
100 of Seville, Cadiz, &c.	103 7
100 of Portugal,	95 4
100 of Liege,	104
100 of Spain,	97 dr
Note, The Spanish Arrobe is 25 Spanish pounds,	
	25 12 6

A TABLE to cast up wages, or expenses, for a year, at so much per day, week, or month

Day.	by week.	by mon.	by year.
s. d.	£. s. d.	£. s. d.	£. s. d.
0 10	0 0 7	0 2 4	1 10 5
0 20	0 1 2	0 4 8	3 0 10
0 30	0 1 9	0 7 0	4 11 3
0 40	0 2 4	0 9 4	6 1 8
0 50	0 2 11	0 11 8	7 12 1
0 60	0 3 6	0 14 0	9 2 6
0 70	0 4 1	0 16 4	10 12 11
0 80	0 4 8	0 18 8	12 3 4
0 90	0 5 3	1 1 0	13 13 9
0 100	0 5 10	1 3 4	15 4 2
0 110	0 6 5	1 5 8	16 14 7
1 00	0 7 0	1 8 0	18 5 0
2 00	0 14 0	2 16 0	36 10 0
3 00	1 1 0	4 4 0	54 15 0
4 00	1 8 0	5 12 0	73 0 0
5 00	1 15 0	7 0 0	91 5 0
6 00	2 2 0	8 8 0	109 10 0
7 00	2 9 0	9 16 0	127 15 0
8 00	2 16 0	11 4 0	146 0 0
9 00	3 3 0	12 12 0	164 5 0
10 00	3 10 0	14 0 0	182 10 0
11 00	3 17 0	15 8 0	200 15 0
12 00	4 4 0	16 16 0	219 0 0
13 00	4 11 0	18 4 0	237 5 0
14 00	4 18 0	19 12 0	255 10 0
15 00	5 5 0	21 0 0	273 15 0
16 00	5 12 0	22 8 0	292 0 0
17 00	5 19 0	23 16 0	310 5 0
18 00	6 6 0	25 4 0	328 10 0
19 00	6 13 0	26 12 0	346 15 0
20 00	7 0 0	28 0 0	365 0 0

Note, In these two Tables the month is only 28 days.

A TABLE to find wages or expenses, for a month, week, or day, at so much by the year.

by yr.	by month.	by week.	by day.
£.	£. s. d.	£. s. d.	£. s. d.
1	0 1 6 $\frac{1}{2}$	0 0 4 $\frac{1}{2}$	0 0 0 $\frac{3}{4}$
2	0 3 0 $\frac{1}{2}$	0 0 9 $\frac{1}{4}$	0 0 1 $\frac{1}{4}$
3	0 4 7 $\frac{1}{4}$	0 1 1 $\frac{1}{4}$	0 0 2 $\frac{1}{2}$
4	0 6 1 $\frac{1}{4}$	0 1 6 $\frac{1}{2}$	0 0 2 $\frac{1}{2}$
5	0 7 8	0 1 11	0 0 3 $\frac{1}{4}$
6	0 9 2 $\frac{1}{2}$	0 2 3 $\frac{1}{2}$	0 0 4 $\frac{1}{2}$
7	0 10 9	0 2 8 $\frac{1}{4}$	0 0 4 $\frac{1}{2}$
8	0 12 3 $\frac{1}{4}$	0 3 0 $\frac{3}{4}$	0 0 5 $\frac{1}{4}$
9	0 13 9 $\frac{1}{4}$	0 3 5 $\frac{1}{2}$	0 0 6
10	0 15 4	0 3 10	0 0 6 $\frac{1}{2}$
11	0 16 10 $\frac{1}{2}$	0 4 2 $\frac{3}{4}$	0 0 7 $\frac{1}{4}$
12	0 18 5	0 4 7 $\frac{1}{4}$	0 0 8
13	0 19 11 $\frac{1}{4}$	0 4 11 $\frac{1}{2}$	0 0 8 $\frac{1}{2}$
14	1 1 5 $\frac{3}{4}$	0 5 4 $\frac{1}{2}$	0 0 9 $\frac{1}{4}$
15	1 3 0 $\frac{1}{4}$	0 5 9	0 0 9 $\frac{1}{2}$
16	1 4 6 $\frac{1}{2}$	0 6 1 $\frac{1}{2}$	0 0 10 $\frac{1}{2}$
17	1 6 1	0 6 6 $\frac{1}{4}$	0 0 11 $\frac{1}{4}$
18	1 7 7 $\frac{1}{2}$	0 6 10 $\frac{3}{4}$	0 0 11 $\frac{1}{2}$
19	1 9 1 $\frac{3}{4}$	0 7 3 $\frac{1}{2}$	0 1 0 $\frac{1}{2}$
20	1 10 8 $\frac{1}{4}$	0 7 8	0 1 1 $\frac{1}{4}$
30	2 6 0 $\frac{1}{4}$	0 11 6	0 1 7 $\frac{1}{2}$
40	3 1 4 $\frac{1}{2}$	0 15 4	0 2 2 $\frac{1}{4}$
50	3 16 8 $\frac{1}{2}$	0 19 2 $\frac{1}{4}$	0 2 9
60	4 12 0 $\frac{3}{4}$	1 3 0 $\frac{1}{4}$	0 3 3 $\frac{1}{2}$
70	5 7 4 $\frac{1}{4}$	1 6 10 $\frac{1}{4}$	0 3 10
80	6 3 9	1 10 8 $\frac{1}{4}$	0 4 4 $\frac{1}{4}$
90	6 18 1	1 14 6 $\frac{1}{4}$	0 4 11 $\frac{1}{4}$
100	7 13 5	1 18 4 $\frac{1}{2}$	0 5 5 $\frac{1}{2}$
200	15 6 10 $\frac{1}{4}$	3 16 8 $\frac{1}{2}$	0 10 11 $\frac{1}{2}$
300	23 0 3 $\frac{1}{4}$	5 15 0 $\frac{1}{2}$	0 16 5 $\frac{1}{2}$
400	30 13 8 $\frac{1}{2}$	7 13 5	1 1 11
500	38 7 1 $\frac{1}{2}$	9 11 9 $\frac{1}{4}$	1 7 4 $\frac{1}{2}$
1000	76 14 3	18 3 6 $\frac{1}{2}$	2 14 0 $\frac{1}{2}$

SIMPLE INTEREST, at 6 per cent. from 1s. to £1000, and from 1 day to 1 year.

Princ. money.	1 day.	2 days.	3 days.	4 days.	5 days.	6 days.	7 days.
s. 1	s. d. q.	s. d. q.	s. d. q.	s. d. q.	s. d. q.	s. d. q.	£ s. d. q.
2	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
3	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
4	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
5	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
6	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
7	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
8	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
9	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
10	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
11	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
12	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
13	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
14	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
15	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
16	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
17	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
18	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0
19	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 1
£ 1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 1
2	0 0 0	0 0 0	0 0 0	0 0 0	0 0 1	0 0 1	0 0 0 2
3	0 0 0	0 0 0	0 0 0	0 0 1	0 0 1	0 0 2	0 0 0 3
4	0 0 0	0 0 1	0 0 1	0 0 2	0 0 3	0 0 3	0 0 1 0
5	0 0 0	0 0 1	0 0 2	0 0 3	0 0 3	0 0 1	0 0 1 1
6	0 0 0	0 0 1	0 0 2	0 0 3	0 0 1	0 0 1	0 0 1 2
7	0 0 1	0 0 2	0 0 3	0 0 1	0 0 1	0 0 1	0 0 1 3
8	0 0 1	0 0 2	0 0 3	0 0 1	0 0 1	0 0 1	0 0 2 0
9	0 0 1	0 0 2	0 0 1	0 0 1	0 0 1	0 0 2	0 0 2 1
10	0 0 1	0 0 3	0 0 1	0 0 1	0 0 1	0 0 2	0 0 2 3
20	0 0 3	0 0 1	0 0 2	0 0 3	0 0 3	0 0 4	0 0 5 2
30	0 0 1	0 0 2	0 0 3	0 0 4	0 0 5	0 0 7	0 0 8 1
40	0 0 1	0 0 3	0 0 4	0 0 6	0 0 7	0 0 9	0 0 11 0
50	0 0 1	0 0 3	0 0 5	0 0 7	0 0 9	0 0 11	0 0 1 3
60	0 0 2	0 0 4	0 0 7	0 0 9	0 0 11	0 0 1 2	0 0 1 4 2
70	0 0 2	0 0 5	0 0 8	0 0 11	0 0 1 1	0 0 1 4	0 0 1 7 1
80	0 0 3	0 0 6	0 0 9	0 0 1 1	0 0 1 3	0 0 1 6	0 0 1 10 0
90	0 0 3	0 0 7	0 0 10	0 0 1 2	0 0 1 5	0 0 1 9	0 0 2 0 3
100	0 0 3	0 0 7	0 0 11	0 0 1 3	0 0 1 7	0 0 1 11	0 0 2 3 2
200	0 0 7	0 0 1 3	0 0 1 11	0 0 2 7	0 0 3 3	0 0 3 11	0 0 4 7 0
300	0 0 11	0 0 2 11	0 0 3 11	0 0 4 11	0 0 5 10	0 0 6 10	0 0 8 10 3
400	0 0 1 3	0 0 3 11	0 0 5 11	0 0 6 6	0 0 7 10	0 0 9 2	0 0 9 2 1
500	0 0 1 7	0 0 3 14	0 0 6 11	0 0 8 2	0 0 9 10	0 0 11 6	0 0 11 6 0
600	0 0 1 11	0 0 4 15	0 0 7 10	0 0 9 10	0 0 11 9	0 0 13 9	0 0 13 9 2
700	0 0 2 3	0 0 5 7	0 0 8 10	0 0 10 2	0 0 11 5	0 0 13 9	0 0 16 1 0
800	0 0 2 7	0 0 5 10	0 0 9 10	0 0 10 6	0 0 13 1	0 0 15 9	0 0 18 4 3
900	0 0 2 11	0 0 6 10	0 0 10 1	0 0 11 9	0 0 14 9	0 0 17 8	0 0 20 8 1
1000	0 0 3 3	0 0 6 2	0 0 10 1	0 0 13 1	0 0 16 5	0 0 19 10	0 0 21 3 0

T A B L E S.

397

SIMPLE INTEREST at £6 per cent. from 1s. to £1000, and from 1 day to 1 year.

Princ. money.	8 days.				10 days.				15 days.				30 days, or 1 month.				2 such months.			
	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.
s. 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3
9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	1	0
12	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	1	0
13	0	0	0	0	0	0	0	1	0	0	0	1	0	0	3	0	0	1	2	0
14	0	0	0	0	0	0	0	1	0	0	0	1	0	0	3	0	0	1	2	0
15	0	0	0	0	0	0	0	1	0	0	0	1	0	0	3	0	0	1	3	0
16	0	0	0	0	0	0	0	1	0	0	0	1	0	0	3	0	0	1	3	0
17	0	0	0	1	0	0	0	1	0	0	0	2	0	0	3	0	0	2	0	0
18	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	2	0	0
19	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	2	0	0
£. 1	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	2	1	0
2	0	0	0	2	0	0	0	3	0	0	1	0	0	0	2	1	0	0	4	2
3	0	0	0	3	0	0	1	0	0	0	1	3	0	0	3	2	0	0	7	0
4	0	0	1	1	0	0	1	2	0	0	2	1	0	0	4	2	0	0	9	1
5	0	0	1	2	0	0	1	3	0	0	2	3	0	0	5	3	0	0	11	3
6	0	0	1	3	0	0	2	1	0	0	3	2	0	0	7	0	0	1	2	0
7	0	0	2	0	0	0	2	3	0	0	4	0	0	0	8	1	0	1	4	2
8	0	0	2	2	0	0	3	0	0	0	4	2	0	0	9	1	0	1	6	3
9	0	0	2	3	0	0	3	2	0	0	5	1	0	0	10	2	0	1	9	1
10	0	0	3	0	0	0	3	3	0	0	5	3	0	0	11	3	0	1	11	2
20	0	0	6	1	0	0	7	3	0	0	11	3	0	1	11	2	0	3	11	1
30	0	0	9	1	0	0	11	3	0	1	5	2	0	2	11	1	0	5	11	0
40	0	1	0	2	0	1	3	3	0	1	11	2	0	3	11	1	0	7	10	2
50	0	1	3	2	0	1	7	2	0	2	5	2	0	4	11	0	0	9	10	1
60	0	1	6	3	0	1	11	2	0	2	11	1	0	5	10	3	0	11	0	0
70	0	1	10	0	0	2	3	2	0	3	5	1	0	6	10	3	0	13	9	2
80	0	2	1	0	0	2	7	2	0	3	11	0	0	7	10	2	0	15	9	1
90	0	2	4	1	0	2	11	1	0	4	5	0	0	8	10	1	0	17	9	0
100	0	2	7	1	0	3	3	1	0	4	11	0	0	9	10	1	0	19	8	2
200	0	5	2	3	0	6	6	3	0	9	10	0	0	19	8	2	1	19	5	1
300	0	7	10	1	0	9	10	1	0	14	9	0	1	9	6	3	2	19	2	0
400	0	10	5	3	0	13	1	2	0	19	8	0	1	19	5	1	3	18	10	2
500	0	13	1	0	0	16	5	0	1	4	7	0	2	9	3	2	4	18	7	2
600	0	15	8	2	0	19	8	2	1	9	6	0	2	19	1	3	5	18	4	1
700	0	18	4	0	1	3	0	0	1	14	5	1	3	9	0	0	6	18	0	3
800	1	0	11	2	1	6	3	1	1	19	4	1	3	18	10	2	7	17	9	2
900	1	3	6	3	1	9	6	3	2	4	4	2	4	8	9	0	8	17	6	1
1000	1	6	2	1	1	12	1	0	2	9	2	1	4	18	7	0	9	17	3	0

SIMPLE INTEREST, at £6 per cent. from 1s. to £1000, and from 1 day to 1 year.

	three months.				four months.				five months.				six months.				seven months.			
	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	s.	s.	d.	q.
s. 1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1
2	0	0	0	1	0	0	0	1	0	0	0	2	0	0	0	2	0	0	0	3
3	0	0	0	2	0	0	0	2	0	0	0	3	0	0	1	0	0	0	1	0
4	0	0	0	3	0	0	0	3	0	0	1	0	0	0	1	1	0	0	1	2
5	0	0	0	3	0	0	1	0	0	0	1	1	0	0	1	3	0	0	2	0
6	0	0	1	0	0	0	1	1	0	0	1	3	0	0	2	0	0	0	2	1
7	0	0	1	0	0	0	1	2	0	0	2	0	0	0	2	1	0	0	2	3
8	0	0	1	1	0	0	1	3	0	0	2	1	0	0	2	3	0	0	3	1
9	0	0	1	2	0	0	2	0	0	0	2	2	0	0	3	0	0	0	3	2
10	0	0	1	3	0	0	2	1	0	0	2	3	0	0	3	2	0	0	4	0
11	0	0	1	3	0	0	2	2	0	0	3	1	0	0	3	3	0	0	4	2
12	0	0	2	0	0	0	2	3	0	0	3	2	0	0	4	1	0	0	4	3
13	0	0	2	1	0	0	3	0	0	0	3	3	0	0	4	2	0	0	5	1
14	0	0	2	1	0	0	3	1	0	0	4	0	0	0	4	3	0	0	5	3
15	0	0	2	2	0	0	3	2	0	0	4	1	0	0	5	1	0	0	6	0
16	0	0	2	3	0	0	3	3	0	0	4	2	0	0	5	2	0	0	6	2
17	0	0	2	3	0	0	4	0	0	0	5	0	0	0	6	0	0	0	7	0
18	0	0	3	0	0	0	4	1	0	0	5	1	0	0	6	1	0	0	7	1
19	0	0	3	1	0	0	4	1	0	0	5	2	0	0	6	2	0	0	7	3
£. 1	0	0	3	2	0	0	4	2	0	0	5	3	0	0	7	0	0	0	8	1
2	0	0	7	0	0	0	9	1	0	0	11	3	0	1	2	0	0	1	4	0
3	0	0	10	2	0	1	2	0	0	1	5	2	0	1	9	0	0	2	0	0
4	0	1	2	0	0	1	6	3	0	1	11	2	0	2	4	0	0	2	9	0
5	0	1	5	2	0	1	11	2	0	2	5	2	0	2	11	0	0	3	5	1
6	0	1	9	0	0	2	4	1	0	2	11	1	0	3	6	0	0	4	1	2
7	0	2	0	2	0	2	9	0	0	3	5	1	0	4	1	0	0	4	9	3
8	0	2	4	0	0	3	1	3	0	3	11	1	0	4	8	0	0	5	6	1
9	0	2	7	2	0	3	6	2	0	4	5	0	0	5	3	0	0	6	2	2
10	0	2	11	0	0	3	11	1	0	4	11	0	0	5	11	0	0	6	10	3
20	0	5	10	0	0	7	10	2	0	9	10	1	0	11	10	0	0	13	9	2
30	0	8	9	0	0	11	9	3	0	14	9	1	0	17	9	0	1	0	8	2
40	0	11	8	0	0	15	9	1	0	19	8	1	1	3	8	0	1	7	7	1
50	0	14	7	0	0	19	8	2	1	4	7	2	1	9	7	0	1	14	6	0
60	0	17	6	0	1	3	7	1	1	9	6	3	1	15	6	0	2	1	5	0
70	1	0	5	0	1	7	7	1	1	14	5	0	2	1	5	0	2	8	3	3
80	1	3	4	0	1	11	6	2	1	19	5	0	2	8	4	0	2	15	2	3
90	1	6	3	0	1	15	5	3	2	4	4	0	2	13	3	0	3	2	1	2
100	1	9	2	0	1	19	5	1	2	9	3	1	2	19	2	0	3	9	0	1
200	2	18	4	0	3	18	10	2	4	18	8	2	5	18	4	0	6	18	0	3
300	4	7	6	0	5	18	3	3	7	8	0	3	8	17	6	0	10	7	1	1
400	5	16	8	0	7	17	9	0	9	17	5	1	11	16	8	0	13	16	1	3
500	7	5	10	0	9	17	2	1	12	6	9	3	14	15	10	0	17	5	2	1
600	8	15	0	0	11	16	7	2	14	16	2	0	17	15	0	0	20	14	2	3
700	10	4	2	0	13	16	0	3	17	5	6	2	20	14	2	0	24	3	3	1
800	11	13	4	0	15	15	6	0	19	15	10	3	23	13	4	0	27	12	3	3
900	13	2	6	0	17	14	11	2	22	4	3	0	26	12	6	0	31	1	4	1
1000	14	11	8	0	19	14	4	3	24	13	7	2	29	11	8	0	34	10	4	3

TABLE OF SIMPLE INTEREST.

393

SIMPLE INTEREST at 6l. per cent. from 1s. to 1000l. and from 1 Day to 1 Year.

Princ. Money	8 Months.			9 Months.			10 Months.			11 Months, or 330 days.			1 Year, or 365 days.		
	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.	l.	s.	d. q.
1	0	0	0 1	0	0	0 2	0	0	0 2	0	0	1 2	0	0	0 2
2	0	0	0 3	0	0	1 0	0	0	1 0	0	0	1 1	0	0	1 1
3	0	0	1 1	0	0	1 2	0	0	1 3	0	0	1 3	0	0	2 0
4	0	0	1 3	0	0	2 0	0	0	2 1	0	0	2 2	0	0	2 3
5	0	0	2 1	0	0	2 2	0	0	2 3	0	0	3 1	0	0	3 2
6	0	0	2 3	0	0	3 0	0	0	3 2	0	0	3 3	0	0	4 1
7	0	0	3 1	0	0	3 2	0	0	4 0	0	0	4 2	0	0	5 0
8	0	0	3 3	0	0	4 1	0	0	4 2	0	0	5 0	0	0	5 3
9	0	0	4 1	0	0	4 3	0	0	5 1	0	0	5 3	0	0	6 1
10	0	0	4 2	0	0	5 1	0	0	5 3	0	0	6 2	0	0	7 0
11	0	0	5 0	0	0	5 3	0	0	6 1	0	0	7 0	0	0	7 2
12	0	0	5 2	0	0	6 1	0	0	7 0	0	0	7 3	0	0	8 0
13	0	0	6 0	0	0	6 3	0	0	7 2	0	0	8 1	0	0	9 1
14	0	0	6 2	0	0	7 1	0	0	8 1	0	0	9 0	0	0	10 0
15	0	0	7 0	0	0	7 3	0	0	8 3	0	0	9 3	0	0	10 3
16	0	0	7 2	0	0	8 2	0	0	9 1	0	0	10 1	0	0	11 2
17	0	0	8 0	0	0	9 0	0	0	10 0	0	0	11 0	0	1	0 0
18	0	0	8 2	0	0	9 3	0	0	10 2	0	0	11 2	0	1	0 3
19	0	0	8 3	0	0	10 0	0	0	11 0	0	1	0 1	0	1	1 2
L. 1	0	0	9 1	0	0	10 2	0	0	11 3	0	1	1 0	0	1	2 1
2	0	1	6 3	0	1	9 1	0	1	11 2	0	2	2 0	0	2	4 3
3	0	2	4 1	0	2	7 3	0	2	11 1	0	3	3 0	0	3	7 0
4	0	3	1 3	0	3	6 2	0	3	11 0	0	4	4 0	0	4	9 2
5	0	3	11 1	0	4	2 0	0	4	11 0	0	5	5 0	0	6	0 0
6	0	4	8 2	0	5	3 3	0	5	10 3	0	6	6 0	0	7	2 1
7	0	5	6 0	0	6	2 2	0	6	10 2	0	7	7 0	0	8	4 3
8	0	6	3 2	0	7	1 0	0	7	8 3	0	8	8 0	0	9	7 0
9	0	7	1 0	0	7	11 3	0	8	10 0	0	9	9 0	0	10	9 2
10	0	7	10 2	0	8	10 2	0	9	10 0	0	10	10 0	0	13	0 0
20	0	15	9 0	0	17	9 0	0	19	8 0	1	1	8 0	1	4	0 0
30	1	3	7 2	1	6	7 2	1	9	6 0	1	12	6 0	1	16	0 0
40	1	11	6 0	1	15	6 0	1	19	4 1	2	3	4 0	2	8	0 0
50	1	19	4 3	2	4	4 2	2	9	2 1	2	14	2 0	3	0	0 0
60	2	7	3 1	2	13	3 0	2	19	0 1	3	5	0 0	3	12	0 0
70	2	15	1 3	3	2	1 2	3	8	10 2	3	15	10 0	4	4	0 0
80	3	3	0 1	3	11	0 0	3	18	8 2	4	6	8 0	4	16	0 0
90	3	10	11 0	3	19	10 2	4	8	6 2	4	17	6 0	5	8	0 0
100	3	18	9 2	4	8	11 0	4	18	4 3	5	8	4 0	6	0	0 0
200	7	17	7 0	8	17	10 0	9	16	9 2	10	6	8 0	12	0	0 0
300	11	16	4 0	13	6	9 0	14	15	2 1	16	5	0 0	18	0	0 0
400	15	15	2 1	17	15	8 0	19	13	7 0	21	13	4 0	24	0	0 0
500	29	14	0 0	22	4	7 0	24	12	0 0	27	1	8 0	30	0	0 0
600	23	12	9 2	26	13	6 0	29	10	4 3	32	10	0 0	36	0	0 0
700	37	11	7 0	31	2	5 0	31	8	5 2	37	18	4 0	42	0	0 0
800	31	10	4 3	35	11	4 0	39	7	3 1	43	6	8 0	48	0	0 0
900	35	9	2 1	40	0	4 0	44	5	7 0	48	15	0 0	54	0	0 0
1000	39	8	0 0	44	9	3 0	49	4	0 0	54	3	4 0	60	0	0 0

PERPETUAL ALMANACK.

February March November	February* August	May	January October	January* April July	September December	June
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

To find on what day of the week any given day in any month will fall, and the contrary.

E X A M P L E.

On what day of the week will the 31st day of January, 1787, fall?
Observe the day of the week annexed to the year in the outer column; then, in the Table, under the given month, in the upper row of figures, you will find the day of the month on which that day falls.—According to this direction, I find that, in January 1787, Thursday is the 4th, 11th, 18th, and 25th, then reckoning on, Friday 26th, Saturday 27th, &c. I find the 31st day falls on Wednesday; or, that the last Wednesday in January is the 31st day.

NOTES. In leap years, January and February must be taken in the columns marked thus, *. Leap years are marked in the outer columns, thus, †.

The years 1800, 1803, and all other 100th years, not to be leap years, except the years 2000, 2400, 2800, and every 400th year following, which must be leap years.

1787 Thursday	1801 Sunday	1815 Wednesday	1829 Sunday	1843 Tuesday	1857 Monday
1788 Saturday†	1802 Monday	1816 Friday†	1830 Monday	1844 Friday†	1858 Tuesday
1789 Sunday	1803 Tuesday	1817 Saturday	1831 Tuesday	1845 Saturday	1859 Wednesday
1790 Monday	1804 Thursday†	1818 Sunday	1832 Thursday†	1846 Sunday	1860 Thursday
1791 Tuesday	1805 Friday	1819 Monday	1833 Friday	1847 Monday	1861 Friday
1792 Thursday†	1806 Saturday	1820 Tuesday	1834 Saturday	1848 Wednesday†	1862 Saturday
1793 Friday	1807 Sunday	1821 Wednesday	1835 Sunday	1849 Thursday	1863 Sunday
1794 Saturday	1808 Tuesday†	1822 Thursday	1836 Tuesday†	1850 Friday	1864 Monday
1795 Sunday	1809 Wednesday	1823 Friday	1837 Wednesday	1851 Saturday	1865 Tuesday
1796 Tuesday†	1810 Thursday	1824 Saturday	1838 Thursday	1852 Monday†	1866 Wednesday
1797 Wednesday	1811 Friday	1825 Sunday	1839 Friday	1853 Tuesday	1867 Thursday
1798 Thursday	1812 Sunday†	1826 Monday	1840 Sunday†	1854 Wednesday	1868 Friday
1799 Friday	1813 Monday	1827 Tuesday	1841 Monday	1855 Thursday	1869 Saturday
1800 Saturday	1814 Tuesday	1828 Wednesday	1842 Tuesday	1856 Friday	1870 Sunday
	1815 Wednesday	1829 Thursday	1843 Wednesday	1857 Saturday	1871 Monday
	1816 Thursday	1830 Friday	1844 Thursday	1858 Sunday	1872 Tuesday
	1817 Friday	1831 Saturday	1845 Friday	1859 Monday	1873 Wednesday
	1818 Saturday	1832 Sunday	1846 Saturday	1860 Tuesday	1874 Thursday
	1819 Sunday	1833 Monday	1847 Sunday	1861 Wednesday	1875 Friday
	1820 Monday	1834 Tuesday	1848 Monday	1862 Thursday	1876 Saturday
	1821 Tuesday	1835 Wednesday	1849 Tuesday	1863 Friday	1877 Sunday
	1822 Wednesday	1836 Thursday	1850 Wednesday	1864 Saturday	1878 Monday
	1823 Thursday	1837 Friday	1851 Thursday	1865 Sunday	1879 Tuesday
	1824 Friday	1838 Saturday	1852 Friday	1866 Monday	1880 Wednesday
	1825 Saturday	1839 Sunday	1853 Saturday	1867 Tuesday	1881 Thursday
	1826 Sunday	1840 Monday	1854 Sunday	1868 Wednesday	1882 Friday
	1827 Monday	1841 Tuesday	1855 Monday	1869 Thursday	1883 Saturday
	1828 Tuesday	1842 Wednesday	1856 Tuesday	1870 Friday	1884 Sunday
	1829 Wednesday	1843 Thursday	1857 Wednesday	1871 Saturday	1885 Monday
	1830 Thursday	1844 Friday	1858 Thursday	1872 Sunday	1886 Tuesday
	1831 Friday	1845 Saturday	1859 Friday	1873 Monday	1887 Wednesday
	1832 Saturday	1846 Sunday	1860 Saturday	1874 Tuesday	1888 Thursday
	1833 Sunday	1847 Monday	1861 Sunday	1875 Wednesday	1889 Friday
	1834 Monday	1848 Tuesday	1862 Monday	1876 Thursday	1890 Saturday
	1835 Tuesday	1849 Wednesday	1863 Tuesday	1877 Friday	1891 Sunday
	1836 Wednesday	1850 Thursday	1864 Wednesday	1878 Saturday	1892 Monday
	1837 Thursday	1851 Friday	1865 Thursday	1879 Sunday	1893 Tuesday
	1838 Friday	1852 Saturday	1866 Friday	1880 Monday	1894 Wednesday
	1839 Saturday	1853 Sunday	1867 Saturday	1881 Tuesday	1895 Thursday
	1840 Sunday	1854 Monday	1868 Sunday	1882 Wednesday	1896 Friday
	1841 Monday	1855 Tuesday	1869 Monday	1883 Thursday	1897 Saturday
	1842 Tuesday	1856 Wednesday	1870 Tuesday	1884 Friday	1898 Sunday
	1843 Wednesday	1857 Thursday	1871 Wednesday	1885 Saturday	1899 Monday
	1844 Thursday	1858 Friday	1872 Thursday	1886 Sunday	1900 Tuesday
	1845 Friday	1859 Saturday	1873 Friday	1887 Monday	1901 Wednesday
	1846 Saturday	1860 Sunday	1874 Saturday	1888 Tuesday	1902 Thursday
	1847 Sunday	1861 Monday	1875 Sunday	1889 Wednesday	1903 Friday
	1848 Monday	1862 Tuesday	1876 Monday	1890 Thursday	1904 Saturday
	1849 Tuesday	1863 Wednesday	1877 Tuesday	1891 Friday	1905 Sunday
	1850 Wednesday	1864 Thursday	1878 Wednesday	1892 Saturday	1906 Monday
	1851 Thursday	1865 Friday	1879 Thursday	1893 Sunday	1907 Tuesday
	1852 Friday	1866 Saturday	1880 Friday	1894 Monday	1908 Wednesday
	1853 Saturday	1867 Sunday	1881 Saturday	1895 Tuesday	1909 Thursday
	1854 Sunday	1868 Monday	1882 Sunday	1896 Wednesday	1910 Friday
	1855 Monday	1869 Tuesday	1883 Monday	1897 Thursday	1911 Saturday
	1856 Tuesday	1870 Wednesday	1884 Tuesday	1898 Friday	1912 Sunday
	1857 Wednesday	1871 Thursday	1885 Wednesday	1899 Saturday	1913 Monday
	1858 Thursday	1872 Friday	1886 Thursday	1900 Sunday	1914 Tuesday
	1859 Friday	1873 Saturday	1887 Friday	1901 Monday	1915 Wednesday
	1860 Saturday	1874 Sunday	1888 Saturday	1902 Tuesday	1916 Thursday
	1861 Sunday	1875 Monday	1889 Sunday	1903 Wednesday	1917 Friday
	1862 Monday	1876 Tuesday	1890 Monday	1904 Thursday	1918 Saturday
	1863 Tuesday	1877 Wednesday	1891 Tuesday	1905 Friday	1919 Sunday
	1864 Wednesday	1878 Thursday	1892 Wednesday	1906 Saturday	1920 Monday
	1865 Thursday	1879 Friday	1893 Thursday	1907 Sunday	1921 Tuesday
	1866 Friday	1880 Saturday	1894 Friday	1908 Monday	1922 Wednesday
	1867 Saturday	1881 Sunday	1895 Saturday	1909 Tuesday	1923 Thursday
	1868 Sunday	1882 Monday	1896 Sunday	1910 Wednesday	1924 Friday
	1869 Monday	1883 Tuesday	1897 Monday	1911 Thursday	1925 Saturday
	1870 Tuesday	1884 Wednesday	1898 Tuesday	1912 Friday	1926 Sunday
	1871 Wednesday	1885 Thursday	1899 Wednesday	1913 Saturday	1927 Monday
	1872 Thursday	1886 Friday	1900 Thursday	1914 Sunday	1928 Tuesday
	1873 Friday	1887 Saturday	1901 Friday	1915 Monday	1929 Wednesday
	1874 Saturday	1888 Sunday	1902 Saturday	1916 Tuesday	1930 Thursday
	1875 Sunday	1889 Monday	1903 Sunday	1917 Wednesday	1931 Friday
	1876 Monday	1890 Tuesday	1904 Monday	1918 Thursday	1932 Saturday
	1877 Tuesday	1891 Wednesday	1905 Tuesday	1919 Friday	1933 Sunday
	1878 Wednesday	1892 Thursday	1906 Wednesday	1920 Saturday	1934 Monday
	1879 Thursday	1893 Friday	1907 Thursday	1921 Sunday	1935 Tuesday
	1880 Friday	1894 Saturday	1908 Friday	1922 Monday	1936 Wednesday
	1881 Saturday	1895 Sunday	1909 Saturday	1923 Tuesday	1937 Thursday
	1882 Sunday	1896 Monday	1910 Sunday	1924 Wednesday	1938 Friday
	1883 Monday	1897 Tuesday	1911 Monday	1925 Thursday	1939 Saturday
	1884 Tuesday	1898 Wednesday	1912 Tuesday	1926 Friday	1940 Sunday
	1885 Wednesday	1899 Thursday	1913 Wednesday	1927 Saturday	1941 Monday
	1886 Thursday	1900 Friday	1914 Thursday	1928 Sunday	1942 Tuesday
	1887 Friday	1901 Saturday	1915 Friday	1929 Monday	1943 Wednesday
	1888 Saturday	1902 Sunday	1916 Saturday	1930 Tuesday	1944 Thursday
	1889 Sunday	1903 Monday	1917 Sunday	1931 Wednesday	1945 Friday
	1890 Monday	1904 Tuesday	1918 Monday	1932 Thursday	1946 Saturday
	1891 Tuesday	1905 Wednesday	1919 Tuesday	1933 Friday	1947 Sunday
	1892 Wednesday	1906 Thursday	1920 Wednesday	1934 Saturday	1948 Monday
	1893 Thursday	1907 Friday	1921 Thursday	1935 Sunday	1949 Tuesday
	1894 Friday	1908 Saturday	1922 Friday	1936 Monday	1950 Wednesday
	1895 Saturday	1909 Sunday	1923 Saturday	1937 Tuesday	1951 Thursday
	1896 Sunday	1910 Monday	1924 Sunday	1938 Wednesday	1952 Friday
	1897 Monday	1911 Tuesday	1925 Monday	1939 Thursday	1953 Saturday
	1898 Tuesday	1912 Wednesday	1926 Tuesday	1940 Friday	1954 Sunday
	1899 Wednesday	1913 Thursday	1927 Wednesday	1941 Saturday	1955 Monday
	1900 Thursday	1914 Friday	1928 Thursday	1942 Sunday	1956 Tuesday
	1901 Friday	1915 Saturday	1929 Friday	1943 Monday	1957 Wednesday
	1902 Saturday	1916 Sunday	1930 Saturday	1944 Tuesday	1958 Thursday
	1903 Sunday	1917 Monday	1931 Sunday	1945 Wednesday	1959 Friday
	1904 Monday	1918 Tuesday	1932 Monday	1946 Thursday	1960 Saturday
	1905 Tuesday	1919 Wednesday	1933 Tuesday	1947 Friday	1961 Sunday
	1906 Wednesday	1920 Thursday	1934 Wednesday	1948 Saturday	1962 Monday
	1907 Thursday	1921 Friday	1935 Thursday	1949 Sunday	1963 Tuesday
	1908 Friday	1922 Saturday	1936 Friday	1950 Monday	1964 Wednesday
	1909 Saturday	1923 Sunday	1937 Saturday	1951 Tuesday	1965 Thursday
	1910 Sunday	1924 Monday	1938 Sunday	1952 Wednesday	1966 Friday
	1911 Monday	1925 Tuesday	1939 Monday	1953 Thursday	1967 Saturday
	1912 Tuesday	1926 Wednesday	1940 Tuesday	1954 Friday	1968 Sunday
	1913 Wednesday	1927 Thursday	1941 Wednesday	1955 Saturday	1969 Monday
	1914 Thursday	1928 Friday	1942 Thursday	1956 Sunday	1970 Tuesday
	1915 Friday	1929 Saturday	1943 Friday	1957 Monday	1971 Wednesday
	1916 Saturday	1930 Sunday	1944 Saturday	1958 Tuesday	1972 Thursday
	1917 Sunday	1931 Monday	1945 Sunday	1959 Wednesday	1973 Friday
	1918 Monday	1932 Tuesday	1946 Monday	1960 Thursday	1974 Saturday
	1919 Tuesday	1933 Wednesday	1947 Tuesday	1961 Friday	1975 Sunday
	1920 Wednesday	1934 Thursday	1948 Wednesday	1962 Saturday	1976 Monday
	1921 Thursday	1935 Friday	1949 Thursday	1963 Sunday	1977 Tuesday
	1922 Friday	1936 Saturday	1950 Friday	1964 Monday	1978 Wednesday
	1923 Saturday	1937 Sunday	1951 Saturday	1965 Tuesday	1979 Thursday
	1924 Sunday	1938 Monday	1952 Sunday	1966 Wednesday	1980 Friday
	1925 Monday	1939 Tuesday	1953 Monday	1967 Thursday	1981 Saturday
	1926 Tuesday	1940 Wednesday	1954 Tuesday	1968 Friday	1982 Sunday
	1927 Wednesday	1941 Thursday	1955 Wednesday	1969 Saturday	1983 Monday
	1928 Thursday	1942 Friday	1956 Thursday	1970 Sunday	1984 Tuesday
	1929 Friday	1943 Saturday	1957 Friday	1971 Monday	1985 Wednesday
	1930 Saturday	1944 Sunday	1958 Saturday	1972 Tuesday	1986 Thursday
	1931 Sunday	1945 Monday	1959 Sunday	1973 Wednesday	1987 Friday
	1932 Monday	1946 Tuesday	1960 Monday	1974 Thursday	1988 Saturday
	1933 Tuesday	1947 Wednesday	1961 Tuesday	1975 Friday	1989 Sunday
	1934 Wednesday	1948 Thursday	1962 Wednesday	1976 Saturday	1990 Monday
	1935 Thursday	1949 Friday	1963 Thursday	1977 Sunday	1991 Tuesday
	1936 Friday	1950 Saturday	1964 Friday	1978 Monday	1992 Wednesday
	1937 Saturday	1951 Sunday	1965 Saturday	1979 Tuesday	1993 Thursday
	1938 Sunday	1952 Monday	1966 Sunday	1980 Wednesday	1994 Friday
	1939 Monday	1953 Tuesday	1967 Monday	1981 Thursday	1995 Saturday
	1940 Tuesday	1954 Wednesday	1968 Tuesday	1982 Friday	1996 Sunday
	1941 Wednesday	1955 Thursday	1969 Wednesday	1983 Saturday	1997 Monday
	1942 Thursday	1956 Friday	1970 Thursday	1984 Sunday	1998 Tuesday
	1943 Friday	1957 Saturday	1971 Friday	1985 Monday	1999 Wednesday
	1944 Saturday	1958 Sunday	1972 Saturday	1986 Tuesday	2000 Thursday
	1945 Sunday	1959 Monday	1973 Sunday	1987 Wednesday	2001 Friday
	1946 Monday	1960 Tuesday	1974 Monday	1988 Thursday	2002 Saturday
	1947 Tuesday	1961 Wednesday	1975 Tuesday	1989 Friday	2003 Sunday
	1948 Wednesday	1962 Thursday	1976 Wednesday	1990 Saturday	2004 Monday
	1949 Thursday	1963 Friday	1977 Thursday	1991 Sunday	2005 Tuesday
	1950 Friday	1964 Saturday	1978 Friday	1992 Monday	2006 Wednesday
	1951 Saturday	1965 Sunday	1979 Saturday	1993 Tuesday	2007 Thursday
	1952 Sunday	1966 Monday	1980 Sunday	1994 Wednesday	2008 Friday
	1953 Monday	1967 Tuesday	1981 Monday	1995 Thursday	2009 Saturday
	1954 Tuesday	1968 Wednesday	1982 Tuesday	1996 Friday	2010 Sunday
	1955 Wednesday	1969 Thursday	1983 Wednesday	1997 Saturday	2011 Monday
	1956 Thursday	1970 Friday	1984 Thursday	1998 Sunday	2012 Tuesday
	1957 Friday	1971 Saturday	1985 Friday	1999 Monday	2013 Wednesday
	1958 Saturday	1972 Sunday	1986 Saturday	2000 Tuesday	2014 Thursday
	1959 Sunday	1973 Monday	1987 Sunday	2001 Wednesday	2015 Friday
	1960 Monday	1974 Tuesday	1988 Monday	2002 Thursday	2016 Saturday
	1961 Tuesday	1975 Wednesday	1989 Tuesday	2003 Friday	2017 Sunday
	1962 Wednesday	1976 Thursday	1990 Wednesday	2004 Saturday	2018 Monday
	1963 Thursday	1977 Friday	1991 Thursday	2005 Sunday	2019 Tuesday
	1964 Friday	1978 Saturday	1992 Friday	2006 Monday	2020 Wednesday
	1965 Saturday	1979 Sunday	1993 Saturday	2007 Tuesday	2021 Thursday
	1966 Sunday	1980 Monday	1994 Sunday	2008 Wednesday	2022 Friday
	1967 Monday	1981 Tuesday	1995 Monday	2009 Thursday	2023 Saturday
	1968 Tuesday	1982 Wednesday	1996 Tuesday	2010 Friday	2024 Sunday
	1969 Wednesday	1983 Thursday	1997 Wednesday	2011 Saturday	2025 Monday
	1970 Thursday	1984 Friday	1998 Thursday	2012 Sunday	2026 Tuesday
	1971 Friday	1985 Saturday	1999 Friday	2013 Monday	2027 Wednesday
	1972 Saturday	1986 Sunday	2000 Saturday	2014 Tuesday	2028 Thursday
	1973 Sunday	1987 Monday	2001 Sunday	2015 Wednesday	2029 Friday
	1974 Monday	1988 Tuesday	2002 Monday	2016 Thursday	2030 Saturday
	1975 Tuesday	1989 Wednesday	2003 Tuesday	2017 Friday	2031 Sunday
	1976 Wednesday	1990 Thursday	2004 Wednesday	2018 Saturday	2032 Monday
	1977 Thursday	1991 Friday	2005 Thursday	2019 Sunday	2033 Tuesday
	1978 Friday	1992 Saturday	2006 Friday	2020 Monday	2034 Wednesday
	1979 Saturday	1993 Sunday	2007 Saturday	2021 Tuesday	2035 Thursday
	1980 Sunday	1994 Monday	2008 Sunday	2022 Wednesday	2036 Friday
	1981 Monday	1995 Tuesday	2009 Monday	2023 Thursday	2037 Saturday
	1982 Tuesday	1996 Wednesday	2010 Tuesday	2024 Friday	2038 Sunday
	1983 Wednesday	1997 Thursday	2011 Wednesday	2025 Saturday	2039 Monday
	1984 Thursday	1998 Friday	2012 Thursday	2026 Sunday	2040 Tuesday
	1985 Friday	1999 Saturday	2013 Friday	2027 Monday	2041 Wednesday
	1986 Saturday	2000 Sunday	2014 Saturday	2028 Tuesday	2042 Thursday
	1987 Sunday	2001 Monday	2015 Sunday	2029 Wednesday	2043 Friday
	1988 Monday	2002 Tuesday	2016 Monday	2030 Thursday	2044 Saturday
	1989 Tuesday	2003 Wednesday	2017 Tuesday	2031 Friday	20

A TABLE for reducing
Troy wt. to Avoirdupois.

Troy.	Avoirdupois.				Tr.	Avoir.			
lb.	lb.	oz.	dr.	pw.	drams.	lb.	lb.	oz.	drams.
4000	3291	6	13,68	19	16,67	6000	7291	8	0 0 0
3000	2528	9	2,26	18	15,79	5000	6076	4	13 8 14
2000	1645	11	6,84	17	14,92	4000	4861	1	6 16 13
1000	822	13	11,42	16	14,04	3000	3645	10	0 0 12
900	740	9	2,28	15	13,16	2000	2430	6	13 8 11
800	658	4	9,14	14	12,29	1000	1215	3	6 16 10
700	576	0	0	13	11,41	900	1093	9	0 0 9
600	493	11	6,85	12	10,53	800	972	2	13 8 8
500	411	6	13,71	11	9,65	700	850	8	6 16 7
400	329	2	4,57	10	8,78	600	729	2	0 0 6
300	246	13	11,42	9	7,9	500	607	7	13 8 5
200	164	9	2,28	8	7,02	400	486	1	6 16 4
100	82	4	9,15	7	6,14	300	364	7	0 0 3
90	74	0	13,62	6	5,27	200	243	0	13 8 2
80	65	13	4,11	5	4,39	100	121	6	6 16 1
70	57	9	9,6	4	3,51	90	109	4	10 0 dr
60	49	5	15,08	3	2,63	80	97	2	13 8 15
50	41	2	4,57	2	1,75	70	85	0	16 16 14
40	32	14	10,05	1	0,88	60	72	11	0 0 13
30	24	10	15,54	0	0,00	50	60	9	3 8 12
20	16	7	5,03	23	,84	40	48	7	6 16 11
10	8	3	10,52	22	,8	30	36	5	10 0 10
9	7	6	7,86	21	,77	20	24	3	13 8 9
8	6	9	5,21	20	,73	10	12	1	16 16 8
7	5	12	2,56	19	,69	9	10	11	5 0 7
6	4	14	15,9	18	,66	8	9	8	13 8 6
5	4	1	13,25	17	,62	7	8	6	1 16 5
4	3	4	10,6	16	,58	6	7	3	10 0 4
3	2	7	7,95	15	,55	5	6	0	18 8 3
2	1	10	5,3	14	,51	4	4	10	6 16 2
1	0	13	2,65	13	,47	3	3	7	15 0 1
oz				12	,44	2	2	5	3 8 3/4
11	12	1,09		11	,4	1	1	2	11 16 1/2
10	10	15,54		10	,36				
9	9	13,99		9	,33				
8	8	12,43		8	,29				
7	7	10,88		7	,26				
6	6	9,32		6	,22				
5	5	7,77		5	,18				
4	4	6,22		4	,15				
3	3	4,66		3	,11				
2	2	3,11		2	,07				
1	1	1,55		1	,04				

A TABLE for reducing Avoirdupois
weight into Troy.

Avoir.	Troy.				Av	Troy.			
lb.	lb.	oz.	pw.	gr.	oz	lb.	oz.	pw.	gr.
6000	7291	8	0	0	15	1	1	13	10,5
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23,5
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12,5
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1,5
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14,5
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3,5
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16,5
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1		0	18	5,5
90	109	4	10	0	dr				
80	97	2	13	8	15		17	2,1	
70	85	0	16	16	14		15	22,76	
60	72	11	0	0	13		14	19,42	
50	60	9	3	8	12		13	15,08	
40	48	7	6	16	11		12	12,74	
30	36	5	10	0	10		11	9,4	
20	24	3	13	8	9		10	6,06	
10	12	1	16	16	8		9	2,72	
9	10	11	5	0	7		8	23,38	
8	9	8	13	8	6		7	20,04	
7	8	6	1	16	5		6	16,7	
6	7	3	10	0	4		5	13,36	
5	6	0	18	8	3		3	10,02	
4	4	10	6	16	2		2	6,68	
3	3	7	15	0	1		1	3,34	
2	2	5	3	8	3/4		0	20,51	
1	1	2	11	16	1/2			13,67	

An Account of the Gregorian or New Style, together with some Chronological Problems, for finding the Epact, Golden Number, Moon's Age, &c.

POPE GREGORY the XIIIth made a reformation of the calendar. The Julian calendar, or old style, had, before that time, been in general use all over Europe. The year, according to the Julian calendar, consists of three hundred and sixty five days and six hours; which six hours being one fourth part of a day, the common years consisted of three hundred and sixty five days, and every fourth year, one day was added to the month of February, which made each of those years three hundred and sixty six days, which are usually called leap years.

This computation, though near the truth, is more than the solar year by 11 minutes, which, in one hundred and thirty one years, amounts to a whole day. By which the Vernal Æquinox was anticipated ten days, from the time of the general council of Nice, held in the year 325 of the Christian Æra, to the time of Pope Gregory; who therefore caused ten days to be taken out of the month of October in 1582, to make the Æquinox fall on the 21st of March, as it did at the time of that Council. And, to prevent the like variation for the future, he ordered that three days should be abated in every four hundred years, by reducing the leap year at the close of each century, for three successive centuries, to common years, and retaining the leap year at the close of each fourth century only.

This was at that time esteemed as exactly conformable to the true solar year; but Dr. Halley makes the solar year to be three hundred and sixty five days, five hours, forty eight minutes, fifty four seconds, forty one thirds, twenty seven fourths, and thirty one fifths: According to which, in four hundred years, the Julian year of three hundred and sixty five days and six hours will exceed the solar by three days, one hour and fifty five minutes, which is near two hours, so that in fifty centuries it will amount to a day.

Though the Gregorian calendar, or new style, had long been used throughout the greatest part of Europe, it did not take place in Great Britain and America till the first of January 1752; and in September following, the eleven days were adjusted, by calling the third day of that month the fourteenth, and continuing the rest in their order.

CHRONOLOGICAL PROBLEMS.

PROBLEM I.

As there are three leap years to be abated in every four centuries: To shew how to find on which century the last year is to be a leap year, and in which it is not.

RULE.—Cut off two cyphers, and divide the remaining figures by 4; if nothing remain, the year is a leap year.

EXAMP.

EXAMP. 1. The year 18|00.

$$\begin{array}{r} 4)18(4 \\ 16 \\ \hline 2 \end{array}$$

EXAMP. 2. The year 19|00.

$$\begin{array}{r} 4)19(4 \\ 16 \\ \hline 3 \end{array}$$

EXAMP. 3. The year 20|00.

$$\begin{array}{r} 4)20(5 \\ 20 \\ \hline 0 \end{array}$$

EXAMP. 4. The year 40|00.

$$\begin{array}{r} 4)40(10 \\ 40 \\ \hline 0 \end{array}$$

The first and second examples, having remainders, shew the years to be common years of three hundred and sixty five days ; but the third and fourth, having no remainders, are leap years of three hundred and sixty six days.

PROBLEM II.

To find, with regard to any other years, whether any given year be leap year, and the contrary.

RULE.

Divide the proposed year by 4, and if there be no remainder, after the division, it is leap year ; but if 1, 2 or 3 remain, it is the first, second or third after leap year.

EXAMP. 1. For the year 1784.

$$\begin{array}{r} 4)1784(446 \\ 16 \\ \hline 18 \\ 16 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

EXAMP. 2. For the year 1786.

$$\begin{array}{r} 4)1786(446 \\ 16 \\ \hline 18 \\ 16 \\ \hline 26 \\ 24 \\ \hline 2 \end{array}$$

$\left. \begin{array}{l} \text{second after} \\ \text{leap year.} \end{array} \right\}$

PROBLEM III.

To find the Dominical Letter for any year, according to the Julian method of calculation.

RULE.

Add to the year its fourth part and 4, and divide that sum by 7 : If nothing remain, the Dominical Letter is G ; but if there be any remainder, it shews the letter in a retrograde order from G, beginning the reckoning with F ; or, if it be subtracted from 7, you will have the index of the letter from A, accounting as follows :

	A	B	C	D	E	F	G
	1	2	3	4	5	6	7

EXAMP. For the year 1786.

Add { Given year = 1786
 { Its fourth = 446
 { And = 4

$$\begin{array}{r} 7)2236(319 \\ \underline{21} \\ 13 \\ \underline{7} \\ 66 \\ \underline{63} \\ 3 \end{array}$$

And $7-3=4=D$, reckoning from A.

PROBLEM IV.

To find the Dominical Letter for any year, according to the Gregorian computation.

RULE.—Divide the year and its fourth part by 7 ; subtract the remainder, after the division, from 7, and this remainder will be the index of the Dominical Letter, as before ; if nothing remain it is G.

EXAMP. 1. For the year 1786. EXAMP. 2. For the year 1788.*

Add { Given year = 1786
 { Its fourth = 446

$$\begin{array}{r} 7)2232(318 \\ \underline{21} \\ 12 \\ \underline{7} \\ 62 \\ \underline{56} \\ 6 \end{array}$$

$$\begin{array}{r} 1788 \\ 447 \\ \hline 7)2235(319 \\ \underline{21} \\ 13 \\ \underline{7} \\ 65 \\ \underline{63} \\ 2 \end{array}$$

And $7-6=1=A$. And $7-2=5=E$.

PROBLEM

* Here it is to be observed, that every leap year has two Dominical Letters ; that, found by this rule, is the Dominical Letter from the twenty fifth day of February to the end of the year ; and the next in the order of the alphabet serves from the first of January to the twenty fourth of February.

In the 2d Example, E is the Dominical letter for the year ; but F, the next in the order of the alphabet, is the Dominical Letter for January and February. From this interruption of the Dominical Letter every fourth year, it is twenty eight years before the Dominical Letter returns to the same order, which, were it not for the leap years, would return to the same every seven years.

This Cycle of twenty eight years is called the Cycle of the Sun.

CHRONOLOGICAL PROBLEMS. 399

PROBLEM V.

To find the Prime, or Golden Number.

RULE.—Add 1 to the given year ; divide the sum by 19, and the remainder, after the division, will be the Prime ; if nothing remain, then 19 will be the Golden Number.

EXAMP. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

1 Golden Number.

The Golden Number, or Lunar Cycle, is a period of nineteen years, invented by *Meton*, an *Athenian*, and from him called the *Metonic Cycle*. The use of this cycle is to find the change of the moon ; because, after nineteen years, the changes of the moon fall on the same days of the month as in the former 19 years ; though not at the same time of the day, there being an anticipation of one hour, twenty seven minutes, forty one seconds, and thirty two thirds ; which, in three hundred and twelve years, amount to a whole day. Hence, the Golden Number will not shew the true change of the moon for more than three hundred and twelve years, without being varied. But the Golden Number is not so well adapted to the *Gregorian*, as the *Julian* calendar : The *Epaet* being more certain in the new style, to find which, the Golden Number is of use.

PROBLEM VI.

To find the Julian Epaet.

RULE.—First find the Golden Number, which multiply by 11, and the product, if less than 30, will be the number required ; if the product exceed 30, then divide it by 30, and the remainder is the Epaet.

EXAMP. I. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

Golden Number = 1 and $1 \times 11 = 11$ the Julian Epaet.

EXAMP.

400 CHRONOLOGICAL PROBLEMS.

EXAMP. 2. For the year 1791.

$$\begin{array}{r}
 1791 \\
 \underline{1} \\
 19)1792(94 \\
 \underline{171} \\
 82 \\
 \underline{76}
 \end{array}$$

6 = Golden Numb. and $6 \times 11 = 66$, therefore $30)66(2$

60
6Epaft.

PROBLEM VII.

To find the Gregorian Epaft.

RULE.—Subtraft 11 from the Julian Epaft : If the subtraction cannot be made, add 30 to the Julian Epaft ; then subtraft, and the remainder will be the Gregorian Epaft ; if nothing remain, the Epaft is 29.

Or, take 1 from the Golden Number, divide the remainder by 3 ; if 1 remain, add 10 to the dividend, which fum will be the Epaft ; if 2 remain, add 20 to the dividend ; but if nothing remain, the dividend is the Epaft.

EXAMP. 1. For the year 1786.

The Julian Epaft being 11

Subtraft 11

$$\begin{array}{r}
 \text{—} \\
 0
 \end{array}$$

Because nothing remains, the Epaft is 29.

Or,

EXAMP. 2. For the year 1786.

The Golden Number being 1

Take from it 1

$$\begin{array}{r}
 \text{—} \\
 0
 \end{array}$$

Divide by 3)0(0

There being no remainder, the Epaft is 29, as before.

EXAMP. 3. For the year 1791.

The Julian Epaft being but 6

Add to it 30

$$\begin{array}{r}
 \text{—} \\
 36
 \end{array}$$

Subtraft 11

$$\begin{array}{r}
 \text{—} \\
 25
 \end{array}$$

Gregorian Epaft = 25

Or,

EXAMP. 4. For the year 1791.

The Golden Numb. being 6

Take from it 1

$$\begin{array}{r}
 \text{—} \\
 5
 \end{array}$$

3)5(1

$$\begin{array}{r}
 \text{—} \\
 3
 \end{array}$$

$$\begin{array}{r}
 \text{—} \\
 2
 \end{array}$$

Therefore, as 2 remains, add 20 to the dividend, and it gives the Epaft 25, as before.

A

A general Rule for finding the Gregorian Epact forever.

Divide the *centuries* of any year of the Christian *Æra* by 4, (rejecting the subsequent numbers ;) multiply the remainder by 17, and to this product add the quotient multiplied by 43 ; divide this sum plus 86 by 25, multiplying the Golden Number by 11, from which subtract the last quotient, and rejecting the *thirties*, the remainder will be the Epact.

EXAMP. For the year 1786.

Rejecting the subsequent numbers 86, it will be 17.

$$4)17(4$$

$$16$$

$$—$$

$$1$$

Multiply by 17

$$—$$

$$17$$

Add $4 \times 43 = 172$

$$—$$

$$189$$

Add 86

$$—$$

$$25)275(11$$

$$25$$

$$—$$

$$25$$

$$25$$

Golden Number = 1

Multiply by 11

$$—$$

$$11$$

Subtract the last quotient = 11

$$—$$

$$00$$

Therefore, as nothing remains, the Epact is 29, as before.

A TABLE of the nineteen Epacts for the Julian and Gregorian Accounts, by the Golden Number.

G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.
1	11	29	7	17	6	13	23	12
2	22	11	8	28	17	14	4	23
3	3	22	9	9	28	15	15	4
4	14	3	10	20	9	16	26	15
5	25	14	11	1	20	17	7	26
6	6	25	12	12	1	18	18	7
						19	29	18

PROBLEM VIII.

To calculate the Moon's Age on any given day.

R U L E.

To the given day of the month, add the Epact and number of the month : If the sum be less than 30, it is the Moon's age ; but if it exceed 30, then take 30 from it, and the remainder will be the Moon's age.

E c c

Note,

Note, The numbers to be added to the following months, are as follow :

To	January	0	July	5
	February	2	August	6
	March	1	September	8
	April	2	October	8
	May	3	November	10
	June	4	December	10

EXAMPLE. For January 25th, 1786.

$$\begin{array}{rcl}
 \text{Add } \left\{ \begin{array}{l} \text{Given day} \\ \text{Epaet} \\ \text{No. of the month} \end{array} \right. & \begin{array}{l} = 25 \\ = 29 \\ = 00 \end{array} & \\
 & \begin{array}{r} - \\ 54 \\ - \end{array} & \\
 \text{Subtract } 30 & & \\
 & \begin{array}{r} - \\ 24 \end{array} & = \text{Moon's age.}
 \end{array}$$

P R O B L E M IX.

To find the times of the New and Full Moon, and the first and last Quarters.

R U L E.

Find the Moon's age on the given day, then, if it be 15, the Moon will be full on that day, and by counting $7\frac{1}{2}$ days backward and forward you will have the first and last quarters, and by counting backward and forward 15 days, you will have the times of the last and next change; but if the age of the Moon be greater than 15, take 15 from it, and the remainder will shew how many days have past since the last full moon, and, counting these backward, you will have the day the last full Moon happened on, and by knowing that, we can find the change, or either of the quarters, as before. Again, if the age of the Moon, on the assumed day, be less than 15, then take that from 15, and the remainder will shew how many days are to run till the next full Moon, which you will have by adding the remainder to the assumed day; and, proceeding as before, you will have the days of the change, and either quarter as above.

EXAMPLE.

CHRONOLOGICAL PROBLEMS.

403

EXAMP. For January 25th, 1786.

$$\text{Add} \begin{cases} \text{Assumed day} & = 25 \\ \text{Epaet} & = 29 \\ \text{Number of the month} & = 00 \end{cases}$$

$$\begin{array}{r} - \\ 54 \\ \text{Subtract } 30 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Moon's age} = 24 \\ \text{Subtract } 15 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Take the days since the last full Moon} = 9 \\ \text{From the assumed day} = 25 \\ \hline \end{array}$$

$$\begin{array}{r} \text{To the day of the full Moon} = 16\text{th.} \\ \text{Add } 15 \\ \hline \end{array}$$

$$\text{New Moon } 31\text{st.}$$

$$\begin{array}{r} \text{From the full Moon } 16 \\ \text{Take } 7\frac{1}{2} \\ \hline \end{array}$$

$$\text{First quarter } 9\text{th.}$$

$$\begin{array}{r} \text{To the full Moon} = 16 \\ \text{Add } 7\frac{1}{2} \\ \hline \end{array}$$

$$\text{Last quarter} = 23\text{d.}$$

PROBLEM X.

The time of the Moon's coming to the South, after the Sun, being given, to find the Age of the Moon.

R U L E.

As 24 hours, the whole difference of time, are to 30, the number of days from change to change : So is the difference of time, to the Moon's age.

EXAMP. I observed the Moon to be on the meridian, or due south, at 6 o'clock in the afternoon ; What is the Moon's age ?
 $24 : 30 :: 6 : 7\frac{1}{2} \text{ days, Ans.}$

PROBLEM XI.

To find the time of the Moon's Southing.

R U L E.

Multiply the Moon's age, on the given day, by 48 minutes, and divide the product by 60, the minutes in an hour, (or multiply by 4, and divide by 5,) and the quotient will shew how many

104 CHRONOLOGICAL PROBLEMS.

ny hours and minutes the Moon is later, in coming on the meridian, than the Sun, and counting so many hours and minutes forward from 12 o'clock, we have the time of the Moon's southing; if the hours and minutes, found as above, be less than 12, then, that will be the time of the Moon's southing after noon; but, if greater than 12, then take 12 from them, and the remainder will be the time of the Moon's southing in the morning.

EXAMP. 1. Required the time of the Moon's southing on the 25th day of January 1786?

Moon's age = 24		Or,
h. m.	48	24
From 19 12	<u>192</u>	4
Take 12 00	96	<u>5)96</u>
7 12	60)1152(19 12	h. m.
Hence, the Moon	60	19½ = 19 12 as before.
souths at 12 minutes	<u>552</u>	
past 7 in the morning.	<u>540</u>	
	12	

EXAMP. 2. For the 9th of February 1786?

Moon's age = 10

48	
— h. m.	
60)480(8 0	afternoon, is the time of the Moon's
480	southing.

Note. From the change to the full, the Moon comes to the south afternoon; but from the full to the change, before noon.

PROBLEM XII.

To find on what day of the week, any given day in any month will fall.

As one of the first seven letters of the alphabet is prefixed to every day in the year, beginning with A, which is always prefixed to the first day of January: And as, in common years, the letter, annexed to the first Sunday in January, shews the Dominical Letter for that year; but every leap year having two Dominical Letters, the first of which serving to the 24th of February, and the other for the rest of the year, consequently, in any common year, the Dominical Letter being known, the first of January may be easily found, reckoning from A according to the natural order of the letters: And in any leap year, the first of its two Dominical Letters will shew as above, counting from A 1, B 2, C 3, &c. and by counting backward, you may have the day of the week, on which the first of January will happen.

RULE.

RULE.—Find the day of the week answering to the first of January that year, then add together the days contained in each month from the beginning of the year to the proposed day of the month inclusively ; divide this sum by 7, and if any thing remain, after the division, then, count so many forward, beginning with that day on which the first of January falls, and you will have the day of the week, on which the proposed day will fall : But if nothing remain, then the day of the week, preceding that day on which the first of January falls, answers to the proposed day.

EXAMPLE.

On what day of the week will the 5th day of May 1786 fall ?

By the preceding observations, and by
 Prob. 4th, the first of January is found
 to fall on Sunday.

Jan.	31
Feb.	28
March	31
April	30
May	5th.

Now, counting forward six days from Sunday, the first of January (inclusively) and the 5th of May falls on Friday.

7	125	(17
	7	
	—	
	55	
	49	
	—	

6 from Jan. 1.

PROBLEM XIII.

To find the Cycle of the Sun.

RULE.—Add 9 to the given year ; divide the sum by 28, and the remainder, after division, is the Cycle required ; but if nothing remain, the Cycle is 28.

EXAMPLE.

For the year 1786 ?

To 1786
Add 9
—
28)1795(64
168
—
115
112
—

The use of this Cycle is to find the Dominical Letter by the following Table.

3 = Cycle required.

<i>A TABLE of the Dominical Letters for the New Style, according to the Cycle of the Sun.</i>							
Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.
1	D C	8	B	15	G	22	E
2	B	9	A G	16	F	23	D
3	A	10	F	17	E D	24	C
4	G	11	E	18	C	25	B A
5	F E	12	D	19	B	26	G
6	D	13	C B	20	A	27	F
7	C	14	A	21	G F	28	E

This Table, by the present rule, will serve but to the end of this century. The leap year being to be omitted in the year 1800, will make it necessary to add 25 to the date of the year, and then dividing by 28, it will give the Cycle right during the next century. And this is a general rule to be observed, that when a leap year has been abated, add 16 to the number which was before added to the year, rejecting 28, when it exceeds it, and this number being added to the year, and the sum divided by 28, the remainder, after division, will be the Cycle for finding the Dominical Letter. Thus, in the nineteenth century, it will be $9 + 16 = 25$, and in the twentieth century $25 + 16 - 28 = 13$, which number will serve two centuries, for the year 2000 is a leap year.

PROBLEM XIV.

To find the year of the Dionysian Period.

RULE.—Add to the given year 457; divide the sum by 532, and the remainder will be the number required.

EXAMPLE.

Required the year of the Dionysian Period for the year 1786?

To 1786

Add 457

532)2243(4
2128

115 = Dionysian Period.

PROBLEM XV.

To find the year of Indiction.

RULE.—Add 3 to the given year; divide the sum by 15, and the remainder, after division, will be the Indiction; if nothing remain, it will be 15.

EXAMPLE.

EXAMPLE.

Required the year of Indiction for 1786 ?

To 1786

Add 3

15)1789(119

15

28

15

139

135

4 = Indiction.

PROBLEM XVI.

To find the Julian Period.

RULE.—Add 4713 to the given year, and the sum will be the Julian Period.

EXAMPLE.

What year of the Julian Period will answer to the year 1786?

To 1786

Add 4713

6499 Ans.

PROBLEM XVII.

To find the Cycle of the Sun, Golden Number, and Indiction, for any current year.

RULE.—To the current year add 4713; divide the sum by 28, 19 and 15, respectively, and the several remainders will be the numbers required; when nothing remains, the divisor is the number required.

EXAMPLE.

What are the Cycle of the Sun, Golden Number, and Indiction, for the year 1786?

1786

4713

28)6499(232

56

89

84

59 Gold. Num. = 1

56

3 Cycle of the Sun.

19)6499(342

57

79

76

39

38

15)6499(433

60

49

45

49

45

Indiction = 4

PROBLEM

PROBLEM XVIII.

Having the Cycle of the Sun, the Golden Number, and Indiction, to find the year of the Christian Æra.

RULE.—Multiply 4845 by the Cycle of the Sun; 4200 by the Golden Number, and 6916 by the Indiction: Add the several products together, and divide the sum by 7980; the remainder, after division, will be the Julian Period, from which subtract 4713, and the remainder will be the year required.

EXAMPLE.

The Cycle of the Sun being 3, Golden Number 1, and Indiction 4; What year of the Christian Æra is it?

4845	4200	6916	27664
<u>3</u>	<u>1</u>	<u>4</u>	4200
14535	4200	27664	14535
			<u>7980</u> 46399(5
			39900
			<u>6499</u>
			Subtract 4713
			<u>1786</u> <i>Ans.</i>

PROBLEM XIX.

To find the time of High Water.

RULE.—Find the Moon's southing, to which add the point of the compass making full sea, on the full and change days, for the place proposed, and the sum will be the time required.

EXAMPLE.

I demand the time of high water at Boston, January 25th 1786, admitting the tide to flow and ebb N. W. and S. E. on the days of change and full?

We have before found the Moon's southing to be 7h. 12m. in the morning.

	<i>h. m.</i>
Therefore to	7 12
Add	4 0 = the point of the compass, and it
	<u>11 12</u>

Gives 11 12 in the morning, for the time of high water.

PROBLEM XX.

To find on what day Easter will happen.

It was ordered by the Nicene Council, that Easter Sunday should be kept on the first Sunday after the first full Moon, which happened upon or after the twenty first day of March, the day on which they thought the Vernal Æquinox happened. Though this was a mistake, for the Vernal Æquinox, that year, fell on the twentieth of March. But yet, the full Moon, which fell on,

or next after the twenty first of March, they called the Paschal full Moon. And by the introduction of the Gregorian, or New Style, the Æquinox will now always happen on the twentieth or twenty first of March. And the feast of Easter is now to be kept on the next Sunday after the Paschal full Moon, or the full Moon which happens after the twenty first of March ; but, if the full Moon happens on a Sunday, Easter day is to be the next Sunday after.

RULE.—Find the age of the Moon on the 21st of March, in the given year, and if it be 14, then find the day of the week answering to it, and the Sunday following is Easter Sunday ; but if the Moon's age on the 21st day of March be not 14, then reckon forward to the day on which the Moon's age is 14, and find the day of the week answering to that day ; the Sunday following will be the day required.

N. B. On leap year take the 20th of March.

EXAMP. When does Easter happen in the year 1786 ?

21 of March	Jan. 31
29 Epact.	Feb. 28
1 No. of the month.	March 31
—	April 13th
51	—
Subt. 30	7) 103 (14
—	7
21 Moon's age.	—
Add 23 { No. of days to the Moon's	33
being 14 days old.	28
—	—
44	5 Therefore,
Take 31 = days in March.	the first of January being Sunday,
—	reckon forward 5 days, including
13th of April, the	Sunday, and you will find the 13th
day of the full Moon, or	of April falls on Thursday, con-
Easter limit.	sequently the next Sunday is the
	16th, which is Easter Sunday.

Easter may be found, for any future time, by the following Table, which is calculated from 1753, the time of the commencement of the New Style in America, and which shews, by the Golden Number, the days of the Paschal full Moons ; by which, and the Dominical Letter, the day on which Easter will fall, may be found.

The Use of the Table.

First find the Golden Number as before taught, which seek in the column of Golden Numbers under the time in which the given year is included ; right against the Golden Number of the year, in the last column but one, you have the day of the month on which the Paschal full Moon happens, which is the limit of Easter ; from thence run your eye down among the Dominical Letters,

Letters, till you come to the Letter of the given year, and againſt it you have the day of the month, on which Eaſter falls that year.

EXAMP. To know when Easter falls in 1786.

The Golden Number for the year being 1, and the Dominical Letter A ; therefore seek in the first column (the given year being included between the years 1753 and 1899) for the Golden Number : Then cast your eye along to the last column but one, under the title, Paschal full☉, and you will find the thirteenth of April to be the day of the full Moon ; against which, in the last column, stands E, which shews it to be Thursday, therefore the next Sunday following is Easter Sunday, which, by going down the column of Letters to the next A, you will find to be the sixteenth of April.

GOLDEN NUMBERS *from 1753 to 1899, and so on to 4199, inclusively.*

															Full ●	Pagchal Days of the Month	Dom. Letter.
1753	1900	2200	2300	2400	2500	2600	2900	3100	3400	3500	3600	3700	3800	4100	March	21	C
to	to	to	to	to	to	to	to	to	to	to	to	to	to	to		22	D
1899	2199	2299	2399	2499	2599	2899	3099	3399	3499	3599	3699	3799	4099	4199		23	E
14	—	6	17	6	17	—	9	—	1	12	1	12	—	4		24	F
3	14	—	6	—	6	17	—	9	—	9	—	1	12	—		25	G
—	3	14	—	14	—	6	17	—	9	—	9	—	1	12			
11	—	3	14	3	14	—	6	17	—	9	—	9	—	1			
—	11	—	3	—	3	14	—	6	17	—	17	—	9	—			
19	—	11	—	11	—	13	14	—	6	17	6	17	—	9	26	A	
8	19	—	11	—	11	—	3	14	—	6	—	6	17	—	27	B	
—	8	19	—	19	—	11	—	3	14	—	14	—	6	17	28	C	
16	—	8	19	8	19	—	11	—	3	14	3	14	—	6	29	D	
5	16	—	8	—	8	19	—	11	—	3	—	3	14	—	30	E	
—	5	16	—	16	—	8	19	—	11	—	11	—	3	14	31	F	
13	—	5	16	5	16	—	8	19	—	11	—	11	—	3	1	G	
2	13	—	5	—	5	16	—	8	19	—	19	—	11	—	2	A	
—	2	13	—	13	—	5	16	—	8	19	8	19	—	11	3	B	
10	—	2	13	2	13	—	5	16	—	8	—	8	19	—	4	C	
—	10	—	2	—	2	13	—	5	16	—	16	—	8	19	5	D	
18	—	10	—	10	—	2	13	—	5	10	5	16	—	8	6	E	
7	18	—	10	—	10	—	2	13	—	5	—	5	16	—	7	F	
—	7	18	—	18	—	10	—	2	13	—	13	—	5	16	8	G	
15	—	7	18	7	18	—	10	—	2	13	2	13	—	5	9	A	
4	15	—	7	—	7	18	—	10	—	2	—	2	13	—	10	B	
—	4	15	—	15	—	7	18	—	10	—	10	—	2	13	11	C	
12	—	4	15	4	15	—	7	18	—	10	—	10	—	2	12	D	
1	12	—	4	—	4	15	—	7	18	—	18	—	10	—	13	E	
—	1	12	—	12	—	4	15	—	7	18	7	18	—	10	14	F	
9	—	1	12	1	12	—	4	15	—	7	—	7	18	—	15	G	
—	9	—	1	—	1	12	—	4	15	—	15	—	7	18	16	A	
17	17	9	—	9	—	1	12	12	4	15	4	15	15	7	17	B	
6	6	17	9	17	9	9	1	1	12	4	12	4	4	15	18	C	
															19	D	
															20	E	
															21	F	
															22	G	
															23	A	
															24	B	
															25	C	

The USE of LOGARITHMS.

1. In MULTIPLICATION.

Given two numbers, viz. 275 and 12,6, to find their product.

RULE.—To the logarithm of 275, viz. 2,43933
 Add the logarithm of 12,6, viz. 1,10037

And their sum is the logar. of their prod. viz. $3465 = 3,53970$

2. In DIVISION.

Let it be required to find the quotient, which arises by dividing one number by another ; suppose 1425 by 57.

From the logarithm of the dividend, viz. 1425 = 3,15381
 Take the logarithm of the divisor, viz. 57 = 1,75587

And the remainder is the logar. of the quot. viz. $25 = 1,39794$

3. In the RULE of THREE.

Three numbers given, to find a fourth, in direct proportion.

RULE.—From the Tables take the logarithms of each of the proposed numbers, then, add the logarithms of the second and third together, and from the sum take the logarithm of the first, and the remainder will be the logarithm of the fourth number.

Let the three proposed numbers be 18, 24, and 33, and the operation will stand thus ;

1,38021 = the logarithm of 24, the 2d term.

1,51851 = the logarithm of 33, the 3d term.

2,89872 = the logarithm of their product.

—1,25527 = the logarithm of the first term 18.

1,64345 = the logarithm of the fourth term required, which, by the Table, answers to the natural number 44, the 4th proportional to the three proposed numbers.

4. In INVOLUTION or RAISING POWERS.

To find any power of any proposed number, or to involve any number to any proposed power, by logarithms.

RULE.—Multiply the logarithm of the given root by the power, viz. by 2 for the square, by 3 for the cube, &c. and the product is the logarithm of the power sought.

Required to find the cube of 12 ?

1,07918 = the logarithm of 12.

× 3 = the third power, or cube.

3,23754 = 1728, the cube of 12.

5. In EVOLUTION or EXTRACTING ROOTS.

To extract any root of any proposed number.

RULE.—Divide the logarithm of the proposed number by the index of the required root, viz. by 2 for the square, by 3 for the cube, &c. and the quotient will be the logarithm of the root required.

Required to find the cube root of 1728 ?

$3,23754 =$ the logarithm of 1728, and $3,23754 \div 3 = 1,07918$ is the logarithm of the cube root of 1728, viz. 12.

6. In COMPOUND INTEREST.

To find the amount of any sum for any time, and at any rate, at Compound Interest.

RULE.—Multiply the logarithm of the ratio (*i. e.* the amount of 1*l.* for 1 year) by the number of years, and to the product add the logarithm of the principal; the sum will be the logarithm of the amount.

What will 45*l.* amount to, forborne 12 years, at 6 per cent. per annum, compound interest ?

Log. of 1,06, the ratio, is ,02533

Multiply by the time 12

,30396

Add log. of 45, the princip. 1,65321

The sum is 1,95717 which is the logarithm of
90,7 = £90 14*s.* Ans.

7. In DISCOUNT at COMPOUND INTEREST.

To find the present worth of any sum of money, due any time hence, at any rate, at Compound Interest.

RULE.—From the logarithm of the sum to be discounted, subtract the logarithm of the rate multiplied by the time; and the remainder is the logarithm of the present worth.

What present money will pay a debt of 90*l.* 14*s.* due 12 years hence, discounting at the rate of 6 per cent. per annum ?

From the logarithm of £90 14 = 1,95717

Subt. prod. of the log. of the ratio \times by the time = ,30396

The remainder 1,65321 is the
logarithm of £45 Ans.

PLAIN GEOMETRY.

Definitions.

1. A *Point* in the Mathematics is considered only as a mark, without any regard to dimensions.

2. A *Line* is considered as length, without regard to breadth or thickness.

3. A *Plain* or *Surface* has two dimensions, length and breadth, but is not considered as having thickness.

4. A *Solid* has three dimensions, length, breadth and thickness, and is usually called a *Body*.

5. A *Line* is either *straight*, which is the nearest distance between two Points; or *crooked*, called a *Curve Line*, whose ends may be drawn further asunder.

6. If two *Lines* are at equal distance from one another in every part, they are called *parallel Lines*, which, if continued infinitely, will never meet.

7. If two *Lines* incline one towards another, they will, if continued, meet in a point: By which meeting is formed an *Angle*.

8. If one *Line* fall directly upon another; so that the *Angles* on both sides are equal, the *Line*, so falling, is called a *perpendicular*, and the *Angles*, so made, are called *right Angles*, and are equal to 90 degrees, each.

9. All *Angles*, except *right Angles*, are called *oblique Angles*, whether they are *acute*, that is, less than a *right Angle*; or *obtuse*, that is, greater than a *right Angle*.

GEOMETRICAL PROBLEMS.

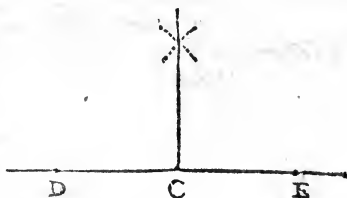
PROBLEM I. To divide a *Line* AB into two equal parts.

Set one foot of the compasses in the point A, and, opening them beyond the middle of the line, describe arches above and below the line; with the same extent of the compasses, set one foot in the point B, and describe two arches crossing the former: Draw a line from the intersection of the arches above the line, to the intersection below the line, and it will divide the line AB into two equal parts.



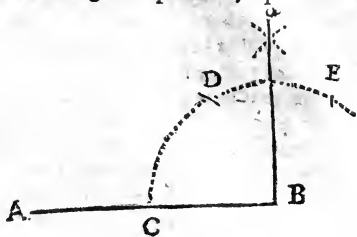
PROBLEM II. To erect a perpendicular on the point C in a give line.

Set one foot of the compasses in the given point C, extend the other foot to any distance at pleasure, as to D, and with that extent make the marks D, and E. With the compasses, one foot in D, at any extent above half the distance of D and E, describe an arch above the line, and with the same extent, and one foot in E, describe an arch, crossing the former; draw a line from the intersection of the arches to the given point C, which will be perpendicular to the given line in the point C.



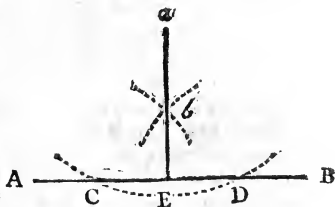
PROBLEM III. *To erect a perpendicular upon the end of a line.*

Set one foot of the compasses in the given point B, open them to any convenient distance, and describe the arch CDE; set one foot in C, and with the same extent, cross the arch at D: With the same extent cross the arch again, from D to E; then with one foot of the compasses in D, and with any extent above the half of DE, describe an arch *a*; take the compasses from D, and, keeping them at the same extent with one foot in E, intersect the former arch *a* in *a*; from thence draw a line to the point B, which will be a perpendicular to AB.



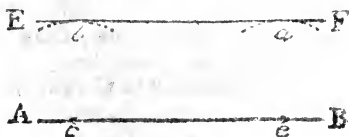
PROBLEM IV. *From a given point, *a*, to let fall a perpendicular to a given line AB.*

Set one foot of the compasses in the point *a*, extend the other so as to reach beyond the line AB, and describe an arch to cut the line AB in C and D; put one foot of the compasses in C, and, with any extent above half CD, describe an arch *b*; keeping the compasses at the same extent, put one foot in D, and intersect the arch *b* in *b*; through which intersection, and the point *a*, draw *aE*, the perpendicular required.



PROBLEM V. *To draw a Line parallel to a given Line AB.*

Set one foot of the compasses in any part of the line, as at *c*; extend the compasses at pleasure, unless a distance be assigned, and describe an arch *b*; with the same extent, in some other part of the line AB, as at *e*, describe the arch *a*; lay a ruler to the extremities of the arches, and draw the line EF, which will be parallel to the line AB.

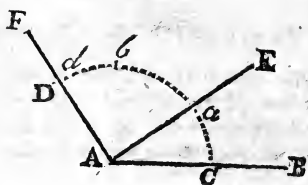


PROBLEM VI. *To make an Angle equal to any number of Degrees.*

It is required to lay off an acute Angle of 35° on a given line AB.

Take

Take 60 degrees from the line of chords in the compasses, set one foot of the compasses in the point A, describe an arch CD, at pleasure; then set one foot of the compasses in the brass centre in the beginning of the line of chords, and bring the other to 35 on the line; with this extent, set



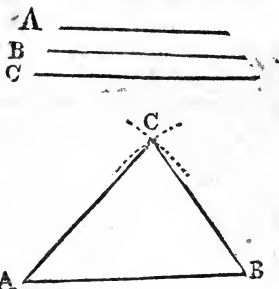
one foot in C, with the other intersect the arch CD, in *a*, and through *a* draw the line AE, so will EAB be an angle of 35 degrees.

If the angle had been obtuse, suppose 125° , then take 90° from the line of chords; set one foot in C, and intersect the arch in *b*; then take 35° from the same line of chords, and set them from *b* to *d*; a line drawn from A through *d* to F will make an angle, FAB, of 125° .

To measure an angle by the line of chords is only to take the distance on the arch between the lines AB and AE, or AB and AF, and laying it on the line of chords.

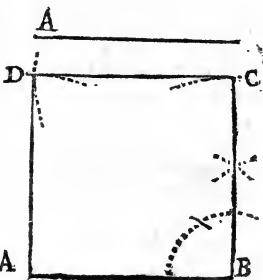
PROBLEM VII. *To make a Triangle, whose sides shall be equal to three given lines, provided any two of them be longer than the third.*

Let A, B, C, be the three given lines; draw a line, AB, at pleasure; take the line C in the compasses, set one foot in A, and with the other make a mark at B; then take the given line B in the compasses, and setting one foot in A, draw the arch C; then take the line A in the compasses, and intersect the arch C in C; lastly, draw the lines AC and BC, and the triangle will be completed.



PROBLEM VIII. *To make a Square, having equal sides, equal to any given line.*

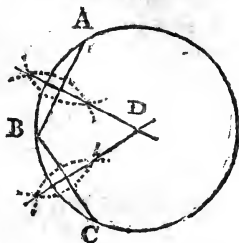
Let A be the given line; draw a line AB equal to the given line; from B raise a perpendicular to C equal to AB, with the same extent, set one foot in C, and describe the arch D; also with the same extent, set one foot in A and intersect the arch D; lastly, draw the line AD and CD, and the square will be completed.



In like manner may a Parallelogram be constructed, only attending to the difference between the length and breadth.

PROBLEM IX. *To describe a Circle, which shall pass through any three given Points, which are not in a straight line.*

Let the three given points be ABC, through which the circle is to pass. Join the points AB and BC with right lines, and bisect these lines; the point D, where the bisecting lines cross each other, will be the centre of the circle required. Therefore, place one point of the compasses in D, extending the other to either of the given points, and the circle, described by that radius, will pass through all the points.



Hence, it will be easy to find the centre of any given circle; for, if any three points are taken in the circumference of the given circle, the centre will be readily found as above. The same may also be observed, when only a part of the circumference is given.

PROBLEM X. *To describe an Ellipsis or Oval mechanically.*

Draw two parallel lines, as L and M, at a moderate distance, by Prob. 5; then draw two others at the same distance, across the former, as N and O; by the crossing of these lines will be made a figure, ABCD, of four sides; extend the compasses at pleasure, and setting one foot in D, describe the arch *cde*; with the same extent, set one foot in B, and describe the arch *fgh*; then, set one foot in C, and contract them so as to reach the point *e*, and describe the arch *lm*; with the same extent, and one foot in A, describe the arch *ih*, and the oval will be completed. In the same manner, with a greater or less extent of the compasses, may a greater or less oval be made by the same four sided figure ABCD.



Of PLAIN TRIGONOMETRY, RIGHT and OBLIQUE ANGLED.

Plain Trigonometry is that science, by which we measure the sides and angles of plain triangles.

SECTION I. *Of Rectangular Trigonometry.*

In a right triangle, the longest side is usually called the hypotenuse, the next longest, the base, and the shortest, the perpendicular.

Logarithmic sines, tangents, and secants, are called the *tabular* sides of a triangle, and are the sines, &c. of the opposite angles. The length of the sides are called the *natural* sides.

All

All the three angles of a triangle are equal to two right angles, or 180° .

The proportion ought to be made between sides and sides ;
and between angles and angles.

When a side is required, any side (whether known, or not) may be made radius; but when an angle is required, then a known side only, must be made radius.

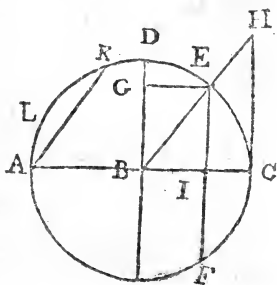
Note. A side is said to be made radius, when one foot of the dividers is set in one end of the side, and such a circle described, of which the side is the femidiameter : Also, that when the hypotenuse is radius, it is the sign of the right angle, or 90° , and the base and perpendicular, usually called the legs, become sides of their opposite angles : But when one of the legs is made radius, the other becomes the tangent of the opposite angle, and the hypotenuse, the secant of the same angle.* Tangent's radius is 45° .

When a side is to be found, the two first terms of the proportion must be tabular sides, and the last a real one; but when an angle is to be found, the two first terms must be real sides, and the third, a tabular one.

The given parts, whether sides or angles, are marked with —, and the part required, with O.

Angles are measured by the arch of a circle. The periphery of every circle, whether great or small, is divided into 360 degrees, each degree into 60 minutes, every minute into 60 seconds, and so on, to thirds, fourths, &c.

Any portion of the periphery of a circle, as ECF, is called an *arch*, and a line drawn from the ends of an arch, as, EIF, is called the *chord* of the arch. Half the chord of any arch, as EI, is called the *fine* of the arch EC, and IC is called the *versed fine* of the same arch EC: So also, EG is the *fine* of the arch ED. A line drawn perpendicular to the diameter of a circle, so as to touch the circle, and not cut it, is called a *tangent*, as CH, which is the tangent of the arch EC, because the line BH, drawn from the centre B, through E, called the *secant*, meets it in the point H.



The complement of an arch is the remainder, after the arch is taken from 90° ; thus KD is the complement of the arch ALK, taken from the arch AD. The cosine or sine complement of an arch is the sine of the complement of that arch, as ED is the complement of EC.

PROBLEM

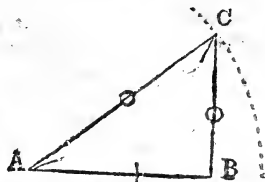
* To work on the scale with a secant, you must take the fines backwards, that is 80 fines for 10 secants, &c.

G 60 60

PROBLEM I. *The Angles and one of the Legs given, to find the Hypothenuse and other Leg.*

EXAMP. In the triangle ABC, right angled at B, suppose the leg AB, 86 equal parts (as feet, yards, miles, &c.) the angle $A = 33^\circ, 40'$, and the angle $C = 56^\circ, 20'$; Required the length of the hypothenuse AC, and the other leg BC?

Geometrically. Draw AB equal to 86, from any line of equal parts, then upon the point B, erect the perpendicular BC; lastly, from the point A, draw the line AC, making with AB an angle $= 33^\circ, 40'$, and that line produced will meet BC in C, and so constitute the triangle. The length of AC and BC may be found by taking them in your compasses, and applying them to the same line of equal parts that AB was taken from.



By Calculation. By making the hypothenuse radius, the legs will become the sines of the opposite angles; and as natural sines are required, the proportions must begin with tabular sines: Therefore, for the hypothenuse,

As the sine of C	$56^\circ, 20'$	9,92027	Here, I add the logarithms of the 2d and 3d terms, and from their sum subtract the first, and the remainder is the logarithm of the side sought, which gives 103,3. The same must be done in all the following cases.
Is to radius	$90^\circ, 00'$	10,	
So is the side AB	86	1,93450	
		11,93450	
		9,92027	
To the side AC	103,3	2,01423	

For the Leg BC.

As the sine of the angle C	$56^\circ, 20'$	9,92027
Is to the sine of the angle A	$33^\circ, 40'$	9,74380
So is the side AB	86	1,93450
To the side BC	57, 28	1,75803

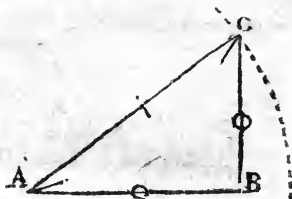
It might have been as easily found by the following proportion:
As $R : S, A :: AC : BC$.

PROBLEM II. *The Angles and Hypothenuse given, to find the Legs.*

EXAMP. In the triangle ABC, suppose the hypothenuse $AC = 146$, the angle $A = 36^\circ, 25'$, and the angle $C = 53^\circ, 35'$; Required the legs AB and BC?

Geometrically. Draw the line AB at pleasure, and make the angle $A = 36^\circ, 25'$; then take $AC = 146$ from any line of equal parts

parts ; lastly, from the point C let fall the perpendicular CB on the line AB : So the triangle is constructed, and AB and BC may be measured from the line of equal parts.



By Calculation. Making AC radius, the legs become fines, as before, and as the angles are given to find the sides, we must begin the proportions with angles, or tabular sides.

For the Leg AB.

As radius	90° 00'	10,
Is to the sine of C	53 ,35	9,90565
So is side AC	146	2,16435
To side AB	117,5	2,07000

For the Leg BC.

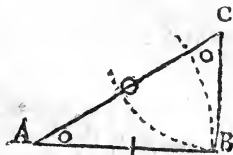
As radius	90° 00'	10,
Is to the sine of A	36 ,25	9,77353
So is side AC	146	2,16435
To side BC	86,67	1,93788

As we had before found AB, the proportion might have been As S, C : S, A :: AB ; BC.

PROBLEM III. and IV. *The two Legs given, to find the Angles and Hypothenufe.*

EXAMP. In the triangle ABC, suppose the leg AB = 94, and BC = 56 ; Required the angles and hypothenufe ?

Geometrically. Draw AB = 94 from any line of equal parts, then, from the point B raise BC perpendicular to AB, and take BC from the former line of equal parts = 56 ; lastly, join the points A and C with the straight line AC, so the triangle is constructed. AC may be found by taking it in your dividers and applying it to the line of equal parts ; and the angles may be measured by the 6th Geometrical Problem.



By Calculation. 1st, For the angle A ; supposing the base AB the radius, then the hypothenufe becomes secant of the angle A, and the perpendicular BC, the tangent of the angle A : And as an angle is required, we must begin the analogy with a natural side.

As AB	94	1,97313
Is to BC	56	1,74819
So is tangent's radius	45° 00'	10,
To the tangent of A	30° 47'	9,77506

The perpendicular might have been made radius, and then the proportion would have been, As BC : AB :: tang. rad. : tang. of C. Now,

Now, as we have found the angle A, and as the angles A and C, taken together, are equal to 90° , therefore from $90^\circ, 00'$
 Take the angle A = $30^\circ, 47'$

And we have the angle C = $59^\circ, 13'$

2d. *For the Hypotenuse.* The base still being radius, we have this analogy for finding the hypotenuse : As T. R : Sec. A :: AB : AC. But this may be done without the help of secants : For, having found the angles, we may now make the hypotenuse radius ; and as a natural side is required, we must begin the proportion with a tabular side ; therefore,

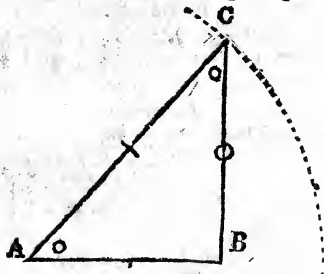
As the sine of C	$59^\circ, 13'$	9.93405
Is to radius	$90^\circ, 00'$	10,
So is AB	94	1,97313
To AC	109,4	2,03908

Or the analogy might have been, As S. C : R :: BC : AC.

PROBLEMS V. and VI. *The Hypotenuse and one of the Legs given, to find the Angles, and other Leg.*

EXAMP. In the triangle ABC, suppose the leg AB = 83, and the hypotenuse AC = 126 ; Required the angles A & C, & the leg BC ?

Geometrically. Draw AB = 83 from any line of equal parts ; and from the point B, raise the perpendicular BC of any length, then, take the length of AC 126 from the same line of equal parts, and setting one foot of the dividers in A, with the other cross the perpendicular BC in C ; lastly, join AC, so the triangle will be constructed, and the angles may be measured as directed in Problem 3d and 4th.



By Calculation. First, for the angle C ; and as an angle is required, we must begin with a side, making the hypotenuse radius.

As AC	126	2,10037
Is to AB	83	1,91908
So is radius	$90^\circ, 00'$	10,
To sine of C	$41^\circ, 12'$	9,81871

From $90^\circ, 00'$
 Take the angle at C = $41^\circ, 12'$

And we have the angle A = $48^\circ, 48'$

For the side BC. As a side is now required, we must begin with an angle ; therefore,

As radius	$90^\circ, 00'$	10,
Is to the sine of A	$48^\circ, 48'$	9,87646
So is AC	126	2,10037
To BC	94.8	1,97683

SECT. 2. *Of oblique angular Trigonometry.*

In any triangle, the sides are proportional to the sines of the opposite angles.

When two angles of any triangle are given, their sum, being subtracted from 180° , leaves the third angle; and when one angle is given, that being subtracted from 180° , leaves the sum of the two unknown angles.

When an angle exceeds 90° , subtract it from 180° and work with the remainder.

When the given and required parts, viz. sides and angles, are opposite.

RULE 1.—As in right angled triangles.

As the sine of any angle is to the sine of any other angle: So is the side opposite to the first angle, to the side opposite to the other angle.

Or, As any one side is to any other side: So is the sine of the angle opposite to the first side, to the sine opposite to the other side.

When any two sides with the angle included between them are given.

RULE 2.—As the sum of any two sides is to their difference; So is the tangent of the half sum of the two opposite angles, to the tangent of half the difference of those two angles; which half difference, being added to the half sum, gives the greater of the two angles, and, being subtracted from the half sum, leaves the less of the two unknown angles.

When the three sides are given, to find the angles.

RULE 3.—As the base of any triangle (or sum of the segments of the base) is to the sum of the other two sides: So is the difference of those sides, to the difference of the two segments of the base, made by letting fall a perpendicular to the base from the angle opposite to it: Half of which difference, being added to half the sum of the two segments, gives the longest, and, being subtracted, leaves the shortest.

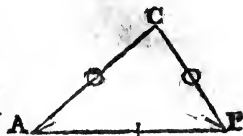
The learner being now somewhat acquainted with the common method of working by logarithms, it will be proper to shew how to perform those proportions without subtracting the first number from the sum of the second and third; which is done by setting down the arithmetical complement of the first term instead of the logarithm. This may be readily done thus; subtract the first figure of the logarithm from 10, and set down the remainder: Then subtract each of the other figures, index and all, from 9, setting down the remainders, and place a dot before the index, as in the case of the logarithm. Thus the arithmetical complement (usually marked Co. Ar.) of the logarithm 9,66004 is 0,33996, and so of any other.

When the arithmetical complement of the first term is used instead of the logarithm, add all the three numbers together, and reject 10 out of the index of their sum, as in those cases where the radius is the first term.

PROBLEM I. *In the oblique angled triangle ABC, given two angles and a side opposite to one of them, to find the two other sides.*

Suppose the angle at A $36^{\circ}, 40'$, the angle at B, $60^{\circ}, 51'$, and the base AB 85,6; Required AC and BC?

Geometrically. Draw the base AB, and from any scale of equal parts, lay thereon 85,6 from A to B; then, from the line of chords, lay off an angle of $36^{\circ}, 40'$ at A, and an angle of $60^{\circ}, 51'$ at B, and the meeting of these two lines in C completes the triangle, and AB and BC may be measured by the same line of equal parts.



From the sum of all the angles	180°, 00'
Take the sum of the angles A and B, viz.	97°, 31'
	82°, 29'

And we have the angle C equal to $82^{\circ}, 29'$

Here we have the angle at C opposite to the given base, and the angles at A and B opposite the two required sides, which may be found by the first rule, as follows :

By Calculation. For the side BC. Having to find a side, we begin with an angle.

As the sine of the angle at C,	$82^{\circ}, 29'$	Co. Ar.	0,00375
Is to the sine of the angle A,	$36^{\circ}, 40'$		9,77609
So is the base AB	85,6		1,93247

To side BC	$51,55$	$1,71231$
------------	---------	-----------

For the side AB.

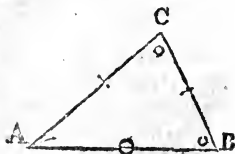
As the sine of C	$82^{\circ}, 29'$	Co. Ar.	0,00375
Is to the sine of B	$60^{\circ}, 51'$		9,94118
So is AB	85,6		1,93247

To AC	$75,4$	$1,87740$
-------	--------	-----------

PROBLEM II. and III. *Two sides, and an angle opposite to one of them, given, to find the two other angles and remaining side.*

In the oblique angled triangle ABC, given the side AC 75,4, the side BC 51,56, and the angle at A $36^{\circ}, 40'$, to find the base AB, and the angles at B and C.

Geometrically. Draw the base AB at pleasure, and on any point assumed, as A, make an angle of $36^{\circ}, 40'$; take 75,4 from the scale of equal parts and set it from A to C; then take 51,56 from the same scale; set one foot of the dividers in C, and with the other intersect the base in B; lastly, draw BC, and the triangle is completed, and the base may be measured by the same scale of equal parts.



By

By Calculation. Here we have the side BC opposite the known angle at A, and the side AC opposite the unknown angle at B, which may be found by Rule 1st.

To find the angle at B. Having to find an angle, we begin with a side.

As BC	51,55	Co. Ar. 8,28778
Is to AC	75,4	1,87737
So is the sine of the angle A	36°,40'	9,77609

To the sine of the angle B 60°,51' 9,94124

From the sum of all the angles 180°,00'

Take the sum of the angles A and B 97°,01'

And we have the angle C equal to 82°,59'

For the base AB. Having to find a side, we begin with an angle.

As the sine of A 36°,40' Co. Ar. 0,22392

Is to the sine of C 82°,29' 9,99625

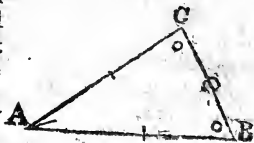
So is BC 51,55 1,71223

To AB 85,6 1,93240

PROBLEM IV. and V. Two sides, and the angle included between them at A, given, to find the two other angles and the other side.

In the oblique angled triangle ABC, given the side AC 75,4, the base AB 85,6, and the included angle at A 36°,40', to find the angles B and C, and the side BC.

Geometrically. Draw the base AB, and, from any scale of equal parts, set off 85,6 from A to B; make an angle at A of 36°,40', and draw AC, and from the same scale of equal parts, set 75,4 from A to C; lastly, draw the line BC, and the triangle is completed: BC may be measured by the same scale of equal parts, and the angles B and C, on the line of chords.



By Calculation. Here we have given the two sides AB and AC, with the angle included between them; and therefore these cases must be solved by Rules 2d and 1st. Now, as the three angles of every triangle are equal to 180°, the angle at A 36°,40' being subtracted from 180° leaves 143°,20', the sum of the two unknown angles B and C, half of which is 71°,40'; and half their difference may be found by the following proportion, according to Rule 2.

As the sum of the two sides AB and AC 161 Co. Ar. 7,79318

Is to their difference 10,2 1,00860

So is the tangent of half the sum of the unknown angles B and C } 71°,40' 10,47969

To the tangent of half their difference 10°,49' 9,28147

To

As AC	75.4	Co. Ar. 8,12263
Is to AD	60,48	1,78161
So is radius	90°,00'	10,
To the sine of C	53°,20'	9.90424

The angle A, being the complement of the angle C, is 36°,40'. Then for the angle C in the right angled triangle BDC.

As BC	51,56	Co. Ar. 8,28778
Is to BD	25,12	1,40002
So is radius	90°,00'	10,
To the sine of C	29°,09'	9,68780

Whence the angle A is 60°,51', being the complement of 29°,09'; and the angle at C, in one triangle, being added to the angle C in the other, is 82°,29'; thus the solution of the problem is finished.

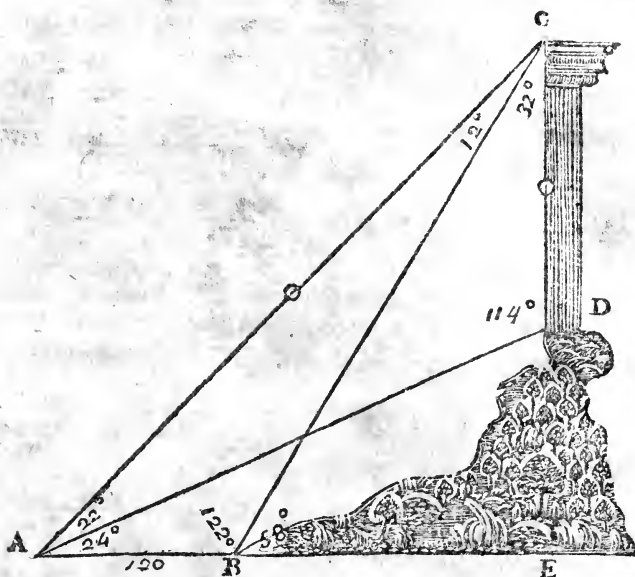
Trigonometry is easily applied to Navigation, and the Mensuration of Heights and Distances. With respect to the former; suppose in the first Problem of right angled Trigonometry, the angle at A is the ship's course, the base to be the true, (or meridional,) difference of latitude, the perpendicular to be the departure, or difference of longitude, and the hypotenuse to be the distance the ship is to run; then we have the course and true (or meridional) difference of latitude given, to find the distance, and departure from the meridian, (or difference of longitude.)

In Problem 2d, we have the course and distance given, to find the true (or meridional) difference of latitude, and the departure, (or difference of longitude.)

With respect to heights and distances; If we suppose, in the first Problem before mentioned, the angle at A to be the angle which the top of any distant object makes with the surface of the earth, where we stand—the base to be the distance of the object, (on level ground,) and the perpendicular, the object's height; then we have the angle A, and the distance AB, to find the height BC; but this will serve only on level ground, and where the object is accessible.

The distance of any *inaccessible* object may be found by Problem 1st, of oblique Trigonometry: For, if we suppose the object at C, then, at *two* stations, as at A and B, take the bearing of the place; also measure the stationary distance AB, and you will then have *two* angles and a side opposite to one of them, to find either of the other sides.

To take the height of an Object standing on a hill, which is inaccessible.



At two stations, as at A and B, take the angles, viz. CAE and CBE, which the top of the object makes with an horizontal line, and that, which the bottom of the object makes with the first station, at A, viz. DAE; then take DAE from CAE, and the remainder is CAD.

Note, When an angle is expressed by three letters, the middle one shews the angle. Now, suppose the stationary distance AB 120, the angle ACB 12° , and angle CBA 122° , then by Problem 1st, of oblique Trigonometry, we have two angles and a side opposite to one of them given, to find the side AC. Therefore,

	<i>Co. Ar.</i>
As S. of ACB	$12^{\circ},00' - 0,68213$
Is to S. of CBA	$122,00' - 9,92842$
So is stationary distance	$120 - 2,07918$
To side AC	$489,5 - 2,68973$

Note, I subtracted 122° from 180° , and worked with the remainder, and in the following, 114° from 180° . Now, having found AC 489,5, suppose the angle CDA 114° , and the angle CAD 22° , and we have two angles and a side opposite to one of them, as before, to find the perpendicular height of the object CD. Therefore,

	deg. min.	Co. Ar.		deg.	Co. Ar.
As S. of CDA	114,00	0,03927	As S. of CDA	114	— 0,03927
Is to S. of CAD	22,00	9,57358	Is to S. of ACD	44	— 9,84177
So is side AC	489,5	2,68973	So is side AC	489,5	— 2,68973
To perpend. hht. CD	200,7	2,30258	To side AD	372,2	— 2,67077

To find the height of the mountain and object together ; we have the right angled triangle ACE, in which are given the hypotenuse AC 489,5, angle CAE 46°, and the angle ACE 44°, whence, by Problem 2d, of right angled Trigonometry, we have these proportions.

As radius	90°	10,00000	As radius	90°	— 10,00000
Is to S. of CAE	46°	9,85693	Is to S. of ACE	44°	— 9,84177
So is hypoth. AC	489,5	2,68973	So is AC	489,5	— 2,68973
To perp. hht. CE	252,1	2,54666	To AE	340	— 2,53150

If you subtract CD from CE, you will have the height of the hill 151,4.

Any figure in Navigation, or Mensuration of Heights and Distances, may be measured Geometrically, as directed in the foregoing Problems of Trigonometry.

MENSURATION of SUPERFICIES and SOLIDS.

SECTION I. Of SUPERFICIES.

Superficies, or surfaces, are measured by the superficial inch, foot, yard, &c. according to the measures peculiar to different artists.

The superficial inch, foot, &c. is one inch, foot, &c. in length and breadth ; and, because 12 inches make 1 foot of Long Measure, therefore, $12 \times 12 = 144$ inches make 1 superficial foot, $3 \times 3 = 9$ feet, a yard, &c.

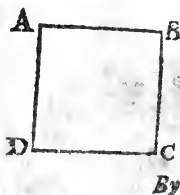
The superficial content of every surface is found by the proper rule of its figure, whether square, triangle, polygon, or circle.

ARTICLE 1. To measure a Square, having equal sides.

RULE.—Multiply the side of the square into itself, and the product will be the area or superficial content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.

Let ABCD represent a square, whose side is 12 feet. Multiply the side 12 by itself, thus,

12 inches.	12 feet.
12 inches.	12 feet.
—	—
Area = 144 inches.	144 feet.



By the Sliding Rule.

Set 1 to the length on B, then, find the breadth on A, and opposite to this on B, you will have the content.

By Gunter's Scale.

Extend the dividers from 1, on the line of numbers, to the length; that distance, laid the same way from the breadth, will point out the answer.

ART. 2. *To measure a Parallelogram, or long Square.*

RULE.—Multiply the length by the breadth, and the product will be the area, or superficial content.

Let ABCD represent a parallelogram, whose length is 16 feet, and breadth, 12 feet. Multiply 16 by 12.

Length 16
Breadth 12



—
192 area.

The content of this figure is found on the sliding rule and scale, as the former.

ART. 3. *When the breadth of a Superficies is given, to find how much in length will make a square foot, yard, &c.*

RULE.—As the breadth is to a foot, yard, &c. so is a foot, yard, &c. to the length required to make a foot, yard, &c. Or, divide 144 by the breadth, and the quotient will be the length required.

How much, in length, of a board $2\frac{1}{2}$ feet wide, will make a square foot?

In. br. In. leng. In. br. In. leng.

As 30 : 12 :: 12 : 4,8

12

—
30)144(4,8 inches, length required.

120

—
240

240

In.

Breadth = 30)144(4,8 inches, *Ans.*

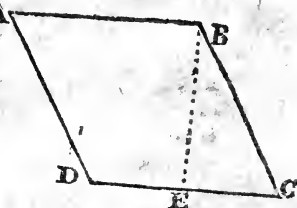
ART. 4. *To measure a Rhombus.*

Definition. A rhombus is a figure with four equal sides, in the form of a diamond on cards, having two angles greater, and two less, than the angles of a square: The former are called *obtuse angles*, and the latter, *acute*, or sharp, angles.

RULE.

RULE.—Multiply the side by the length of a perpendicular let fall from one of the obtuse angles to the side opposite such angle.

Let ABCD represent a rhombus, each of whose sides is 15 feet : A perpendicular let fall from the obtuse angle, at B, on the side DC, will intersect it in the point E, so will BE be 12 feet ; and this being multiplied into the given side, the product will be the area of the rhombus.



$$\begin{array}{r} \text{Side} = 16 \\ \text{Perp.} = 12 \\ \hline 192 \text{ area.} \end{array}$$

By the Sliding Rule.

Set 1 on A to the length on B ; find the perpendicular height on A, against which on B is the content.

By Gunter.

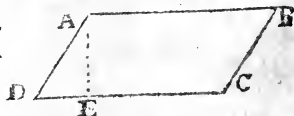
The extent from 1 to the perpendicular height will reach from the length to the content.

ART. 5. *To find the Area of a Rhomboides.*

Definition. A rhomboides is a figure whose opposite sides and opposite angles are equal.

RULE.—Multiply one of the longest sides by the perpendicular let fall from one of the obtuse angles on one of the longest sides.

Let ABCD represent a rhomboides ; the longest sides AB and CD being 16,5 feet, and the perpendicular AE, 9,7 feet.



$$\begin{array}{r} \text{Side} = 16,5 \\ \text{Perp.} = 9,7 \\ \hline \end{array}$$

$$\begin{array}{r} 1155 \\ 1485 \\ \hline \end{array}$$

Ans. 160,05 feet.

The content is found on the sliding rule, and scale, as in the last figure.

ART. 6. *To measure a Triangle.*

RULE.—If it be a right angled triangle, multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area : But, if it be an oblique angled triangle (whether obtuse, or acute) multiply half the base by the length of the perpendicular let fall on the base from the angle opposite to it, and the product will be the area. The longest side of a triangle is usually called the base, except in a right angled

gled triangle, where the longest of the two legs, which include the right angle, is called the base.

In the right angled triangle ABC right angled at C ; the base AC is 18,8 feet, and the perpendicular BC = 12,6.

Base = 18,8 Or, Perp. = 12,6
Perp. = 6,3 $\frac{1}{2}$ Base = 9,4

564
1128

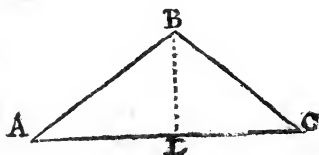
118,44 area.

504
1134

118,44 area.



The oblique angled triangle ABC being given, let fall a perpendicular from the angle at B on the base AC, and that perpendicular is the height of the triangle. The base AC being 15,6, and the perpendicular BD = 9, to find the area.



7,8 = half the base.

9 = height of the triangle.

70,2 = area.

By the Sliding Rule.

Set 1 on A to the length of the base on B, and opposite to half the length of the perpendicular, on A, you will have the content on B.

By Gunter.

The extent from 1 to half the length of the perpendicular will reach from the length of the base to the content.

In this place it may be proper to instruct the learner in one of the properties of a right angled triangle : viz. That the square of the longest side of a right angled triangle, usually called the hypotenuse, is equal to the sum of the squares of the two other sides, usually called the legs, which is of great use, for, by this mean, any two sides of a right angled triangle being given, the other may be found by common Arithmetic. Thus, in the right angled triangle ABC, the base AC and perpendicular BC being given, the hypotenuse AB may be found by extracting the square root of the sum of the squares of the base and perpendicular.

Base

Base 18,8	Perp. 12,6	353,44 = square of the base.
18,8	12,6	158,76 = square of the perp.
<hr/>	<hr/>	<hr/>
1504	756	• • •
1504	252	512,20 (22,63 hypotenuse.
188	126	4
<hr/>	<hr/>	<hr/>
353,44	158,76	42)112
		84
		<hr/>
		446)2820
		2676
		<hr/>
		4523)14400
		13569
		<hr/>
		831

And, if the hypotenuse and one of the legs be given, the other may be found by subtracting the square of the given leg from the square of the hypotenuse.

There are some numbers, the sum of whose squares make a perfect square, of which sort are 3 and 4, whose squares, being added together, make 25, which is the square of 5 : Therefore, if the base of a triangle be 4, and the perpendicular 3, the hypotenuse will be 5 ; and if any of these numbers be multiplied by any other number, those products will be the sides of right angled triangles, as 6, 8, 10, and 15, 20, 25, &c. Thus, artificers, when they set off the corner of a building, usually measure 6 feet on one side, and 8 feet on the other, then laying a ten feet pole across, it makes the corner a true right angle.

ART. 7. *There is another method of finding the area of triangles, the three sides being given.*

RULE.—Add the three sides together, then take the half of that sum, and out of it subtract each side severally ; multiply the half of the sum and these remainders continually, and the square root of this product will be the area of the triangle.

In the oblique triangle ABC, the base AC is given 15,6, the side AB is 10,4, and the side BC is 9,2, to find the area.

15,6	17,6	17,6	17,6
10,4	—15,6	—10,4	— 9,2
9,2	<hr/>	<hr/>	<hr/>
	2	7,2	8,4
<hr/>			
35,2 sum.			
<hr/>			
17,6 = half the sum.			



$$\begin{array}{r}
 17,6 \\
 \times 2 \\
 \hline
 35,2 \\
 7,2 \\
 \hline
 704 \\
 2464 \\
 \hline
 253,44 \\
 8,4 \\
 \hline
 101376 \\
 202752 \\
 \hline
 2128,896
 \end{array}$$

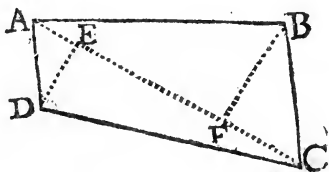
$$\begin{array}{r}
 2128,8960 (46,139 = \text{area.} \\
 16 \\
 \hline
 86)528 \\
 516 \\
 \hline
 921)1289 \\
 921 \\
 \hline
 9223)36860 \\
 27669 \\
 \hline
 92269)919100 \\
 830421 \\
 \hline
 88679
 \end{array}$$

ART. 8. To measure a Trapezium.

Definition. A trapezium is an irregular figure of four unequal sides, and unequal angles.

RULE.—Draw a diagonal line from one of the angles to the opposite angle, as AC, and then will the trapezium be divided into two triangles, of which the diagonal is the common base: Then letting fall perpendiculars from the other opposite angles on the diagonal, add those perpendiculars together, and multiply half that sum into the diagonal, or half of the diagonal into the sum of the perpendiculars, and that product will be the area of the trapezium.

In the trapezium ABCD, the diagonal AC is 24, the perpendicular DE 6, and the perpendicular BF 10. The sum of the perpendiculars is 16, whose half is 8, which being multiplied into 24, will give the area.



$$\begin{array}{r}
 24 \\
 \times 8 \\
 \hline
 192 = \text{area.}
 \end{array}$$

By the Sliding Rule.

Set 1 on A to $\frac{1}{2}$ the sum of the perpendiculars on B, and opposite the length of the diagonal on A, you will have the area on B.

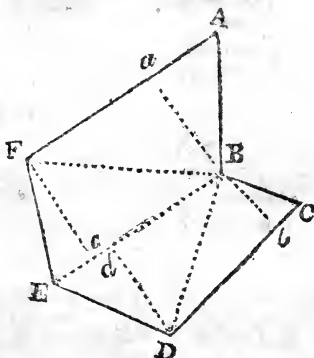
By Gunter.

The extent from 1 to $\frac{1}{2}$ the sum of the perpendiculars will reach from the length of the diagonal to the area.

ART. 9. To measure any irregular Figure.

RULE.—Divide the figure into triangles, by drawing diagonals from one angle to another ; then measure all the triangles by either of the rules, already taught, at Article 6 or 7, and the sum of the several areas of all the triangles will be the area of the given figure.

The irregular figure ABCDEF being given, divide it into triangles by the diagonals FB, EB, and DB : Then may the triangles be measured by letting fall perpendiculars on their respective bases, as Ba, Bb, Dc, Fd, and multiplying those perpendiculars by half their respective bases.



In the triangle AFB the base FA is 100, and the perpendicular Ba 49 ; in the triangle FBE the base BE is 92, and the perpendicular Fd 52 ; in the triangle EBD, the base BE is the same as before, and the perpendicular Dc 44 ; and in the triangle DCB, the base DC is 80, and the perpendicular Bb 38 ; by which the area of each may be found by Art. 6, as follows.

50 = half AF.	46 = half BE.	2450
49 = perp. aB.	52 = perp. Fd.	2024
<hr/>	<hr/>	<hr/>
2450 = area of AFB.	92	2392
	320	1520
	<hr/>	<hr/>
46 = half BE.		8386 = area of the
44 = perp. Dc.	2392 = area of FBE.	figure ABCDEF.
<hr/>	<hr/>	
184	38 = perp. Bb.	
184	40 = half DC.	
<hr/>	<hr/>	
2024 = area of EBD.	1520 = area of DCB.	

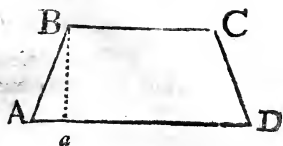
In dividing any irregular figure into triangles, the triangles will be less, by two, and the diagonals less by three, than the number of the sides of the figure.

ART. 10. To measure a Trapezoid.

Definition. A trapezoid is the segment of a triangle, cut by a line parallel to the base.

RULE.—Add the parallel sides together, and multiply half that sum by the perpendicular breadth.

In the trapezoid ABCD, the side AD is 24, the side BC is 16, and the perpendicular breadth Ba is 10, to find the area by adding the sides BC and AD — and multiplying 200 = area. half their sum by the perpendicular breadth Ba.



By the Sliding Rule.

Set 1 on A to the equated length on B, and against the breadth on A you will have the area on B.

By Gunter.

The extent from 1 to the breadth will reach from the equated length to the area.

ART. 11. To measure any regular Polygon.

Definition. A regular polygon is a figure whose sides and angles are all equal; they are usually denominated from the number of their sides:

Thus, A figure having	$\left\{ \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} \right\}$	equal sides and angles is a	$\left\{ \begin{array}{l} \text{Trigon.} \\ \text{Tetragon.} \\ \text{Pentagon.} \\ \text{Hexagon.} \\ \text{Heptagon.} \\ \text{Octagon.} \\ \text{Enneagon.} \\ \text{Decagon.} \\ \text{Endecagon.} \\ \text{Dodecagon.} \end{array} \right\}$
-----------------------	---	-----------------------------	---

RULE.—Multiply the length of one of the sides by the number of sides; then this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the sides, and the product will be the area of the polygon.

In the pentagon ABCDE, each side is 95, and the perpendicular FG 65.36, to find the area.

95 = length of a side.

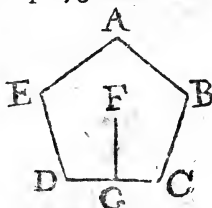
5 = number of sides.

475 = sum of the sides.

32,68 = half of the perpendicular.

$$\begin{array}{r} 3800 \\ 2850 \\ 950 \\ 1425 \\ \hline 15523,00 \end{array}$$

15523,00 = area of the pentagon.



By

By the Sliding Rule.

Set 1 on A to $\frac{1}{2}$ the perpendicular on B, and against the sum of the sides on A you will have the area on B.

By Gunter.

The extent, from 1 to $\frac{1}{2}$ the length of the perpendicular, will reach from the sum of the sides to the content.

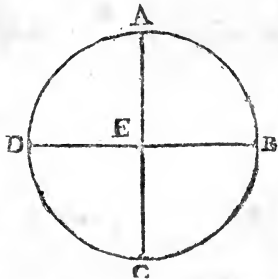
But, for the more ready measuring regular polygons, the following Table, containing multipliers for all regular figures from the Triangle to the Dodecagon, will be of use to the learner.

Number of sides.	Names.	Multipliers.	Number of sides.	Names.	Multipliers.
3	Trigon.	,433013	8	Octagon.	4,828427
4	Tetragon.	1,	9	Enneagon.	6,181827
5	Pentagon.	1,720477	10	Decagon.	7,694209
6	Hexagon.	2,589076	11	Endecagon.	8,51425
7	Heptagon.	3,633959	12	Dodecagon.	9,330125

If the square of the side of a polygon be multiplied by the multiplier of the like figure, the product will be the area of the figure sought.

To measure a Circle and its Parts.

In the annexed circle ABCD, the arch line ABCD is called the *periphery*, the length of which is called the *circumference*: Any line, as DB or AC, passing through the centre E, cuts the circle into two equal parts, called *femicircles*, or half circles; and such lines are called *diameters* of the circle: If two diameters be drawn through a circle, at right angles to each other, then, the four equal divisions of the circle are called *quadrants*: Half the diameter, as EB, is called the *radius*, or *semidiameter*.



ART. 12. *The Diameter of a Circle being given, to find the Circumference.**

RULE.—This may be done by either of the following proportions in whole numbers, as 7 is to 22, or more exactly, as 113 is to

* Note 1. If the diameter of any circle
 be { multiplied } by { 3,14159 } the product { is the circumference.
 { divided } { ,31831 } the quotient {

2. If the diameter of any circle
 be { multiplied } by { ,886227 } the product { is the side of an equal square.
 { divided } { 1,128379 } the quotient {

3. If

to 355; or in decimals, as 1 is to 3,14159; so is the diameter of a circle to the circumference.—EXAMP. A circle whose diameter is 12, to find the circumference.

As

3. If the diameter of any circle
 be { multiplied } by { .866024 } the product { is the side of the equilateral
 { divided } { 1,547 } the quotient { triangle inscribed.

4. If the diameter of any circle
 be { multiplied } by { .707106 } the product { is the side of the square
 { divided } { 1,414213 } the quotient { inscribed.

5. If the square of the diameter of any circle
 be { multiplied } by { .785398 } the product { is the area.
 { divided } { 1,273241 } the quotient {

6. If the circumference of any circle
 be { multiplied } by { .31831 } the product { is the diameter.
 { divided } { 3,14159 } the quotient {

7. If the circumference of any circle
 be { multiplied } by { .282094 } the product { is the side of the
 { divided } { 3,544907 } the quotient { square equal.

8. If the circumference of any circle
 be { multiplied } by { .2756646 } the product { is the side of the equilateral
 { divided } { 3,6275939 } the quotient { triangle inscribed.

9. If the circumference of any circle
 be { multiplied } by { .225079 } the product { is the side of the
 { divided } { 4,442877 } the quotient { square inscribed.

10. If the square of the circumference of any circle
 be { multiplied } by { .079577525 } the product { is the area.
 { divided } { 12,56636217 } the quotient {

11. If the area of any circle
 be { multiplied } by { 1,273241 } the product { is the square of
 { divided } { .785398 } the quotient { the diameter.

12. If the area of any circle
 be { multiplied } by { 12,56636217 } the product { is the square of the
 { divided } { .079577525 } the quotient { circumference.

13. When the diameter of one circle is 1, and the diameter of another is 2, the circumference of the first is equal to the area of the second, = 3,141592.

14. If the circumference be 4, the diameter and area are equal, = 1,273241.

15. If the diameter be 4, the circumference and area are equal, = 12,566368.

Hence, because circles are the most capacious of all figures, if the *fourth* part of a circle be *squared*, it will not be equal to the *area* of that circle (as is commonly supposed) although the *four* sides added together are equal to the *circumference* of that circle.

In a circle, whose diameter is 24, circumference 75,4, and area 452,4, the *fourth* part of the circumference is 18,85, the *square* of which is only 355,3225, that is 97,0775 less than the truth; and the larger the circle is, the greater will the error be.

For further proof of this matter; If a cylindrical pint, beer measure, whose content is 35,25 cubic inches, be beaten into a perfectly *square* form, it will contain only 28,902 cubic inches, which is less than the truth by 6,3484+; the area of the circle is 8,7615859288, and the area of the square only 6,8813320653076624.

Hence appears the reason, why taking the *fourth part* of the girth in measuring a cylinder (or a round stick of timber) is false.

16. If the diameter of one circle be double to that of another, the *area* of the first circle will be *four times* the area of the second.

$$\begin{array}{r} \text{As } 7 : 22 :: 12 \\ \hline 12 \end{array}$$

$$\begin{array}{r} \text{As } 113 : 355 :: 12 \\ \hline 12 \end{array}$$

$$\begin{array}{r} \text{As } 1 : 3,14159 :: 12 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 7 \overline{)264} (37,71 = \text{cir-} \\ 21 \text{ cumference.} \end{array}$$

$$\begin{array}{r} 113 \overline{)4260} (37,699 \text{ cir.} \\ 339 \end{array}$$

$$37,69908 \text{ cir.}$$

$$\begin{array}{r} 54 \\ 49 \\ \hline \end{array}$$

$$\begin{array}{r} 870 \\ 791 \\ \hline \end{array}$$

$$\begin{array}{r} 50 \\ 49 \\ \hline \end{array}$$

$$\begin{array}{r} 790 \\ 678 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1120 \\ 1017 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \end{array}$$

$$\begin{array}{r} 103 \end{array}$$

Note, 3,14159 may be contracted to 3,1416 without any sensible difference.

ART. 13. *The Circumference of a Circle being given, to find the Diameter.*

RULE.—As 22 is to 7 ; or 355 to 113 ; or as 1 to 3,1831, So is the circumference of a circle to the diameter.

EXAMP. The circumference of a circle being 326, to find the diameter.

$$\begin{array}{r} \text{As } 22 : 7 :: 326 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 355 : 113 :: 326 \\ \hline 326 \end{array}$$

$$\begin{array}{r} 1 : 3,1831 :: 326 \\ \hline 326 \end{array}$$

$$\begin{array}{r} 22 \overline{)2282} (103,72 \text{ diam-} \\ 22 \text{ eter.} \end{array}$$

$$\begin{array}{r} 678 \\ 226 \end{array}$$

$$\begin{array}{r} 190986 \\ 63662 \end{array}$$

$$\begin{array}{r} 339 \end{array}$$

$$\begin{array}{r} 95493 \end{array}$$

$$\begin{array}{r} 82 \end{array}$$

$$\begin{array}{r} 66 \end{array}$$

$$\begin{array}{r} 355 \overline{)36838} (103,76 \text{ diameter.} \\ 355 \end{array}$$

103,76906 = diam.
This proportion is the most accurate.

$$\begin{array}{r} 160 \end{array}$$

$$\begin{array}{r} 154 \end{array}$$

$$\begin{array}{r} 1338 \end{array}$$

$$\begin{array}{r} 1065 \end{array}$$

$$\begin{array}{r} 60 \end{array}$$

$$\begin{array}{r} 44 \end{array}$$

$$\begin{array}{r} 2730 \end{array}$$

$$\begin{array}{r} 2485 \end{array}$$

$$\begin{array}{r} 16 \end{array}$$

$$\begin{array}{r} 245 \end{array}$$

ART. 14. *To find the Area of a Circle.*

RULE.—Multiply half the diameter by half the circumference, and the product is the area.

If the diameter be given, find the circumference by Art. 12.

If the circumference be given, find the diameter by Art. 13.

EXAMP. A circle, whose diameter is 12, and circumference is 37,7, given, to find the area ?

$$18,85 = \text{half the circumference.}$$

$$6 = \text{half the diameter.}$$

$$113,1 = \text{area of the given circle.}$$

ART.

ART. 15. *The Diameter being given, to find the Area of a Circle without finding the Circumference.*

RULE.—Multiply the square of the diameter by ,7854, and the product will be the area of the circle, whose diameter was given.

EXAMP. The diameter of a circle being 12, to find the area?

$$12 \times 12 = \overset{7854}{144}$$

$$\begin{array}{r} 31416 \\ 31416 \\ \hline 7854 \end{array}$$

$$113,0976 = \text{area.}$$

By the Sliding Rule.

Set 1 on A to the diameter on B, then find ,7854 (which expresses the area of a circle whose diameter is 1) on A, against which on B is a 4th number, then find this 4th number on A, against which on B is the area.

By Gunter.

The extent from 1 to the length of the diameter reaches from ,7854 to a 4th number, and from that 4th number to the area.

ART. 16. *The Circumference of a Circle being given, to find the Area, without finding the Diameter.*

RULE.—Multiply the square of the circumference by ,07958, and the product will be the area of the circle.

EXAMP. The circumference of a circle being 37,7, to find the area?

37,7	1421,29
37,7	,079 58
<hr/>	<hr/>
2639	1137032
2639	710645
1131	1279161
<hr/>	<hr/>
1421,29 = square.	994903

$$113,1062582 = \text{area of the circle.}$$

ART. 17. *The Dimensions of any of the parts of a Circle being given, to find the side of a Square equal to the Circle.*

RULE.—If the area of the circle be given, extract the square root of the area, which will be the side of a square equal to the circle: If the diameter or circumference be given, find the area by Art. 15 or 16, and then extract the square root, as before. And this is a general rule to find the side of a square equal to any superficial figure, regular or irregular: For the square root of the area

area of any figure whatever, is the side of a square equal to the given figure. But, with regard to circles, if the diameter be given; multiply it by ,886, and the product will be the side of an equal square: Or, As 13,545 is to 12, or 1354 to 1200: So is the diameter of a circle to the side of a square equal to the given circle. And, if the circumference be given, multiply it by ,282 for the side of an equal square. Or, divide it by 3,545, and the quotient will be the side of an equal square.

EXAMP. 1.

Let the diameter of a circle be 12, to find the side of a square equal to the circle?
 $,886 \times 12 = 10,632 =$ side of the square.
 Or,
 As 13,545 : 12 :: 12 : 10,631
 $=$ the side.

EXAMP. 2.

The circumference being 37,7 to find the side of an equal square?
 $37,7 \times ,282 = 10,631 =$ side of the square.
 Or, $37,7 \div 3,545 = 10,634$

ART. 18. *The Area of a Circle being given, to find the Diameter.*

RULE — Multiply the given area by 1,2732, and the product will be the square of the diameter; then, extracting the square root of the product, you will have the diameter.

EXAMP. The area of a circle being 113,09, to find the diameter.

$$\begin{array}{r} 1,2732 \\ 113,09 \\ \hline 114588 \\ 381960 \\ 12732 \\ 12732 \\ \hline 143,986188 \end{array}$$

$$\begin{array}{r} 143,986188(11,999 = 12 = \text{diameter,} \\ 1 \\ \hline 21)43 \\ 21 \\ \hline 229)2298 \\ 2061 \\ \hline 2389)23761 \\ 21501 \\ \hline 23989)226088 \\ 215901 \\ \hline 10187 \text{ remainder.} \end{array}$$

ART. 19. *The Area of a Circle being given, to find the Circumference.*

RULE.—Multiply the given area by 12,566, and extract the square root of the product, which root will be the circumference required.

EXAMP. The area of a circle being 113,03 to find the circumference.

$$12,566$$

$$\begin{array}{r} 12,566 \\ 113,03 \\ \hline \end{array}$$

$$\begin{array}{r} 37698 \\ 376980 \\ 12566 \\ 12566 \\ \hline \end{array}$$

$$1420,33498$$

$$\begin{array}{r} \dots\dots\dots \\ 1420,3349(37,68 = \text{circumference.} \\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 67)520 \\ 469 \\ \hline \end{array}$$

$$\begin{array}{r} 746)5133 \\ 4476 \\ \hline \end{array}$$

$$\begin{array}{r} 7528)65749 \\ 60224 \\ \hline \end{array}$$

5525 remainder.

ART. 20. *The Side of a Square being given, to find the Diameter of a Circle equal to the Square, whose side is given.*

RULE.—Multiply the given side by 1,128, and the product will be the diameter of a circle, whose area is equal to the area of the given square. Or, If the side of the square be divided by ,886, the quotient will be the diameter. Or, As 12 to 13,54, So is the side of any square to the diameter of an equal circle.

EXAMP. The side of a square being 10,635, to find the diameter of a circle equal to that square ?

$10,635 \times 1,128 = 12$ nearly. Or, $10,635 \div ,886 = 12 = \text{diameter.}$
Or, As 12 : 13,54 :: 10,635 : 12 nearly.

ART. 21. *The Side of a Square being given, to find the Circumference of a Circle equal to the given Square.*

RULE.—Multiply the given side by 3,545 and the product will be the circumference required. Or, divide it by ,282, and the quotient will be the circumference.

EXAMP. The side of a square being 10,631, to find the circumference of a circle equal to that square. Or,

$10,631 \times 3,545 = 37,686 = \text{circum.}$,282)10,631(37,698 circum.

ART. 22. *To find the Area of a Semicircle, the Diameter being given.*

RULE.—Find the area of the circle by Art. 15, and take the half of it.

In the same manner may the area of a quadrant, or a quarter of a circle, be found, by taking a fourth part of the area of the whole circle.

But with regard to measuring a sector, or a segment of a circle, it will be necessary first to shew how to find the length of the arch line of a sector, and the diameter of the circle to a given segment.

ART. 23. *A Segment of a Circle being given, to find the Length of the Arch Line.*

RULE.—Divide the segment into two equal parts ; then measure the chord of the half arch, from the double of which subtract the

the chord of the whole segment; and one third of that difference, being added to the double of the chord of the half arch, will give the length of the arch line.

EXAMP. In the segment ABCD, the whole chord ADC is 216, and the chord AB or BC 126, to find the arch line ABC.

$$126 = \text{chord AB or BC.}$$

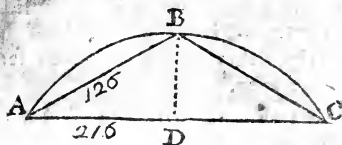
2

$$252 = \text{double.}$$

$$216 = \text{ADC, to be subtracted.}$$

$$3)36 = \text{difference.}$$

$$12 = \frac{1}{3} \text{ difference.}$$



$$252 = \text{double of AB.}$$

$$12 = \frac{1}{3} \text{ difference, added.}$$

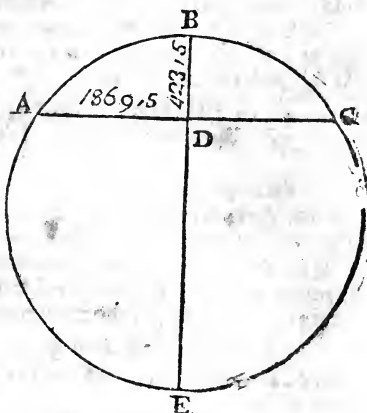
$$264 = \text{length of the arch ABC.}$$

ART. 24. *The Chord and versed Sine of a Segment being given, to find the Diameter of a Circle.*

RULE.—Multiply half the chord by itself, and divide the product by the versed sine; then add the quotient to the versed sine, and the sum will be the diameter of the circle.

EXAMP. In the segment ABCD, the chord AC is 1869,5, and the versed sine BD 423,5, to find the diameter.

$$\begin{array}{r} 934,75 \left\{ \begin{array}{l} \text{half the} \\ \text{chord AC} \end{array} \right. \\ \hline 934,75 \\ 467375 \\ 654325 \\ 373900 \\ 280425 \\ 841275 \end{array}$$



$$423,5)873757,5625(2063,1 = \text{DE.}$$

$$8470$$

$$26757$$

$$25410$$

$$13475$$

$$12705$$

$$7706$$

$$4235$$

$$3471$$

$$423,5 = \text{BD, add.}$$

$$2486,6 = \text{diameter BDE.}$$

K k k

ART.

ART. 25. To measure a Sector.

Definition. A sector is a part of a circle, contained between an arch line and two radii or semidiameters of the circle.

RULE.—Find the length of half the arch by Art. 23 : Then multiply this by the radius or semidiameter, and the product will be the area.

EXAMP. 1. In the sector ABCD, given the radius AD or DC 72 feet, the chord AC = 126 feet, and the chord AB or BC = 70, to find the area of the sector.

First.

$$70 = \text{chord AB or BC.}$$

2

$$\hline 140$$

$$126 = \text{AC, subtract.}$$

$$\hline 3)14$$

$$\hline 4,66$$

$$\hline 140$$

$$\hline 144,66 = \text{length of the arch ABC,}$$

(by Art. 23.

$$72,33$$

EXAMP. 2. In the sector ABCD, greater than a semicircle, given the radius AE or ED = 112, the chord BD (of half the arch ABD) = 204, and the chord BC (of half the arch BCD) = 120, to find the area of the sector.

$$120 = \text{BC.}$$

2

$$\hline 240$$

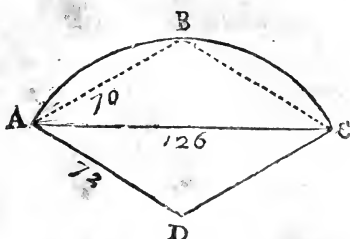
$$204 \text{ subtract.}$$

$$\hline 3)36$$

$$\hline 12$$

$$240 \text{ Add.}$$

$$\hline 252 = \left\{ \begin{array}{l} \text{Length of the arch} \\ \text{BCD, by Art. 23.} \end{array} \right.$$



Secondly.

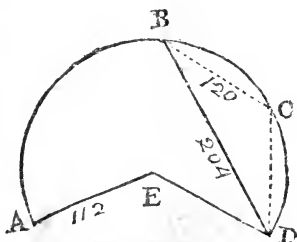
$$72,33 = \text{half the arch.}$$

$$72 = \text{radius.}$$

$$\hline 14466$$

$$\hline 50631$$

$$\hline 5207,76 = \text{area.}$$



$$252 = \text{half of the arch ABD.}$$

$$112 = \text{radius.}$$

$$\hline 504$$

$$252$$

$$\hline 252$$

$$28224 = \text{area of the sector.}$$

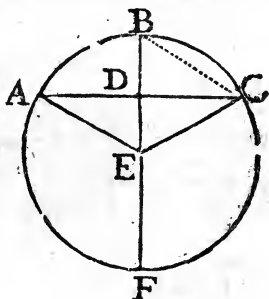
ART.

ART. 26. To find the Area of a Segment of a Circle.

Definition. A segment of a circle is any part of a circle cut off by a right line drawn across the circle, which does not pass through the centre, and is always greater or less than a semicircle.

EXAMP. 1. To find the area of the segment ABC, whose chord AC is 172, the chord of half the arch ABC, viz. BC = 104, and the versed sine BD = 58,48.

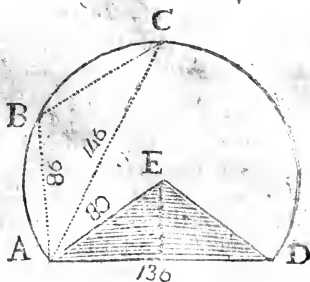
RULE.—By Art. 23, find the length of the arch line ABC, and by Art. 24, the diameter BF; then multiply half the chord of the arch ABC by half the diameter, and the product will be the area of the sector ABCE: Then find the area of the triangle AEC, whose base AC is 172, and perpendicular height 34, found by subtracting the versed sine BD from half the diameter; and the area of the triangle AEC, being subtracted from the area of the sector ABCE, will leave the area of the segment ABC.



104 = BC.	86 = half ADC.
2	86
208	516
172 = AC, subtract.	688
3)36	58,48)7396,00(126,47 = DEF.
12	5848 58,48 = BD, add.
208 add.	15480 184,95 = diameter BF.
220 = arch line ABC.	11696
110 = half arch	37840
	35088
92,475 = radius.	27520
110	23392
924750	41280
92475	40936
10172,25 = area of the sector.	344
86 = half the base = AD.	10172,25 = area of the sector.
34 = perpendicular DE.	2924 = area of triangle.
344	7248,25 = area of the seg.
258	
924 = area of the triangle.	

EXAMP.

EXAMP. 2. In the segment, ABCD greater than a femicircle, given the chord of the whole segment $AD = 136$, the chord AC of half the arch $ACD = 146$, the chord AB or BC of one fourth of the arch $ACD = 86$, and the radius AE or $ED = 80$, to find the area of the segment ABCD.



First find the area of the sector ABCDE, by Art. 25, at the second Example; then find the area of the triangle AED, by Art. 6, and, adding the area of the triangle to the area of the sector, you will have the area of the segment.

86 = chord AB.

2

172

146 = chord AC, subtract.

3) 26

8,666

172 = double of AB, add.

180,666 = arch line ABC.

80 = radius.

14453,280 = area of the sector.

68 = half the base AD.

42 = perpendicular E 136.

136

272

2856 = area of the triangle AED.

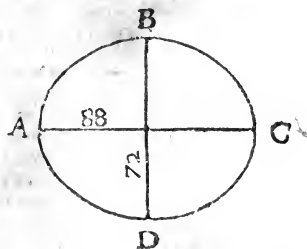
14453,28 = area of the sector, add.

17309,28 = area of the segment.

ART. 27. To find the Area of an Ellipsis.

Definition. An ellipsis, or oval is a curve which returns into itself like a circle, but has two diameters, one longer than the other, the longest of which is called the transverse, and the shortest, the conjugate diameter.

RULE.—Multiply the two diameters of the ellipsis together; then, multiplying the product by ,7854, this last product will be the area of the ellipsis.



EXAMP. In the ellipsis ABCD, the transverse diameter AC is 88, and the conjugate diameter BD is 72, to find the area.

88
72
—
176
616
—
6336
57854
—
25344
31680
50688
44352
—

The content is found by the sliding rule and Gunter, in the same way as the circle, only using the product of the two diameters as the diameter of a circle.

4976,2914 = area.

Menfuration of Superficies is easily applied to *Surveying*: Thus, take the angles of the plot with a good compass, then measure the sides with Gunter's chain, which note down in links (or chains and links, which is done by separating the two right hand figures of your links by a comma, your chain being 100 links) then cast up the contents, according to the rule of the figure, cutting off the five right hand figures of the product, and those at the left hand, if any, are acres; then, multiply the five figures, cut off, by 4, by 40, and by $272\frac{1}{4}$, cutting off as before, and those at the left hand will be roods, poles and feet, respectively.

SECTION II. Of SOLIDS.

Solids are measured by the solid inch, foot or yard, &c. 1728 of these inches, that is $12 \times 12 \times 12$, make 1 cubic or solid foot.

The solid content of every body is found by rules adapted to their particular figures.

ART. 28. To measure a Cube.*

Definition. A cube is a solid of six equal sides, each of which is an exact square.

The

* Here follows a Table of the Proportions, which the following Solids have to the Cube and Cylinder, having the same Base and Altitude.

	Solid Inches.
1. A Cube, whose side is 12 inches, contains	1728
2. A Prism, having an equilateral triangle, whose side is 12 inches for its Base, and its Altitude 12 inches, contains	784,24
3. A Square Pyramid, whose height and the side of its base, are each 12 inches, is $\frac{1}{3}$ of the above cube, and therefore contains	576
4. A Triangular Pyramid, whose height and side of its triangular base are each 12 inches, is near $\frac{1}{7}$ of the cube, and contains	249,413
5. A Cylinder, whose diameter and height are each 12 inches, is $\frac{1}{4}$ of the above cube, and contains	1357,17
6. A Sphere or Globe, whose axis or diameter is 12 inches, equal to the side of the cube, is $\frac{1}{21}$ of it, and contains	904,78
7. A Cone, whose base and altitude are each 12 inches, equal to the side of the cube, is $\frac{5}{19}$ of it, and contains	452,38829

The solid foot is composed of 1728 inches : For a solid, that is 1 foot, or 12 inches every way, that is $12 \times 12 \times 12$, contains 1728 inches.

RULE.

- | | |
|---|--------------------------|
| 8. A <i>Parabolic Conoid</i> , whose diameter at the base, and height are each 12 inches, being $\frac{1}{2}$ its circumscribing cylinder, contains | } Solid Inch.
678,583 |
| 9. A <i>Hyperbolic Conoid</i> , whose height, and diameter at the base, are each 12 inches, is $\frac{5}{8}$ of its circumscribing cylinder, and contains | |
| 10. A <i>Parabolic Spindle</i> , whose height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumscribing cylinder, and contains | } 723,824 |
| | |

Hence arises a different method of finding their contents.

General Rule. If the base of the solid, whose content you would find, be rectilinear, consider it as a *Parallelopipedon* ; if curved, as a *Cylinder*, and find the content accordingly : Then take such a part of the content, thus found, as is specified in the preceding Table, which, if the parts be taken in inches, will be the solid content of the given figure, in inches, which, divided by 1728, will give the cubic feet.

EXAMP. 1. There is a triangular prism, the side of whose base is 48 inches, and whose perpendicular height is 108 inches ; What is its solid content ?

The base being right lined, I consider it as a *parallelopipedon*, the side of whose base is 48 inches, and whose length is 108 inches, and as 784,24 is contained 2,20340712 times in a cubic foot ; 2,20340712 is a divisor, to divide the content of the *parallelopipedon* by ; therefore $48 \times 48 \times 108 \div 2,20340712 = 112930,56$ solid inches = 65,353 solid feet.

Had the dimensions been given in feet, it would have been $4 \times 4 \times 9 \div 2,20340712 = 65,353$ feet.

EXAMP. 2. There is a square pyramid, whose height is 12 feet, and the side of whose base is 3,5 feet ; What is its content ?

$$3,5 \times 3,5 \times 12 \div 3 = 49 \text{ feet, Anf.}$$

EXAMP. 3. There is a triangular pyramid, whose height is 15 feet, and the side of whose base is 5 feet ; What is its content ?

$$5 \times 5 \times 15 \div 7 = 53,57 \text{ feet, Anf.}$$

EXAMP. 4. There is a cylinder, whose diameter is 2,5 feet, and whose length is 24 feet ; What is its content ?

Here, the diameter is to be considered as the side of the base of a *parallelopipedon*. Therefore,

$$2,5 \times 2,5 \times 24 \times 11 \div 14 = 117,857 \text{ feet, Anf.}$$

EXAMP. 5. There is a spherical balloon, whose diameter is 50 feet ; How many cubic feet of air does it contain ?

Here, the diameter is to be considered as the side of a cube. Therefore,

$$50 \times 50 \times 50 \times 11 \div 21 = 65476,19 \text{ feet, Anf.}$$

EXAMP. 6. There is a cone whose height is 15 feet, and the diameter of whose base is 5 feet ; What is its content ?

Here, the diameter of the base is to be considered as the side of the base of a *parallelopipedon*, and its height, as the length. Therefore,

$$5 \times 5 \times 15 \times 5 \div 19 = 98,684 \text{ feet, Anf.}$$

EXAMP. 7. There is a parabolic conoid, whose diameter at the base, is 2,9 feet, and whose height is 6 feet ; What is the content ?

This solid, being $\frac{1}{2}$ of a cylinder ; we must first find the content as of that of a cylinder, and then halve it. Therefore,

$$2,9 \times 2,9 \times 6 \times 11 \div 14 = 39,647, \text{ and } 39,647 \div 2 = 19,823, \text{ Anf.}$$

EXAMP. 8. There is a hyperbolic conoid, whose diameter at the base is 2,9 feet and whose height is 6 feet ; What is the content ?

First, find the content of a cylinder.

$$2,9 \times 2,9 \times 6 \times 11 \div 14 = 39,647, \text{ and } 39,647 \times \frac{5}{12} = 16,519 \text{ feet, Anf.}$$

EXAMP. 9. There is a parabolic spindle, whose middle diameter is 2,9 feet, and whose length is 6 feet ; Required the content ?

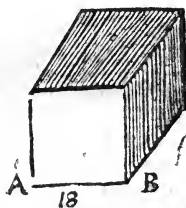
First, find the content of a cylinder.

$$2,9 \times 2,9 \times 6 \times 11 \div 14 = 39,647, \text{ and } 39,647 \times \frac{8}{15} = 21,145 \text{ feet, Anf.}$$

RULE.—Multiply the side by itself, and that product by the same side, and this last product will be the solid content of the cube.

EXAMP. The side of a cube AB, being 18 inches, or 1 foot and 6 inches, to find the content ?

1 foot, 6 inches = 1,5 foot.	18 inches.
1,5	18
—	—
75	144
15	18
—	—
2,25	324
1,5	18
—	—
1125	2592
225	324
—	—
3,375	1728)5832(3,375
	5184
	—



In this operation, the inches are changed into the decimal parts of a foot.

6480
5184
—
12960
12096
—
8640
8640

I have done this two different ways, that the learner may see they come out the same. The content in inches is 5832, which being divided by 1728, the inches in a solid foot, and the division continued by annexing cyphers, it comes out the same as the decimal operation.

Note, The area of the surface, or superficial content of the cube and parallelopipedon is found by adding the areas of the several quadrilateral figures which compose them.

ART. 29. To measure a Parallelopipedon.

Definition. A parallelopipedon is a solid of three dimensions, length, breadth, and thickness; as a piece of timber exactly squared, whose length is more than the breadth and thickness. The ends are called bases, which are equal.

RULE.—Find the area of the base, then multiply that by the length, and it will give the solid content.

EXAMP. 1. The side AB is 1,75 foot, and the length AD 9,5 feet, to find the solid content ?

1,75 =

1,75 = 1 foot, 9 inc.

1,75

875

1225

175

3,0625 = area of base.

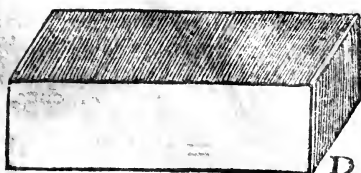
9,5

153125

275625

29,09375 = solid content.

B
5/75
A



9,5

EXAMP. 2. A vessel 3,5 feet each side within, and 5 feet deep, to find the content ?

3,5

3,5

175

105

12,25

5

61,25 = the content.

If a piece of timber, or any other thing, be of an equal bigness through its whole length, though there be a difference between the breadth and thickness, if the breadth and thickness are multiplied together, and that product multiplied by the length, this last product will be the solid content.

EXAMP. 3. A piece of timber being 1 foot and 6 inches, or 18 inches broad, 9 inches thick, and 9 feet 6 inches, or 114 inches long, to find the content ?

1 foot, 6 inches = 1,5 foot.

9 inches = ,75 foot.

Breadth = 18 inches.

Depth = 9 inches.

75

105

1,125

9 Ft. 6 inches = 9,5

5625

10125

10,6875 = content.

162

Length = 114 inches.

648

162

162

1728) 18468 (10,6875 = content, as before.

11880

10368

15120

13824

12960

12096

8640

8640

In this operation the inches are changed into the decimal fractions of a foot.

Note;

Note. When the end is given in inches and the length in feet, find the area at the end in inches, multiply that by the length in feet, and divide this product by 144 (the square inches in a foot) and the quotient will be feet.

Take the last example.

By the Sliding Rule.

Foot.

$$1,5 = 18 \text{ inches.}$$

$$:75 = 9 \text{ inches.}$$

$$162 \text{ area in inches.}$$

$$9,5 \text{ feet} = \text{length.}$$

$$810$$

$$1458$$

$$144)1539(10,6875 = \text{con-}$$

$$144$$

[tent,

$$990$$

$$864$$

$$1260$$

$$1152$$

$$1080$$

$$1008$$

$$720$$

$$720$$

Set 12 inches on the girt line D to the side of the square end on C, then, against the length on D, you will have the answer on C.

By Gunter.

Extend the compasses from 12 inches to the length of the side of the square end; that distance, twice turned over from the length, will reach to the content.

When the side of a square solid is given, in inches, to find how much in length will make a foot solid.

RULE.—As the given side is to 12, so is 12 to a fourth number, and so is that fourth number to the required length.—Or divide 1728 by the area at the end, and the quotient will be the length making a solid foot.

If the given side is in foot measure, then,

RULE.—As the given side is to 1; so is 1 to a fourth number, and so is that fourth number to the required length.

When two sides of an unequal square solid (that is, of unequal breadth) are given, to find what length will make any number of solid feet:

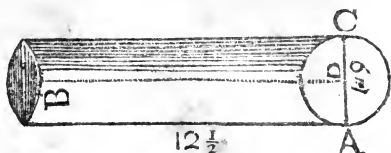
RULE.—Multiply the proposed number of feet by 144: Divide that product by the product of the breadth and depth, and the quotient will be the length required.

ART. 30. *To measure a Cylinder.*

Definition. A cylinder is a round body, whose bases are circles, like a round column, or a rolling stone of a garden.

RULE.—The diameter of the base being given, find the area of the end by Art. 15, then, multiplying the area of the base by the length, that product will be the content of the cylinder.

EXAM. The diameter of the base AC being 1 foot and 9 inches, and the length BD 12 feet and 6 inches, to find the content.



1,75 = diam. of the base.

1,75

875

1225

175

3,0625

,7854

122500

153125

245000

214375

2,40528750 = area of the base.

2,405 = area of the base.

12,5 = length.

12025

4810

2405

30,0625 = content.

If the square of the diameter of a cylinder be multiplied by ,7854, and the solidity divided by that product, the quotient will be the length.

The learner may, for his practice, reduce all the dimensions to inches, and find the solid content in inches, which being divided by 1728, the quotient will be the solid content in feet : Or, if he finds the area at the end in inches, and multiplies that by the length in feet, and divides by 144, the quotient will be feet.

This is a general rule for finding the content of any straight solid body, of equal bigness from end to end, of whatever form the bases are : For, if the area of the base be multiplied by the length, the product will be the solid content.

By the Sliding Rule.

Set 13,5, the square root of 183,34 (which is a gauge point arising from the division of 144 by ,7854) found on D, to the diameter

iameter found on C, and opposite to the length, on D, you will find the content on C.

Or, As 42,54 is to the circumference ; So is the length in feet to a fourth number, and so is that fourth number, to the answer.

Note. The superficial content of a cylinder is found by multiplying the circumference of one of the bases into the length, and to the product adding the areas of the two bases, or ends.

When the diameter is given in inches, to find what length will make a solid foot.

RULE.—As the given diameter is to 13,531 : So is 12 to a 4th number, and so is that 4th number to the required length. If the diameter be given in foot measure : *Rule.* As the given diameter is to 1,128 : So is 1 to a 4th number, and so is that 4th number to the required length. Or, divide 1728 by the area at the end in inches, and the quotient will be the required length.

To find how much a Cyindric or round Tree, that is equally thick from end to end, will hew to, when made square.

RULE.—Multiply twice the square of its femidiameter by the length, then divide the product by 144, and the quotient will be the answer.

If the diameter of a round stick of timber be 24 inches from end to end, and its length 20 feet ; How many solid feet will it contain, when hewn square ; and what will be the content of the flabs which reduce it to a square ?

$$\frac{12 \times 12 \times 2 \times 20}{144} = 40 \text{ feet, the solidity when hewn square.}$$

$$\frac{24 \times 24 \times ,7854 \times 20}{144} = 62,8 \text{ feet, or } 2 \times 2 \times ,7854 \times 20 = 62,8$$

the total solidity, whence $62,8 - 40 = 22,8$ feet, the solidity of the flabs.

ART. 31. To measure a Prism.

Definition. A prism is a body with two equal or parallel ends, either square, triangular, or polygonal, and three or more sides, which meet in parallel lines, running from the several angles at one end, to those of the other.

RULE.—Prisms of all kinds, whether square, triangular or polygonal, are measured by one general rule, viz. Find the superficial content, or area at the base (or end) by the proper rule of Sect. 1. and this multiplied by the length, or height of the prism, will give the solid content.

EXAMP^r

EXAMP. The side of a stick of timber, AB, hewn three square, is 10 inches, and the length, AC, is 12 feet, to find the content ?

$$\text{Side} = 10 \text{ inches.}$$

$$\frac{1}{2} \text{ Perpend.} = 4,33 \text{ inches.}$$

$$43,3 = \text{area at the end.}$$

$$12 \text{ feet} = \text{length.}$$

$$144)519,6(3,6 \text{ feet, content.}$$

$$432$$

$$876$$

$$864$$

$$12$$

$$12$$



Note, The superficial content is found by adding the areas of the several quadrilateral and triangular figures, which compose it.

ART. 32. To measure a Pyramid.

Definition. Solids, which decrease gradually from the base till they come to a point, are generally called pyramids, and are of different kinds, according to the figure of their bases ; thus, if it has a square base, it is called a *square pyramid* : If a triangular base, a *triangular pyramid* : If the base be a circle, a *circular pyramid*, or simply a *cone*. The point, in which the top of a pyramid ends, is called a *Vertex*, and a line drawn from the vertex, perpendicular to the base, is called the height of the pyramid.

RULE.—Find the area of the base, whether triangular, square, polygonal or circular, by the rules in superficial measure ; then, multiply this area by one third of the height, and the product will be the solid content of the pyramid.

EXAMP. 1. In a triangular pyramid, the height BE, being 48, and each side of the base 13 : The base being a triangle, let the perpendicular height DE be 11 ; to find the content.

$$5,5 = \text{half ED.}$$

$$13 = \text{base AC.}$$

$$165$$

$$55$$

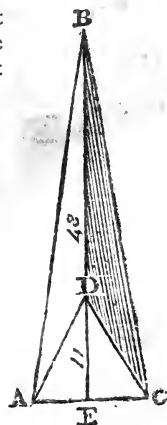
$$71,5 = \text{area of the base.}$$

$$16 = \frac{1}{3} \text{ of the height EB.}$$

$$4290$$

$$715$$

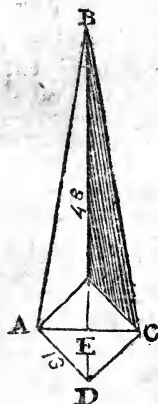
$$1144,0 = \text{content.}$$



EXAMP.

EXAMP. 2. In a quadrangular pyramid, the height BE being 48, and each side of the base 13, to find the content.

$$\begin{array}{r}
 13 \\
 13 \\
 \hline
 39 \\
 13 \\
 \hline
 169 = \text{area of the base.} \\
 16 = \frac{1}{3} \text{ of the height EB.} \\
 \hline
 1014 \\
 169 \\
 \hline
 2704 = \text{content.}
 \end{array}$$



EXAMP. 3. To measure a Cone.—The diameter AC being 13, and the height BD 48, to find the content.

$$\begin{array}{r}
 134 \\
 13 \\
 \hline
 39 \\
 13 \\
 \hline
 169 \\
 57854 \\
 \hline
 676 \\
 845 \\
 1352 \\
 1183 \\
 \hline
 132,7326 = \text{area of the base.} \\
 16 = \frac{1}{3} \text{ of the height.} \\
 \hline
 7963956 \\
 1327326 \\
 \hline
 2123,7216 = \text{content.}
 \end{array}$$



Note, The superficial content of all pyramids is found by taking the sum of the several areas, which compose them. That of a cone, by multiplying the circumference of the base into half the line joining the vertex and any point in that circumference, and adding the area of the base to the product.

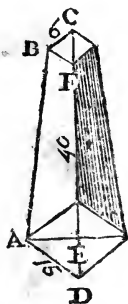
ART. 33. To measure the Frustrum of a Pyramid.

Definition. The frustrum of a pyramid is what remains after the top is cut off by a plane parallel to the base, and is in the form of a log greater at one end than the other, whether round, or hewn three or four square, &c.

RULE.—If it be the frustrum of a square pyramid, multiply the side of the greater base by the side of the less; to this product add one third of the square of the difference of the sides, and the *sum* will be the mean area between the bases; but, if the base be any other regular figure, multiply this *sum* by the proper multiplier of its figure in the Table, Art. 11. and the product will be the mean area between the bases: Lastly, multiply this by the height, and it will give the content of the frustrum.

EXAMP. 1. In the frustrum of a square pyramid the side of the greater base $AD = 15$, the side of the less, $BC = 6$, and the height $EF = 40$, to find the content.

$15 = AD.$	15
$6 = BC.$	6
—	—
Prod. $= 90$	$9 = \text{difference.}$
Add 27	9
—	—
117	$3)81 = \text{square of the dif-}$
$\times 40$	—
—	$[ference.$
$4680 = \text{content.}$	$27 = \frac{1}{3} \text{ of the square.}$



Or, If it be a tapering square stick of timber, take the girth of it in the middle; square $\frac{1}{4}$ of the girth (or multiply it by itself in inches) then say, as 144 (inches) to that product; so is the length, taken in feet, to the content in feet.

EXAMP. 2. What is the content of a tapering square stick of timber, whose side of the largest end is 12 inches, of the least end, 8 inches, and whose length is thirty feet?

One fourth of the girth in the middle $= 10$, and $10 \times 10 = 100$, the area in the middle; then, As 144 : 100 :: 30 feet : 20,83 feet, the content.

By the Sliding Rule.

Set 12 on D to $\frac{1}{4}$ of the circumference on C, and against the length on D is the answer on C.

By Gunter.

The extent from 12 to $\frac{1}{4}$ of the circumference doubled, or twice turned over, will reach from the length to the content.

EXAMP.

EXAMP. 3. In the frustrum of a triangular pyramid, the side of the greater base $AC = 15$, as before, the side of the less $BD = 6$, and the height $EF = 40$, to find the content.

$15 = AC.$	15
$6 = BD.$	6
<hr/>	<hr/>
$9 = \text{difference of the sides.}$	90
9	Add 27
<hr/>	<hr/>
$3) 81 = \text{square of the difference.}$	117
<hr/>	$,433 \text{ multiplier.}$
$27 = \frac{1}{3} \text{ of the square.}$	<hr/>
	351
	351
	468
	<hr/>
	$50,661 = \text{mean area.}$
	$40 = \text{height.}$
	<hr/>
	$2026,440 = \text{content.}$



Or, If it be a tapering three square stick of timber, you may find the area midway from end to end, then, As 144 is to that area ; so is the length, taken in feet, to the content in feet.

EXAMP. 4. To measure the Frustrum of a Cone.

RULE.—Multiply the diameters of the two bases together, and to the product add one third of the square of the difference of the diameters : Then multiplying this sum by ,7854, it will be the mean area between the two bases, which being multiplied by the length of the frustrum, will give the solid content.

Or, To the areas of the top and bottom add the square root of the product of those areas, and the sum, multiplied by *one third* of the height of the frustrum, will give the solidity.

When figures run uniformly taper ; but not to a point (they being considered as portions of the cone or pyramid) we may find the solidity by supplying what is wanting to complete the figure, and then deducting the part cut off.

A general rule for completing every straight sided solid, whose ends are parallel and similar.

As the difference of the top and bottom diameters is to the perpendicular height, (or depth, which is the same :) So is the longest diameter to the altitude of the whole cone or pyramid,

The

The former cone in Art. 32, Examp. 3, being cut off in the middle, the greater diameter AC is 13, the less BD $6\frac{1}{2}$, and height EF 24, to find the content of the frustum.

AC = 13 inches.

BD = $6\frac{1}{2}$ inches.

$\frac{6\frac{1}{2}}{78}$

Add $\frac{84\frac{1}{2}}{14,083}$

$\frac{98,583}{78,54}$

394332

492915

788664

690081

77,427 | 0882 = mean area.

24 feet = length.

309708

154854

1858,248 = content.

13

6,5

6,5 = difference.

6,5

325

390

3)42,25 = { square of the differ.

14,083 = $\frac{1}{3}$ of the square.

144)1858,248 (12,973 feet, content.

144

418

288

1302

1296

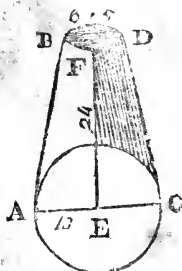
1064

1008

568

432

136



ART. 34. To measure a Sphere or Globe.

Definition. A sphere or globe is a round solid body, in the middle of which is a point, from which all lines drawn from the surface are equal.

RULE.—Multiply the cube of the diameter by ,5236, and the product will be the solid content.

Or, Multiply the circumference by the diameter, which will give the superficial content; then multiply the surface by *one sixth* of the diameter, and it will give the solidity.

Or, Multiply the cube of the diameter by 11, and the product divided by 21, will give the solidity.

EXAMP. The diameter, AB, of a globe, is 4.5 feet; to find the solid content.

$$\begin{array}{r}
 4,5 \\
 \underline{4,5} \\
 225 \\
 \underline{180} \\
 20,25 \\
 \underline{4,5} \\
 10125 \\
 \underline{8100} \\
 91,125 \\
 \underline{,5236} \\
 546750 \\
 \underline{273375} \\
 182250 \\
 \underline{455625} \\
 47,7130500
 \end{array}$$



Note. If the circumference, or greatest circle of the sphere, be given, multiply the cube of it by ,016887 for the content.

The surface of the globe may be found by multiplying the square of the diameter by 3,1416 ; or by multiplying the area of its greatest circle by 4, or the square of the circumference by ,3183.

When the solidity of a globe is given, the diameter may be found by dividing the solidity by ,5236, and extracting the cube root of the quotient.

Or, If the circumference be required, divide the solidity by ,016887, and the cube root of the quotient will give it.

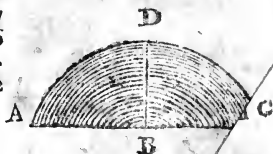
ART. 35. *To measure the Solidity of a Frustrum or Segment of a Globe.*

Definition. The frustrum of a globe is any part cut off by a plane.

RULE.—To three times the square of the semidiameter of the base, add the square of the height ; then multiplying that sum by the height, and the product by ,5236, you will have the solid content.

EXAMP. The height BD being 9 inches, and the diameter of the base AC 24 inches : To find the content.

$$\begin{array}{rcl}
 12 = \text{semidiam.} & 4617 & \\
 12 & \underline{,5236} & \\
 144 = \text{square.} & 27702 & \\
 \times 3 & 13851 & \\
 432 & 5234 & \\
 \text{Add } 9 \times 9 = 81 = \left\{ \begin{array}{l} \text{square of} \\ \text{the hht.} \end{array} \right. & 23085 & \\
 513 & & \\
 \times 9 = \text{height.} & & \\
 \hline
 4617 & & \\
 & 2417,4612 = \text{solid content} &
 \end{array}$$



To measure the Surface of a Frustrum or Segment of a Globe.

RULE.—Find the diameter of the globe by Art. 24, and the surface of the whole globe, by Art. 34; then, As the diameter of the globe is to the height of the frustrum; So is the surface of the globe, to the surface of the frustrum; then, by Art. 15, find the area of the base; add these two together, and the sum will be the whole surface of the frustrum.

ART. 36. *To measure the middle Zone of a Globe.*

Definition. This part of a globe is somewhat like a cask, two equal segments being wanting, one on each side of the axis.

RULE.—To twice the square of the middle diameter, add the square of the end diameter; multiply that sum by ,7854, and that product, multiplied by one third of the length, will give the solidity.

Or, To four times the square of the middle diameter add twice the square of the end diameter, that sum multiplied by ,7854, and that product by one sixth of the length, will give the solidity.

Note. This rule is applicable to the frustrum of a cone or pyramid.

If the middle diameter of a zone be 20 inches, the end diameters each 16 inches, and length 12 inches; Required its solidity?

$$20 \times 20 \times 2 + 16 \times 16 \times ,7854 \times 4 = 3317,5296 \text{ Ans.}$$

ART. 37. *To measure a Spheroid.*

Definition. A spheroid is a solid body like an egg, only both its ends are the same.

RULE.—Multiply the square of the diameter of the greatest circle, viz. the diameter of the middle (DB in the figure) by the length AC, and that product by ,5236, and you will have the solidity.

EXAMP. The diameter BD being 20, and the length AC 30, to find the content.

$$20 \times 20 \times 30 \times ,5236 = 6283,2 \text{ Ans.}$$

ART. 38. *To measure the middle Frustrum of the Spheroid.*

Definition. This is a casklike solid, wanting two equal segments to complete the spheroid.

RULE.—The same as in Article 36.

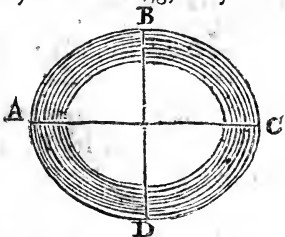
If the middle and end diameters of the middle frustrum of a spheroid be 40 and 30 inches, and its length 50; What is its solidity?

$$50 \div 3 = 16,6, \text{ then } 40 \times 40 \times 2 + 30 \times 30 \times ,7854 \times 16,6 = 53669 \text{ Ans.}$$

ART. 39. *To measure a Segment, or Frustrum of a Spheroid.*

Definition. This is a part of a spheroid made by a plane, parallel to its greatest circular diameter.

RULE.



RULE.—To four times the square of the middle diameter add the square of the base diameter, then multiply that sum by ,7854, and the product by *one sixth* of the altitude, and it will give the solidity.

If the base diameter of the end frustrum of a spheroid be 36, diameter at the middle of the height 30, and the height 20 inches ; Required its solidity ?

$$\overline{30 \times 30} \times 4 + \overline{36 \times 36} \times ,7854 \times 3,3 = 12817,728 \text{ Ans.}$$

N. B. $20 \div 6 = 3,3$.

ART. 40. *To measure a Parabolic Conoid.*

Definition. This solid may be generated by turning a semi-parabola about its abscissa or altitude.

RULE.—As a parabolic conoid is half of its circumscribing cylinder, of the same base and altitude ; multiply the area of the base by half the height, for the solidity.

If the diameter of the base of a parabolic conoid be 40 inches, and its height 42 ; What is the solidity ?

$$40 \times 40 \times ,7854 \times 21 = 26389,44 \text{ Ans.}$$

ART. 41. *To measure the lower Frustrum of a Parabolic Conoid.*

Definition. This solid is made by a plane passing through the conoid, parallel to its base.

RULE.—Multiply the sum of the squares of the diameters of the bases by ,7854, and that product by half the height, for the solidity.

If the diameters of a frustrum of a parabolic conoid be 40 and 30 inches, and its height 20 inches ; Required its solidity ?

$$\overline{40 \times 40} + \overline{30 \times 30} \times ,7854 \times 10 = 19635 \text{ Ans.}$$

ART. 42. *To measure a Parabolic Spindle.*

Definition. This solid is formed by an obtuse parabola, turned about its greatest ordinate.

RULE.—This solid being eight fifteenths of its least circumscribing cylinder, multiply the area of its middle or greatest diameter by eight fifteenths of its perpendicular length, and it will give its solidity.

If the diameter at the middle of a parabolic spindle be 20 inches, and its length 60 ; Required its solidity ?

$$20 \times 20 \times ,7854 \times 32 (= 60 \times 8 \div 15) = 10053,12 \text{ Ans.}$$

ART. 43. *To measure the middle Zone, or middle Frustrum of a Parabolic Spindle.*

Definition. This is a casklike solid, wanting two equal ends of said spindle.

RULE.—To the sum and half sum of the squares of the two diameters add three tenths of the difference of their squares, which multiply by a third of the length, and the product will be the solidity.

If the middle and end diameters of the middle frustrum of a parabolic spindle be 40 and 30 inches, and its length 60; What is its solidity?

$$\begin{array}{rcl}
 40 \times 40 & = & 1600 \\
 30 \times 30 & = & 900 \\
 \hline
 \text{Sum} & = & 2500 \\
 \text{Half sum} & = & 1250
 \end{array}
 \quad
 \begin{array}{l}
 1600 - 900 = 700 \text{ the diff. of the squares.} \\
 700 \times .3 = 210 = \text{three tenths of do. then,} \\
 2500 + 1250 + 210 \times 20 (= \frac{1}{3} \text{ of } 60) = 79200 \text{ Ans.}
 \end{array}$$

ART. 44. *To measure a Cylinderoid, or Prismoid.*

Definition. A cylinderoid is a solid somewhat like the frustrum of a cone, one base may be an ellipsis, and the other a disproportional ellipsis or circle.

A prismoid is a solid somewhat like the frustrum of a pyramid, but its bases are disproportional.

RULE.—The same as for the frustrum of a cone or pyramid: Or, to the areas of both bases add a mean area, that is, the square root of the product of the two bases, then multiply that sum by a third of the height or length, and it will give the solidity.

If the diameters of the greater base of a cylinderoid be 30 and 20 inches, the diameter of the less base 12, and length 60 inches; What is the solidity?

$$\begin{array}{rcl}
 30 \times 20 & = & 600 \\
 12 \times 12 & = & 144 \\
 \hline
 \sqrt{144 \times 600} & = & 293,9 \\
 \hline
 & & 1037,9
 \end{array}
 \left\{
 \begin{array}{l}
 1037,9 \times 7854 \times 20 (= 60 \div 3) = \\
 16308,33 \text{ Ans.}
 \end{array}
 \right.$$

If the diameters of the greater base of a prismoid be 30 and 20 inches, the less base 20 by 10 inches, and length 30 inches; What is its solidity?

$$\begin{array}{rcl}
 30 \times 20 & = & 600 \\
 20 \times 10 & = & 200 \\
 \hline
 \sqrt{600 \times 200} & = & 346,4 \\
 \hline
 & & 1146,4
 \end{array}
 \left\{
 \begin{array}{l}
 1146,4 \times 10 (= 30 \div 3) = 11464 \text{ so-} \\
 \text{lidity in inches.}
 \end{array}
 \right.$$

ART. 45. *To measure a Solid Ring.*

RULE.—Measure the internal diameter of the ring, and its girth, or circumference; then multiply the girth by 3,1831, and the product will be the diameter of the wire, which add to the internal diameter; multiply this sum by 3,1416, and the product will be the length of a cylinder equal to the ring of the same base. Then the area of a section of the ring multiplied by the length of the said cylinder will give the solidity of the ring.

If

If an iron ring be 12 inches in girth, and its internal diameter be 20 inches ; What is its solidity ?

$3,1831 \times 12 = 3,8$ = ring's diameter. $20 + 3,8 \times 3,1416 = 74,77$
the length of a cylinder equal to the ring : And

$$3,8 \times 3,8 \times ,7854 \times 74,77 = 847,97 = \text{solidity.}$$

ART. 46. To measure the Solidity of any irregular Body, whose Dimensions cannot be taken.

Take any regular vessel, either square or round, and put the irregular body into it ; pour so much water into the vessel as will exactly cover the body, and measure the dry part from the top of the vessel to the water ; then take out the body, and measure again from the top of the vessel to the water, and subtract the first measure from the second, and the difference is the fall of the water : Then, if the vessel be square, multiply the side by itself, and that product by the fall of the water, and you will have the content of the body ; but if it be a long square, multiply the length by the breadth, and that product by the fall of the water ; or, lastly, if it be a round vessel, multiply the square of the diameter by ,7854, and that product by the fall of the water, and you will have the content.

EXAM. 1. A body being put into a vessel 18 inches square, on taking out the body, the water sunk 9 inches ; Required the content of the body ?
 $18 \text{ inch.} = 1,5 \text{ foot.}$
 $9 \text{ inch.} = ,75 \text{ foot.}$
 $1,5 \times 1,5 \times ,75 = 1,6875 \text{ foot, content.}$

EXAM. 2. A body put into a cistern 4 feet by 3, on taking it out, the water fell 6 inches ; Required the content of the body ?
 $4 \times 3 \times ,5 = 6 \text{ feet, content.}$

EXAM. 3. A body being put into a round tub, whose diameter was 1,5 foot, on taking out the body, the water fell 1,5 foot ; What was the content of the body ?
 $1,5 \times 1,5 \times ,7854 \times 1,5 = 2,65 \text{ feet, content.}$

Of the five Regular Bodies.

There are five solids contained under equal regular sides, which, by way of distinction, are called the five regular bodies.

These are the Tetraedron, the Hexaedron or Cube, the Octaedron, the Dodecaedron, and the Eicosiedron. The measuring of the cube was shewn at Art. 28. I shall now shew how to measure the other four, by the following Table, which is the shortest method.

A Table of the solid and superficial content of each of the five bodies, the sides being unity, or 1.

Names of the Bodies.	Solidity.	Superficies.
Tetraedron.	0.11785	1.73205
Hexaedron.	1.	6.
Octaedron.	0.4714	3.464
Eicosiedron.	2.181695	8.66025
Dodecaedron.	7.663119	20.6457

All like solid bodies being in proportion to one another as the cubes of their like sides, the solid content of any of these bodies may be found by multiplying the cubes of their sides by the numbers in the second column under *Solidity*; and their superficies, by multiplying the squares of their sides into the numbers in the third column, under *Superficies*.

Of the Tetraedron.

This solid is contained under four equal and equilateral triangles, that is, it is a triangular pyramid of four equal faces, the side of whose base is equal to the slant height of the pyramid, from the angles to the vertex.

ART. 47. *The side of a Tetraedron being 3, to find the solid and superficial content.*

Cube $= 3 \times 3 \times 3 = 27$, and $27 \times 11785 = 3,18195 = \text{solidity}$.
 Square $= 3 \times 3 = 9$, and $9 \times 1,73205 = 15,58845 = \text{superficies}$.

Of the Octaedron.

This solid is contained under eight equal and equilateral triangles, which may be conceived to consist of two quadrangular pyramids of equal bases joined together, the sides of whose bases are equal to the given sides of the triangles, under which it is contained.

ART. 48. *The side of an Octaedron being 3, to find the solid and superficial content.*

Cube $= 3 \times 3 \times 3 = 27$, and $27 \times 4714 = 12,7278 = \text{solidity}$.
 Square $= 3 \times 3 = 9$, and $9 \times 3,464 = 31,176 = \text{superficies}$.

Of the Dodecaedron.

This solid is contained under 12 equilateral pentagons, and may be conceived to consist of twelve pentagonal pyramids, of equal bases and altitude, whose vertices meet in the centre of the dodecaedron.

ART. 49. *The side of a Dodecaedron being 3, to find the solid and superficial content.*

Cube $= 3 \times 3 \times 3 = 27$, and $27 \times 7,663119 = 206,904$.
 Square $= 3 \times 3 = 9$, and $9 \times 20,6457 = 185,8113$.

Of the Eicosiedron.

This solid is contained under twenty equal and equilateral triangles, and may be conceived to consist of twenty equal triangular pyramids, whose vertices all meet in the centre.

ART. 50. *The side of an Eicosiedron being 3, to find the solid and superficial content.*

Cube $= 3 \times 3 \times 3 = 27$, and $27 \times 2,18169 = 58,90563 = \text{solidity}$.
 Square $= 3 \times 3 = 9$, and $9 \times 8,66025 = 77,94225 = \text{superficies}$.

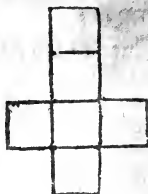
As the figures of some of these bodies would give but a confused idea of them, I have omitted them; but the following figures

ures, cut out in pasteboard, and the lines cut half through, will fold up into the several bodies.

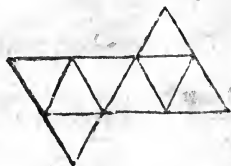
Tetraedron.



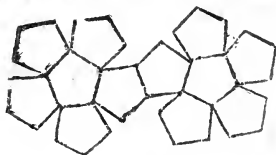
Hexaedron.



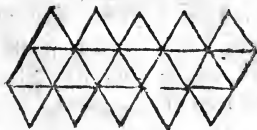
Octaedron.



Dodecaedron.



Eicofiedron.



Of CASK GAUGING.

Among the many different canons, drawn from Stereometry, for gauging *casks*, the following is as exact as any.

Take the dimensions of the cask in inches, viz. the diameter at the bung and head, and length of the cask : Subtract the head diameter from the bung diameter, and note the difference.

If the staves of the cask be much curved or bulging *between the bung and the head*, multiply the difference by ,7 ; if not quite so curve, by ,65 ; if they bulge yet less, by ,6 ; and if they are almost or quite straight, by ,55, and add the product to the head diameter ; the sum will be a mean diameter, by which the cask is reduced to a cylinder.

Square the mean diameter, thus found, then multiply it by the length ; divide the product by 359 for ale or beer gallons, and by 294 for wine gallons.

Note 1. The length is most conveniently taken by callipers, allowing, for the thickness of both heads, 1 inch, $1\frac{1}{2}$ inch, or 2 inches, according to the size of the cask : But if you have no callipers, do thus ; measure the length of the stave ; then take the depth of the chimes, which, with the thickness of the head, being subtracted from the length of the stave, leaves the length within.

Note 2. You must take the head diameter, close to its outside, and, for small casks, add 3 tenths of an inch ; for casks of 30, 40, or 50 gallons, 4 tenths, and for larger casks, 5 or 6 tenths, &c.

the sum will be very nearly the head diameter within. In taking the bung diameter, observe, by moving the rod backward and forward, whether the stave, opposite the bung, be thicker or thinner than the rest, and if it be, make allowance accordingly.

By the Sliding Rule.

On D is 18,94, the gauge point for ale or beer gallons, marked AG, and 17,14, the gauge point for wine gallons, marked WG : Set the gauge point to the length of the cask on C, and against the mean diameter, on D, you will have the answer in ale or wine gallons accordingly as which gauge point you make use of.

By the Scale.

Take the extent from the gauge point to the mean diameter, set one foot of the dividers in the length, and, turning them twice over, they will point out the content.

ART. 51. Required the content, in ale and wine gallons, of a cask, whose bung diameter is 35 inches, head diameter 27 inches, and length 45 inches ?

<p>Bung diameter = 35</p> <p>Head diameter = 27</p> <hr style="width: 100px; margin: 0 auto;"/> <p>Difference = 8</p> <p style="text-align: right;">,7</p> <hr style="width: 100px; margin: 0 auto;"/> <p style="text-align: right;">5,6</p> <p>Add the head dia. = 27</p> <hr style="width: 100px; margin: 0 auto;"/> <p>Mean diameter = 32,6</p> <p style="text-align: right;">32,6</p> <hr style="width: 100px; margin: 0 auto;"/> <p style="text-align: right;">1956</p> <p style="text-align: right;">652</p> <p style="text-align: right;">978</p> <hr style="width: 100px; margin: 0 auto;"/> <p>Squared 1062,76</p>	<p>Square of the diam. = 1062,76</p> <p>Length = 45</p> <hr style="width: 100px; margin: 0 auto;"/> <p style="text-align: right;">531380</p> <p style="text-align: right;">425104</p> <hr style="width: 100px; margin: 0 auto;"/> <p style="text-align: right;">359)47824,20(133,21</p> <p style="text-align: right;">294)47824,2(162,66</p> <p style="text-align: right;">[ale gall.</p> <p style="text-align: right;">wine gall.</p>
---	--

ART. 52. A round mash tub is 42 inches diameter at the top, within, and 36 inches at the bottom, and the perpendicular height 48 inches ; Required the content in beer and wine gallons?

This being the lower frustrum of a cone, to the product of the diameters add $\frac{1}{3}$ of the square of their difference ; multiply this sum by the length, and it will give the solidity in such parts as the dimensions are taken in. If they be taken in inches, divide by 359 for beer, and 294 for wine gallons.

$$42 \times 36 + \frac{42 - 36 \times 42 - 36}{3} \times 48 \div \begin{cases} 359 = 203\frac{1}{2} \text{ ale gallons.} \\ 294 = 248\frac{3}{4} \text{ wine gallons.} \end{cases}$$

ART.

ART. 53. Let the difference of diameters of this tub be 6 inches, the height 48 inches, and the content $203\frac{1}{4}$ gallons, to find the diameters?

Multiply the content, if beer measure, by 359; if wine measure, by 294, and divide the product by the length: From the quotient subtract $\frac{1}{3}$ of the square of the difference of the diameters; to this remainder add the square of $\frac{1}{2}$ the difference of the diameters, and extract the square root of the sum; from the square root subtract $\frac{1}{2}$ the difference of the diameters, and it will give the least diameter to great exactness, to which add the difference of the diameters, and the sum is the greatest diameter.

$$\sqrt{\frac{203,75 \times 359}{48} - \frac{6 \times 6}{3} + 3 \times 3 - 3} = 36, \text{ and } 36 + 6 = 42.$$

The diameters are 36 and 42.

The content of any vessel, in gallons, &c. may be thus found: Measure the inside of the vessel, according to the rule of the figure, and find the content in cubic inches; then,

Divide by $\left\{ \begin{array}{l} 1728 \\ 282 \\ 231 \\ 2150,425 \end{array} \right\}$ and the quotient will $\left\{ \begin{array}{l} \text{cubic feet.} \\ \text{ale or beer gallons.} \\ \text{wine gallons.} \\ \text{bushels.} \end{array} \right.$ be the content in

ART. 54. To ullage a Cask, lying on one side, by the Gauging Rod, when the Bung Diameter, and the Content, one, or both, are greater or less than the Table on the Rod is made for.

RULE.—As the bung diameter of the cask to be measured, is to the bung diameter that the table is made for; so are the dry inches of the cask, to a fourth number, which find in the table on the rod, and note the number of gallons answering to it. Then, as the content of the cask that the table is made for, is to the content of the cask to be measured; so is the number of gallons answering to the aforesaid fourth number, to the number of gallons your cask wants of being full.

ART. 55. To find a Ship's Burthen, or to Gauge a Ship.

There is such diversity in the forms of ships, that no general rule can be applied to answer all varieties; however, the following rules are practised.

RULE 1.—Multiply the breadth at the main beam, half the breadth, and length together: Divide the product by 94, and the quotient is the tons.

RULE 2.—Divide the continued product of the length, breadth and depth, in feet, by 100, for ships of war, and 95 for merchant ships, in which nothing is allowed for guns, &c. and the quotient is the tons.

RULE 3.—Take the length, from the stern post to the upper part of the stem; subtract two thirds of her breadth from that length: Multiply the remainder by the whole breadth, and that product by half the breadth, in feet, and divide by 100 for war, and 94 for merchant tonnage.

RULE 4.—The weight of a ship's burthen is half the weight of water she can hold.

What is the tonnage of a ship, whose length is 97 feet, breadth 31 feet, and depth $15\frac{1}{2}$ feet?

By Rule 1st.

$\frac{1}{2}$ breadth 15,5
Breadth 31

155
465

Length 97
480,5

33635
48245

94)46608,5(495,83 tons.
376

900
846

548
470

785
752

330
282

48

100)46608,5(466 tons.
400

660
600

608
600

85

By Rule 2d.

Length 97
Breadth 31

97
291

Depth = 15,5
3007

15035
15035
3007

95)46608,5(490,61 tons.
380

860
855

585
570

150
95

55

By Rule 3.

Length = 97
Subtract $\frac{2}{3}$ of breadth = 20,66

Multiply by the breadth

76,33

31

7633

22899

Multiply by $\frac{1}{2}$ breadth

2366,23

15,5

1183115

1183115

236623

94)36676,565/390,176 tons.

Allowing the Cubit, as it is found by modern travellers, to be 22 inches, the content of Noah's Ark is as follows, viz.

Feet.

Length of the keel, 300 } Its burthen as a man of war
Breadth by the mid ship beam 50 } 277,29 tons. [tons.
Depth in the hold 30 } As a merchant ship, 29188,6

QUESTIONS in MENSURATION.

1. The largest of the Egyptian pyramids is square at the base, and measures 693 feet on a side; How much ground does it cover?

$$\frac{693 \times 693}{272,25} = 1764 \text{ poles, and } \frac{1764}{160} = 11 \text{ acres and 4 poles, Anf.}$$

2. What difference is there between a floor 20 feet square, and two others, each 10 feet square?

$$20 \times 20 - 10 \times 10 + 10 \times 10 = 200 \text{ feet, Anf.}$$

3. There is a square of 2500 yards in area; What is each side of the square, and the breadth of a walk along one side and one end, which may take up just one half of the square?

$$\sqrt{2500} = 50 \text{ yards, each side. } \sqrt{\frac{2500}{2}} = 35,35, \text{ and } 50 - 35,35 = 14,65 \text{ yards, breadth of the walk, Anf.}$$

4. A pine plank is 16 feet and 5 inches long, and I would have just a square yard slit off; At what distance from the edge must the line be drawn?

$$\text{A square yard} = 1296 \text{ inches, and } 16 \text{ feet } 5 \text{ inches} = 197 \text{ inches. Therefore, } \frac{1296}{197} = 6\frac{114}{197} \text{ inches, Anf.}$$

5. If

5. If the area of a triangle be 900 yards, and the perpendicular 40 yards ; Required the length of the base ?

$$\frac{900 \times 2}{40} = 45 \text{ yards, } \textit{Ans.}$$

6. If the three sides of a plain triangle be 24, 16 and 12 perches ; Required its area ?

$$\frac{24+16+12}{2} = 26 ; 26-24 = 2 ; 26-16 = 10 ; 26-12 = 14,$$

and $\sqrt{26 \times 14 \times 10 \times 2} = 85,32$ perches, = area. Again, As $24 : 16+12 :: 16-12 : 4,6+$, the difference of the segments of the

base ; then, $12 - \frac{4,6+}{2} = 9,6$, and $\sqrt{12 \times 12 - 9,6 \times 9,6} = 7,11$

the perpendicular on the longest side ; whence $24 \div 2 \times 7,11 = 85,32$, area, as above.

7. Required the area of a circular garden, whose diameter is 12 rods ?

$$12 \times 12 \times ,7854 = 113,0976 \text{ poles, } \textit{Ans.}$$

8. The wheel of a perambulator turns just once and an half in a rod ; What is its diameter ?

$$16,5 \times \frac{2}{3} = 11, \text{ circumf. and } 11 \times ,31831 = 3\frac{1}{2} \text{ feet, } \textit{Ans.}$$

9. Agreed for a platform to the curb of a round well, at $7\frac{1}{2}d.$ per square foot ; the inward part, round the mouth of the well, is 36 inches diameter, and the breadth of the platform was to be $15\frac{1}{2}$ inches ; What will it come to ?

$$36 + 15,5 \times 2 = 67 \text{ the greatest diameter ; } 67 \times 67 \times ,7854 - 30 \times 30 \times ,7854 = \frac{2507,8722}{144} = 17,4157 \text{ square feet, at } 7\frac{1}{2}d. \text{ per foot, } = 10s. 10\frac{6}{10}d. \textit{Ans.}$$

10. Required the difference between the area of a circle, whose radius (or semidiameter) is 50 yards, and its greatest inscribed square ?

$50 \times 2 = 100$ the diameter, and $100 \times 100 \times ,7854 = 7854$ the area of the circle ; then, $50 \times 50 \times 2 = 5000$ the area of the greatest inscribed square, and $7854 - 5000 = 2854$ *Ans.*

11. There is a section of a tree 25 inches over ; I demand the difference of the areas of the inscribed and circumscribed squares, and how far they differ from the area of the section ?

$$25 \times 25 - 12,5 \times 12,5 \times 2 = 312,5 \text{ the difference of the squares. } 25 \times 25 - 25 \times 25 \times ,7854 = 134,125 \text{ the circumscribed square, more than the section, and } 25 \times 25 \times ,7854 - 12,5 \times 12,5 \times 2 = 178,375 \text{ inscribed square, less than the area of the section.}$$

12. Four men bought a grindstone of 60 inches diameter ; How much of its diameter must each grind off, to have an equal share of the stone, if one first grind his share, and then another, till the stone is ground away, making no allowance for the eye ?

RULE.

RULE.—Divide the square of the diameter by the number of men, subtract the quotient from the square, and extract the square root of the remainder, which is the length of the diameter after the first man has ground his share ; this work being repeated by subtracting the same quotient from the remainder, for every man, to the last ; extract the square root of the remainders, and subtract those roots from the diameters, one after another ; the several remainders will be the answers.

$$\begin{array}{r} 60 \\ 60 \\ \hline 4)3600 \end{array}$$

Quot. = 900

$$\begin{array}{r} \text{From } 3600 \\ \text{Take } 900 \end{array}$$

$$\sqrt{2700} = 51,9615, \text{ to be taken from } 60.$$

$$\text{Subt. } 900$$

$$\sqrt{1800} = 42,4264, \text{ from } 51,9615$$

$$\text{Subt. } 900$$

$$\sqrt{900} = 30, \text{ from } 42,4264$$

$$\begin{array}{r} \text{From } 60 \\ \text{Take } 51,9615 \end{array}$$

$$\text{Remains } 8,0385 = 1\text{st share.}$$

$$\begin{array}{r} \text{From } 51,9615 \\ \text{Take } 42,4264 \end{array}$$

$$\text{Rem. } 9,5351 = 2\text{d share.}$$

$$\begin{array}{r} \text{From } 42,4264 \\ \text{Take } 30 \end{array}$$

$$\text{Rem. } 12,4264 = 3\text{d share.}$$

$$\text{And } 30 \text{ inch.} = 4\text{th share.}$$

13. If a cubic foot of iron were hammered, or drawn, into a square bar, an inch about, that is, $\frac{1}{4}$ of an inch square ; Required its length, supposing there is no waste of metal ?

$$\frac{12 \times 12 \times 12}{25 \times 25 \times 4} = 6912 \text{ inches,} = 576 \text{ feet, } \text{Ans.}$$

14. Required the axis of a globe, whose solidity may be just equal to the area of its surface ?

$$\frac{7854 \times 4}{5236} = 6 \text{ inches, } \text{Ans.}$$

15. A joist is $7\frac{1}{2}$ inches wide, and $2\frac{1}{4}$ thick ; but I want one just twice as large, which shall be $3\frac{1}{4}$ inches thick ; What will be the breadth ?

$$\frac{7.5 \times 2,25 \times 2}{3,75} = 9 \text{ inches, } \text{Ans.}$$

16. I have a square stick of timber 18 inches by 14 ; but one of a third part of the timber in it, provided it be 8 inches deep, will serve ; How wide will it be ?

$$\frac{18 \times 14}{3} \div 8 = 10\frac{1}{2} \text{ inches, } \text{Ans.}$$

17. A had a beam of oak timber 18 inches square throughout, and 25 feet long, which he bartered with B, for an equilateral triangular

triangular beam of the same length, each side 24 inches ; Required the balance, at 1s. 4d. per foot ?

$$\frac{18 \times 18 \times 25}{144} = 56,25 \text{ solidity of the square beam.}$$

The perpendicular let fall on one of the sides of the triangular beam is 21 inches, and the half perp. = 10,5 ; then, $\frac{10,5 \times 24}{144} = 1,75$ foot, area at the end, and $1,75 \times 25 = 43,75$ feet, solidity of the triangular beam ; therefore $56,25 - 43,75 = 12,5$ feet, at 1s. 4d. per foot = 16s. 8d. balance due to A, *Ans.*

18. What is the difference between a solid half foot, and half a foot solid ?

$$\frac{12 \times 12 \times 6}{6 \times 6 \times 6} = 4, \text{ therefore, one is but } \frac{1}{4} \text{ of the other.}$$

19. A lent B a solid stack of hay, measuring 20 feet every way ; sometime afterward, B returned a quantity, measuring every way 10 feet ; What proportion of the hay remains due ?

$$20 \times 20 \times 20 - 10 \times 10 \times 10 = 7000 \text{ feet} = \frac{7}{8} \text{ Ans.}$$

20. A ship's hold is $75\frac{1}{2}$ feet long, $18\frac{1}{2}$ wide, and $7\frac{1}{4}$ deep ; How many bales of goods $3\frac{1}{2}$ feet long, $2\frac{1}{4}$ deep, and $2\frac{3}{4}$ wide, may be stowed therein, leaving a gang way the whole length, of $3\frac{1}{4}$ feet wide ?

$$\frac{75,5 \times 18,5 \times 7,25 - 75,5 \times 7,25 \times 3,25}{3,5 \times 2,25 \times 2,75} = 385,44 \text{ bales, Ans.}$$

21. If a stick of timber be $20\frac{1}{2}$ feet long, 16 inches broad, and 8 inches thick, and $3\frac{1}{2}$ solid feet be sawed off one end ; How long will the stick then be ?

$$20\frac{1}{2} - \frac{1728 \times 3,5}{16 \times 8} = 16 \text{ feet, } 6\frac{3}{4} \text{ inches, Ans.}$$

22. The solid content of a square stone is found to be $136\frac{1}{2}$ feet ; its length is $9\frac{1}{2}$ feet ; What is the area of one end ? and if the breadth be 3 feet 11 inches ; What is the depth ?

$$\frac{136,5 \times 1728}{9,5 \times 12} = \text{area } 2069,0526 \text{ inches, and } \frac{2069,0526}{47} = 44,022 \text{ inches, Ans.}$$

23. I would have a cubic box made capable of receiving just 50 bushels, the bushel containing 2150,425 solid inches ; What will be the length of the side ?

$$\sqrt[3]{2150,4 \times 50} = 47,55 \text{ inches.}$$

24. A statute bushel is to be made 8 inches high, and $18\frac{1}{2}$ inches diameter, to contain 2176 cubic inches ; (though the content of the dimensions is but 2150,425 inches) I demand what the diameter of the bushel must be, the height being 8 inches ; and what

what the height, the diameter being $18\frac{1}{2}$ inches, to contain 2176 cubic inches?

Solidity.

Height = $8)2176$ and $\sqrt{272 \times 1,273} = 18,6$ diameter. $18,5 \times 18,5 \times ,7854 = 268,80315 = \text{area, and the}$
 Area = 272 solidity $2176 \div 268,8 = 8,0956$ inches, hht.

25. There is a garden rolling stone 66 inches in circumference, and $3\frac{1}{2}$ cubic feet are to be cut off from one end, perpendicular to the axis: Where must the section be made?

Area = $\frac{1728 \times 3,5}{412,5} = 14,65$ inches from one end, *Anf.*

26. I would have a syringe of $1\frac{1}{2}$ inch diameter in the bore to hold a quart, wine measure; What must be the length of the piston, sufficient to make an injection with?

$1,5 \times 1,5 \times ,7854 = 1,76715$, and $231 \div 4 = 57,75$ the cubic inches in a quart, then, $\frac{57,75}{1,76715} = 32,679$ inches, *Anf.*

27. If a round pillar, 9 inches diameter, contain 5 feet; Of what diameter is that column, of equal length, which measures 10 times as much?

As $5 : 9 \times 9 :: 5 \times 10 : 810$, and $\sqrt{810} = 28,46$ inches, *Anf.*

28. There is a square pyramid, each side of whose base is 30 inches, and whose perpendicular height is 120 inches, to be divided by sections parallel to its base into 3 equal parts: Required the perpendicular height of each part?

$30 \times 30 \times 40 = 36000$ the solidity in inches, now $\frac{2}{3}$ thereof is 24000, and $\frac{1}{3}$ is 12000. Therefore,

As $36000 : 120 \times 120 \times 120 :: \left\{ \begin{array}{l} 24000 \\ 12000 \end{array} \right\} : \left\{ \begin{array}{l} 1152000 \\ 576000 \end{array} \right\}$ Then,

$\sqrt[3]{1152000} = 104,8$. Also, $\sqrt[3]{576000} = 83,2$. Then, $120 - 104,8 = 15,2$ length of the thickest part, and $104,8 - 83,2 = 21,6$ length of the middle part, consequently 83,2 is the length of the top part.

29. Suppose the diameter of the base of a conical ingot of gold to be 3 inches, and its height 9 inches; What length of wire may be expected from it, without loss of metal, the diameter of the wire being one hundredth part of an inch?

$3 \times 3 \times ,7854 \times 9 = 21,2058$ the solidity of the cone.

$\frac{21,2058}{,01 \times ,01 \times ,7854} = 270000$ inch. = 4 miles, and 460 yds. *Anf.*

30. Suppose a pole to stand on an horizontal plane 75 feet in height above the surface: At what height from the ground must it be cut off, so as that the top of it may fall on a point 55 feet from the bottom of the pole, the end, where it was cut off, resting on the stump, or upright part?

As

As the whole length of the pole is equal to the sum of the hypotenuse and perpendicular of a triangle, (the 55 feet on the ground being the base) this, as well as the following question, may be solved by this

RULE.—From the square of the length of the pole (that is, of the sum of the hypotenuse and perpendicular) take the square of the base ; divide the remainder by twice the length of the pole, and the quotient will be the perpendicular, or height at which it must be cut off.

$$\frac{75 \times 75 - 55 \times 55}{75 \times 2} = 17\frac{1}{3} \text{ feet, Ans.}$$

31. Suppose a ship sails from lat. 43° , north, between north and east, till her departure from the meridian be 45 leagues, and the sum of her distance and difference of latitude to be 135 leagues ; I demand her distance sailed, and latitude come to ?

$$\frac{135 \times 135 - 45 \times 45}{135 \times 2} = 60 \text{ leagues, and } 60 \times 3 = 180 \text{ miles} = 3$$

degrees the difference of latitude, $135 - 60 = 75$ leagues the distance. Now, as the vessel is sailing from the equator, and consequently the latitude is increasing : Therefore,

To the latitude sailed from $43^{\circ}, 00' N.$

Add the difference of latitude $3, 00$

And the sum is the lat. come to $= 46, 00 N.$

A N

INTRODUCTION TO ALGEBRA,

DESIGNED FOR THE

USE OF ACADEMIES.

DEFINITIONS.

- A**LGEBRA is the art of computing by symbols.
1. *Like quantities* are those which consist of the same letters.
 2. *Unlike quantities* are those which consist of different letters.
 3. *Given quantities* are those whose values are known.
 4. *Unknown quantities* are those whose values are unknown.
 5. *Simple quantities* are those which consist of one term only.
 6. *Compound quantities* are those which consist of several terms.
 7. *Positive or affirmative quantities* are those to be added.
 8. *Negative quantities* are those to be subtracted.
 9. *Like signs* are all $+$ or all $-$.
 10. *Unlike signs* are $+$ and $-$.
 11. *The coefficient* of any quantity is the number prefixed to it.
 12. *A binomial quantity* is one consisting of two terms; a *trinomial*, of three terms; and a *quadrinomial*, of four terms, &c.
 13. *A residual quantity* is a binomial, where one of the terms is a negative.

In the computation of problems, the first letters of the alphabet are put for known quantities, and letters of the latter part of the alphabet for those which are unknown.

AXIOMS.

1. If equal quantities be added to, subtracted from, multiplied or divided by, equal quantities, the wholes, remainders, products and quotients will be respectively equal.
2. The equal powers or roots of equal quantities are equal.
3. Two quantities respectively equal to a third, are equal to each other.
4. The whole is equal to all its parts taken together.

ADDITION.

CASE I. *To add quantities which are alike, and have like signs.**

RULE.—Add all the coefficients together, and to their sum adjoin the letters common to each term, prefixing the common sign.

54

* When a leading quantity has no sign before it, $+$ is always understood; and a quantity without any coefficient prefixed to it, is supposed to have unity, or 1.

5a	-6br	8bru	5r ² + ru	7ar - u
7a	-3br	7bru	3r ² + 2ru	8ar - 3u
8a	-2br	3bru	r ² + 3ru	6ar - 2u
10a	-7br	4bru	7r ² + 8ru	4ar - 3u
2a	-br	5bru	r ² + ru	ar - u
a	-5br	bru	2r ² + 3ru	3ar - 2u
<hr/>				
33a	-24br	28bru	19r ² + 18ru	29ar - 12u

CASE II. To add quantities which are like, but have unlike signs.

RULE 1. Add all the affirmative coefficients into one sum, and all the negative ones into another.

2. Subtract the least sum from the greatest, and to the difference prefix the sign of the greatest, with the common quantity.

$$\begin{array}{rclcl}
 -3a + 8ar^2 + 6r^{\frac{1}{3}} + 8u & -2ru + 8 & + 8r^2 - u + 3\sqrt{r} \\
 +7a + 7ar^2 - 3r^{\frac{1}{3}} + 7u & -3ru + 7 & -10r^2 - 3u + 2\sqrt{r} \\
 +8a - 3ar^2 - 13r^{\frac{1}{3}} + 8u & + ru - 10 & - 4r^2 - 2u + \sqrt{r} \\
 -a - 4ar^2 + 2r^{\frac{1}{3}} - 3u & + 5ru - 7 & + 9r^2 + 6u - 10\sqrt{r} \\
 -2a + 4ar^2 + r^{\frac{1}{3}} - u & - ru + 2 & + r^2 * - \sqrt{r} \\
 \hline
 +9a + 12ar^2 - 7r^{\frac{1}{3}} + 19u & * & + 4r^2 * - 5\sqrt{r}
 \end{array}$$

CASE III. To add quantities which are unlike, and have unlike signs.

RULE.—Collect the like quantities together by the last rule, and set down those which are unlike, one after another, with their proper signs.

$$\begin{array}{rcl}
 2r & 2r - r^2 & \\
 3u & 3a - r & 12ar - r^2 - 6 + \sqrt{ar - r^2} \\
 -a & 2ar + 6r^2 & -6ar + r^2 - r + 10 \\
 r^2 & 3\sqrt{r} - 2ar & 3u - ar - 4 - 2\sqrt{ar - r^2} \\
 \hline
 2r + 3u - a + r^2 & r + 5r^2 + 3a + 3\sqrt{r} & 5ar - \sqrt{ar - r^2} - r + 3u
 \end{array}$$

SUBTRACTION.

RULE.—Change the signs of all the quantities to be subtracted, and then add them together, as in Addition.

$$\begin{array}{rcl}
 3a^2 - 2b & 6r^2 - 8u + 2 & 35ru - 2 + 8r - u^{\frac{1}{2}} & 8ar - 2\sqrt{ru} - 10 \\
 2a^2 - 3b & r^2 + 9u - 20 & 24ru - 8 - 8r - 3u & 10r - 6\sqrt{ru} - ar \\
 \hline
 a^2 + b & 5r^2 - 17u + 22 & 11ru + 6 + 16r - u^{\frac{1}{2}} + 3u & 9ar + 4\sqrt{ru} - 10 - 10r
 \end{array}$$

MULTIPLICATION.

M U L T I P L I C A T I O N.

CASE I. *To multiply simple quantities.*

RULE.—Multiply the coefficients of the two terms together, and to the product annex all the letters in those terms.

Note. Like signs produce +, and unlike signs —.

$2a$	$-2a$	$5a$	$-9r$	$6a^2r$	$-r^2u$	$-7ru$
$3b$	$+4b$	$-6r$	$-5b$	$5r$	ru^2	$-ru$
$6ab$	$-8ab$	$-3oar$	$+45br$	$30a^2r^2$	$-r^3u^3$	$+7r^2u^2$

CASE II. *When one of the factors is a compound quantity.*

RULE.—Find the products of the multiplier and every particular term of the multiplicand separately, and place them one after another with their proper signs.

$4a-2b$	$6ru-8$	$8a^2-2r+6$	$3u-8+2ru$
$3a$	$2r$	$3ru$	ru
$12a^2-6ab$	$12r^2u-16r$	$24a^2ru-6r^2u+18ru$	$3ru^2-8ru+2r^2u^2$

CASE III. *When both the factors are compound quantities.*

RULE.—Multiply every particular term of the multiplier into every term of the multiplicand respectively, and set down the products one after another with their proper signs, and their sum will be the whole product.

$r+u$	$r-u$	$3r^2-2ru+5$
$r+u$	$r-u$	$r^2+2ru-3$
r^2+ru	r^2-ru	$3r^4-2r^3u+5r^2$
$+ru+u^2$	$-ru+u^2$	$+6r^3u-4r^2u^2+10ru$
$r^2+2ru+u^2$	$r^2-2ru+u^2$	$-9r^2+6ru-15$
		$3r^4+4r^3u-4r^2-4r^2u^2+16ru-15$

When two surd numbers are to be multiplied together, multiply them without any regard to the radical sign, and prefix the radical sign to the product. Thus,

$$\sqrt{3} \times \sqrt{2} = \sqrt{6}; \sqrt{a} \times \sqrt{b} = \sqrt{ab}, \&c.$$

D I V I S I O N.

CASE I. *When the divisor is a simple quantity.*

RULE 1.—Place the dividend above a line, and the divisor under it, like a vulgar fraction.

2. Expunge

2. Expunge those letters which are common to both the factors, and divide the coefficients of all the terms by any number which will divide them without a remainder.

Note. Like signs make +, and unlike signs —, as in Multiplication.

$$\frac{a}{a} = 1; \frac{8bc}{2b} = 4c; \frac{abc}{bcd} = \frac{a}{d}; \frac{10ab+15ac}{20ad} = \frac{2b+3c}{4d}; \frac{ab+b^2}{2b} =$$

$$\frac{a+b}{2}; \frac{12ru}{6r^2} = \frac{2u}{r}; \frac{30ar-54au}{12ab} = \frac{5r-9u}{2b}; \frac{10r^2u-15u^2-5u}{5u} =$$

$$2r^2-3u-1; \frac{3a^2-15+6a+3b}{3a} = a - \frac{5}{a} + 2 + \frac{b}{a}.$$

CASE II. When the divisor and dividend are both compound quantities.

RULE 1.—Range the terms of both the quantities according to the dimensions of some letter in them, so that the first term may have the highest power of that letter, and the second term the next highest power; and so on.

2. Divide the first term of the dividend by the first term of the divisor, and place the result in the quotient.

3. Multiply the whole divisor by the quotient term last found, and subtract the result from the dividend.

4. To this remainder bring down the next term of the dividend and divide as before, and so on, as in common arithmetic,

$$\begin{array}{r} a+r \overline{) a^3+5a^2r+5ar^2+r^3(a^2+4ar+r^2)} \\ a^3+a^2r \end{array}$$

$$\begin{array}{r} 4a^2r+5ar^2 \\ 4a^2r+4ar^2 \end{array}$$

$$\begin{array}{r} ar^2+r^3 \\ ar^2+r^3 \end{array}$$

*

$$\begin{array}{r} r-3 \overline{) r^3-9r^2+27r-27(r^2-6r+9)} \\ r^3-3r^2 \end{array}$$

$$\begin{array}{r} -6r^2+27r \\ -6r^2+18r \end{array}$$

$$\begin{array}{r} 9r-27 \\ 9r-27 \end{array}$$

*

$$\begin{array}{r} a-r \overline{) a^3-r^3(a^2+ar+r^2)} \\ a^3-a^2r \end{array}$$

$$\begin{array}{r} a^2r-r^3 \\ a^2r-ar^2 \end{array}$$

$$\begin{array}{r} ar^2-r^3 \\ ar^2-r^3 \end{array}$$

*

ALGEBRAIC

ALGEBRAIC FRACTIONS.

PROBLEM I. To reduce a mixed quantity to an improper fraction.

RULE.—Multiply the integer by the denominator of the fraction, and to the product add the numerator, and the denominator being placed under this sum will give the improper fraction required.

$$r + \frac{r^2}{a} = \frac{ar + r^2}{a}; \quad a - \frac{b}{c} = \frac{ac - b}{c}; \quad 1 - \frac{2r}{a} = \frac{a - 2r}{a};$$

$$a - r + \frac{a^2 - ar}{r} = \frac{a^2 - r^2}{r}.$$

PROB. II. To reduce an improper fraction to a whole, or mixed quantity.

RULE.—Divide the numerator by the denominator, for the integral part, and place the remainder over the denominator, for the fractional part.

$$\frac{ar + a^2}{r} = a + \frac{a^2}{r}; \quad \frac{au + 2u^2}{a + u} = u + \frac{u^2}{a + u}; \quad \frac{ab - a^2}{b} = a - \frac{a^2}{b};$$

$$\frac{a^2 + 2r^2}{a - r} = a + r + \frac{r^2}{a - r}.$$

PROB. III. To reduce fractions of different denominators, to those of the same value, which shall have a common denominator.

RULE.—Multiply every numerator separately into all the denominators but its own, for new numerators, and all the denominators together, for a common denominator.

1. Reduce $\frac{a}{b}$ and $\frac{b}{c}$ to fractions of equal values, having a common denominator.

$$\begin{array}{l} a \times c = ac \text{ new numer.} \\ b \times b = b^2 \text{ new numer.} \end{array} \quad \frac{ac}{bc} \text{ and } \frac{b^2}{bc} = \text{fractions required.}$$

$$b \times c = bc \text{ common denominator.}$$

2. Reduce $\frac{a}{b}$, $\frac{b}{c}$ & $\frac{c}{d}$ to equivalent fractions, having a common denominator.

$$\begin{array}{l} a \times c \times d = acd \\ b \times b \times d = b^2 d \\ c \times b \times c = c^2 b \end{array} \quad \frac{acd}{bcd}, \frac{b^2 d}{bcd} \text{ and } \frac{c^2 b}{bcd}, \text{ Ans.}$$

$$b \times c \times d = bcd.$$

PROB. IV. To find the greatest common measure of a fraction.

RULE 1. Range the quantities according to the dimensions of some letter, as was shewn in division.

2. Divide

2. Divide the greater term by the less, and the last divisor by the last remainder, and so on, till nothing remain, and the divisor last used, will be the common measure required.

Note. All the letters or figures, which are common to the divisor, and dividend, must be cancelled in the divisor before they be used in the operation.

To find the greatest common measure of $\frac{cr+r^2}{ca^2+a^2r}$

$$* \quad cr+r^2)ca^2+a^2r$$

$$\text{Or, } c+r)ca^2+a^2r(a^2) \\ ca^2+a^2r$$

Therefore the greatest common measure is $c+r$.

*

2. To find the greatest common measure of $\frac{b^3-b^2r}{r^2+2br+b^2}$

$$r^2+2br+b^2)r^3-b^2r(r) \\ r^3+2br^2+b^2r$$

$$+ -2br^2-2b^2r)r^2+2br+b^2$$

$$\text{Or, } r+b)r^2+2br+b^2(r+b) \\ r^2+br$$

Therefore $r+b$ is the greatest common divisor.

$$br+b^2$$

$$br+b^2$$

*

PROB. V. To reduce a fraction to its lowest terms.

RULE 1.—Find the greatest common measure, as in the last problem.

2. Divide both the terms of the fraction by the common measure thus found.

1. Reduce

* Here I find that r is common to both divisor and dividend, I therefore cancel r in the divisor, that is, I divide $cr+r^2$ by r , and $c+r$ is the quotient : Thus,

$$r)cr+r^2(c+r, \text{ for the divisor.}$$

cr

$$+r^2$$

$$+r^2$$

*

+ Here $-2br$ is common to the divisor and dividend ; I therefore first divide $-2br^2-2b^2r$ by r , and the quotient is $-2br-2b^2$, thus,

$$r)-2br^2-2b^2r(-2br-2b^2) \\ -2br^2$$

I then divide $-2br-2b^2$ by $-2b$, and the quotient is $r+b$, thus,

$$-2b)-2br-2b^2(r+b, \text{ for the divisor.}$$

$$-2br$$

$$-2b^2$$

$$-2b^2$$

*

1. Reduce $\frac{cr+r^2}{ca^2+a^2r}$ to its lowest terms.

$$\text{Or, } c+r \left) \begin{array}{r} cr+r^2 \\ ca^2+a^2r \\ \hline ca^2+a^2r \end{array}$$

*

Therefore, $c+r$ is the greatest common measure,
and $c+r \left) \frac{cr+r^2}{ca^2+a^2r} \left(\frac{r}{a^2} = \text{fraction required.} \right.$

2. Reduce $\frac{r^3-b^2r}{r^2+2br+b^2}$ to its lowest terms.

$$\begin{array}{r} r^3-b^2r \\ r^2+2br+b^2 \overline{) r^3-b^2r(r} \\ \underline{r^3+2br^2+b^2r} \\ -2br^2-2b^2r \end{array}$$

$$\text{Or, } r+b \left) \begin{array}{r} r^2+2br+b^2 \\ \underline{r^2+br} \\ br+b^2 \end{array}$$

$$\begin{array}{r} br+b^2 \\ \underline{br+b^2} \\ 0 \end{array}$$

Therefore $r+b$ is the greatest common measure, and

*

$$r+b \left) \frac{r^3-b^2r}{r^2+2br+b^2} \left(\frac{r^2-br}{r+b} = \text{fraction required.} \right.$$

PROB. VI. To add fractional quantities together.

RULE I. Reduce the fractions to a common denominator.

2. Add all the numerators together, and under their sum write the common denominator.

1. Add $\frac{r}{2}$ and $\frac{r}{3}$.

$$\begin{array}{l} r \times 3 = 3r \\ r \times 2 = 2r \end{array}$$

$$\underline{2 \times 3 = 6}$$

$$\frac{3r}{6} + \frac{2r}{6} = \frac{5r}{6} = \text{sum.}$$

2. Add $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$.

$$\begin{array}{l} a \times d \times f = adf \\ c \times b \times f = cbf \\ e \times b \times d = ebd \end{array} \quad \frac{adf}{bdf} + \frac{cbf}{bdf} + \frac{ebd}{bdf} = \frac{adf+cbf+ebd}{bdf} = \text{sum.}$$

$$\underline{b \times a \times f = bdf.}$$

3. Add

3. Add $a - \frac{3r^2}{b}$ and $b + \frac{r-2b}{c}$.

$$3r^2 \times c = 3cr^2$$

$$\frac{r-2b}{c} \times b = \frac{br-2b^2}{c}$$

$$b \times c = bc$$

$$a - \frac{3cr^2}{bc}$$

$$b + \frac{br-2b^2}{bc}$$

$$a + b + \frac{br-3cr^2-2b^2}{bc} = \text{sum.}$$

PROB. VII. To subtract one fractional quantity from another.

RULE 1.—Reduce the fractions to a common denominator.

2. Subtract the numerators, and under their difference write the common denominator.

1. Required the difference of $\frac{r}{3}$ and $\frac{2r}{11}$?

$$r \times 11 = 11r$$

$$\frac{2r \times 3}{3 \times 11} = \frac{6r}{33}$$

$$3 \times 11 = 33$$

$$\frac{11r}{33} - \frac{6r}{33} = \frac{5r}{33} = \text{difference.}$$

2. What is the difference of $\frac{r-a}{3b}$ and $\frac{2a-4r}{5c}$?

$$\frac{r-a}{3b} \times 5c = \frac{5cr-5ac}{15bc}$$

$$\frac{2a-4r}{5c} \times 3b = \frac{6ab-12br}{15bc}$$

$$3b \times 5c = 15bc$$

$$\frac{5cr-5ac}{15bc} - \frac{6ab-12br}{15bc} = \frac{5cr-5ac-6ab+12br}{15bc} = \text{difference.}$$

PROB. VIII. To multiply fractional quantities.

RULE.—Multiply the numerators together for a new numerator, and the denominators, for a new denominator.

1. Multiply $\frac{r}{6}$ and $\frac{2r}{9}$ together.

$$\left. \begin{array}{l} r \times 2r \\ 6 \times 9 \end{array} \right\} = \frac{2r^2}{54} = \frac{r^2}{27} = \text{product.}$$

2. Find the product of $\frac{r}{2}$, $\frac{4r}{5}$ and $\frac{10r}{21}$.

$$\left. \begin{array}{l} r \times 4r \times 10r \\ 2 \times 5 \times 21 \end{array} \right\} = \frac{40r^3}{210} = \frac{4r^3}{21} = \text{product.}$$

3. Find

3. Find the product of $\frac{r}{a}$ and $\frac{r+a}{a+c}$.

$$\left. \begin{array}{l} r \times \frac{r+a}{a+c} \\ a \times \frac{r+a}{a+c} \end{array} \right\} = \frac{r^2+ar}{a^2+ac} = \text{product.}$$

PROB. IX. To divide one fractional quantity by another.

RULE.—Invert the divisor, and proceed as in multiplication.

1. Divide $\frac{r}{3}$ by $\frac{2r}{9}$. $\frac{r}{3} \times \frac{9}{2r} = \frac{9r}{6r} = \frac{3}{2} = 1\frac{1}{2} = \text{quotient.}$

2. Divide $\frac{2a}{b}$ by $\frac{4c}{d}$. $\frac{2a}{b} \times \frac{d}{4c} = \frac{2ad}{4bc} = \frac{ad}{2bc} = \text{quotient.}$

3. Divide $\frac{r+a}{2r-2b}$ by $\frac{r+b}{5r+a}$.

$$\frac{r+a}{2r-2b} \times \frac{5r+a}{r+b} = \frac{5r^2+6ar+a^2}{2r^2-2b^2} = \text{quotient.}$$

I N V O L U T I O N.

Involution is the raising of powers from any proposed root; or the method of finding the square, cube, biquadrate, &c. of any given quantity.

RULE.—Multiply the quantity into itself as often as is denoted by the index, and the last product will be the power required.

Or, Multiply the index of the quantity by the index of the power, and the result will be the same as before.

Note. When the sign of the root is + all the powers of it will be +; and when the sign is —, all the odd powers will be —, and all the even powers +.

$$\text{Root} = a \begin{cases} a^2 = \text{square.} \\ a^3 = \text{cube.} \\ a^4 = 4\text{th power.} \\ a^5 = 5\text{th power.} \end{cases}$$

$$\text{Root} = a^2 \begin{cases} a^4 = \text{square.} \\ a^6 = \text{cube.} \\ a^8 = 4\text{th pow.} \\ a^{10} = 5\text{th pow.} \end{cases}$$

$$\text{Root} = -3a \begin{cases} + 9a^2 = \text{square.} \\ - 27a^3 = \text{cube.} \\ + 81a^4 = 4\text{th power.} \\ - 243a^5 = 5\text{th power.} \end{cases}$$

$$\text{Root} = -2ar^2 \begin{cases} + 4a^2r^4 = \text{square.} \\ - 8a^3r^6 = \text{cube.} \\ + 16a^4r^8 = 4\text{th pow.} \\ - 32a^5r^{10} = 5\text{th pow.} \end{cases} \quad \text{Root} = \frac{r}{a} \begin{cases} \frac{r^2}{a^2} = \text{square.} \\ \frac{r^3}{a^3} = \text{cube.} \\ \frac{r^4}{a^4} = \text{biquad.} \end{cases}$$

$$\text{Root} = -\frac{2ar^2}{3b} \left\{ \begin{array}{l} + \frac{4a^2r^4}{9b^2} = \text{square.} \\ - \frac{8a^3r^6}{27b^3} = \text{cube.} \\ + \frac{16a^4r^8}{81b^4} = \text{biquadrate.} \end{array} \right.$$

$$\begin{array}{r} r+a \\ r+a \\ \hline \end{array}$$

$$\begin{array}{r} r^2+ar \\ +ar+a^2 \\ \hline \end{array}$$

$$\begin{array}{r} r^2+2ar+a^2 = \text{square.} \\ r+a \\ \hline \end{array}$$

$$\begin{array}{r} r^3+2ar^2+a^2r \\ +ar^2+2a^2r+a^3 \\ \hline \end{array}$$

$$\begin{array}{r} r^3+3ar^2+3a^2r+a^3 = \text{cube.} \\ r+a \\ \hline \end{array}$$

$$\begin{array}{r} r^4+3ar^3+3a^2r^2+a^3r \\ +ar^3+3a^2r^2+3a^3r+a^4 \\ \hline \end{array}$$

$$\begin{array}{r} r^4+4ar^3+6a^2r^2+4a^3r+a^4 = \text{biquadrate.} \\ \hline \end{array}$$

Of the Composition and Resolution of a Square raised from a Binomial Root.

A binomial is a quantity consisting of two parts or members, connected together by the sign $+$ or $-$, as $r+a$, $r-a$, $r+\frac{b}{2}$, $r-\frac{b}{2}$, and a Square raised from a binomial root is nothing else but the square of such a quantity: Thus, the square of $r+\frac{b}{2}$ is $r^2+br+\frac{b^2}{4}$, and that of $r-\frac{b}{2}$ is $r^2-br+\frac{b^2}{4}$.

$$r+\frac{b}{2}$$

$$r + \frac{b}{2}$$

$$r + \frac{b}{2}$$

$$\begin{array}{r} r^2 + \frac{br}{2} + \frac{b^2}{4} \\ + \frac{br}{2} \\ \hline \end{array}$$

$$r^2 + \frac{2br}{2} + \frac{b^2}{4} = r^2 + br + \frac{b^2}{4}$$

$$r - \frac{b}{2}$$

$$r - \frac{b}{2}$$

$$\begin{array}{r} r^2 - \frac{br}{2} + \frac{b^2}{4} \\ - \frac{br}{2} \\ \hline \end{array}$$

$$r^2 - \frac{2br}{2} + \frac{b^2}{4} = r^2 - br + \frac{b^2}{4}$$

The difference between these two squares arises from the different sign of b , and that only affects the second member; for the third member $\frac{bb}{4}$ will be the same, whether the quantity b be affirmative or negative; therefore, if those cases be thrown into one, it will stand thus: The square of $r \pm \frac{b}{2}$; viz. $+ br$ when the root is $r + \frac{b}{2}$, and $- br$ when the root is $r - \frac{b}{2}$. Now, of the three members, which compose this square, the first, r^2 is the square of r , the second, $\pm br$ is the root of that square multiplied into the coefficient $\pm b$, and the third member, $\frac{b^2}{4}$ is the square of $\pm \frac{b}{2}$, that is, the square of half the coefficient of the second member; whence may be deduced the following observations.

1. Any quantity consisting of two members, as $r^2 \pm br$, whereof one, as r^2 is a square, and the other $\pm br$ is the root of that square multiplied into some given coefficient $\pm b$, it may be considered as an imperfect square raised from a binomial root, and may easily be completed by adding $\frac{b^2}{4}$, that is, by adding the square of half the coefficient of r in the second term; thus $r^2 + 6r$, when completed, is $r^2 + 6r + 9$; $r^2 + 3r$ becomes $r^2 + 3r + \frac{9}{4}$, because half the coefficient is $\frac{3}{2}$. Again, $r^2 + \frac{2r}{3}$ becomes $r^2 + \frac{2r}{3} + \frac{1}{9}$, because half the coefficient is $\frac{1}{3}$, the square of which is $\frac{1}{9}$: Lastly, $r^2 - \frac{br}{a}$ becomes $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$: For the coefficient is $-\frac{b}{a}$, its half $-\frac{b}{2a}$, and the square $\frac{b^2}{4a^2}$.

2. The

2. The root of such a square, when completed, that is, the root of $r^2 \pm br + \frac{b^2}{4}$ will always be $r \pm \frac{b}{2}$, that is, it will always be the square root of the first, together with half the coefficient of the second : Thus the square root of $r^2 + 6r + 9$ will be $r + 3$, that of $r^2 + 3r + \frac{9}{4}$ will be $r + \frac{3}{2}$, that of $r^2 + \frac{2r}{3} + \frac{1}{9}$ will be $r + \frac{1}{3}$, and lastly, that of $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$ will be $r - \frac{b}{2a}$.

SIR ISAAC NEWTON'S RULE for raising a binomial or residual quantity to any power whatever.

1. To find the terms without the coefficients.

The index of the first, or leading quantity, begins with that of the given power, and decreases continually by 1, in every term to the last ; and in the following quantity the indices of the terms are 0, 1, 2, 3, 4, &c.

2. To find the unciæ or coefficients.

The first is always 1, and the second is the index of the power : And, in general, if the coefficient of any term be multiplied by the index of the leading quantity, and the product be divided by the number of terms to that place, it will give the coefficient of the term next following.

Note. The whole number of terms will be one more than the index of the given power ; and when both terms of the root are $+$, all the terms of the power will be $+$; but if the second term be $-$, then all the odd terms will be $+$, and the even terms $-$.

1. Let $a+r$ be involved to the fifth power.

The terms without the coefficients will be $a^5, a^4r, a^3r^2, a^2r^3, ar^4, r^5$, and the coefficients will be 1, 5, $\frac{5 \times 4}{2}, \frac{10 \times 3}{3}, \frac{10 \times 2}{4}, \frac{5 \times 1}{5}$, or 1, 5, 10, 10, 5, 1, and therefore the 5th power is $a^5 + 5a^4r + 10a^3r^2 + 10a^2r^3 + 5ar^4 + r^5$.

2. Let $r-a$ be involved to the 6th power.

The terms without the coefficients will be $r^6, r^5a, r^4a^2, r^3a^3, r^2a^4, ra^5, a^6$, and the coefficients will be 1, 6, $\frac{6 \times 5}{2}, \frac{15 \times 4}{3}, \frac{20 \times 3}{4}, \frac{15 \times 2}{5}, \frac{6 \times 1}{6}$ or 1, 6, 15, 20, 15, 6, 1 ; and therefore the 6th power of $r-a$ is $r^6 - 6r^5a + 15r^4a^2 - 20r^3a^3 + 15r^2a^4 - 6ra^5 + a^6$.

E V O L U T I O N.

Evolution is the reverse of Involution, and teaches to find the roots of any given powers.

CASE I. To find the roots of simple quantities.

RULE.—Extract the root of the coefficient, for the numerical part, and divide the index of the letters by the index of the power, and it will give the root required.

1. The square root of $9r^2 = 3r^{\frac{2}{2}} = 3r$.
2. The cube root of $8r^3 = 2r^{\frac{3}{3}} = 2r$.
3. The square root of $3a^2r^6 = a^{\frac{2}{2}}r^{\frac{6}{2}} \sqrt{3} = ar^3 \sqrt{3}$.
4. The cube root of $-125a^3r^6 = -5a^{\frac{3}{3}}r^{\frac{6}{3}} = -5ar^2$.
5. The biquadrate root of $16a^4r^8 = 2a^{\frac{4}{4}}r^{\frac{8}{4}} = 2ar^2$.

CASE II. To find the square root of a compound quantity.

RULE 1.—Range the quantities according to the dimensions of some letter, and set the root of the first term in the quotient.

2. Subtract the square of the root, thus found, from the first term, and bring down the two next terms to the remainder, for a dividend.

3. Divide the dividend by double the root, and set the result in the quotient.

4. Multiply the divisor and quotient by the term last put in the quotient, and subtract the product from the dividend, and so on, as in common Arithmetic.

1. Extract the square root of $4a^4 + 12a^3r + 13a^2r^2 + 6ar^3 + r^4$.

$$\begin{array}{r}
 4a^4 + 12a^3r + 13a^2r^2 + 6ar^3 + r^4 \\
 \underline{4a^4} \\
 12a^3r + 13a^2r^2 + 6ar^3 + r^4 \\
 \underline{4a^2 + 3ar} \quad 12a^3r + 13a^2r^2 \\
 \quad \underline{12a^3r + 9a^2r^2} \\
 \quad \quad 4a^2 + 6ar + r^2 \quad 4a^2r^2 + 6ar^3 + r^4 \\
 \quad \quad \quad \underline{4a^2r^2 + 6ar^3 + r^4}
 \end{array}$$

*

2. Extract the square root of $r^4 - 4r^3 + 6r^2 - 4r + 1$.

$$\begin{array}{r}
 r^4 - 4r^3 + 6r^2 - 4r + 1 \\
 \underline{r^4} \\
 -4r^3 + 6r^2 - 4r + 1 \\
 \underline{2r^2 - 2r} \quad -4r^3 + 6r^2 \\
 \quad \underline{-4r^3 + 4r^2} \\
 \quad \quad 2r^2 - 4r + 1 \quad 2r^2 - 4r + 1 \\
 \quad \quad \quad \underline{2r^2 - 4r + 1}
 \end{array}$$

*

CASE III. To find the roots of powers in general.

RULE 1.—Find the root of the first term, and place it in the quotient.

2. Subtract the power, and bring down the second term for a dividend.

3. Involve the root, last found, to the next lowest power, and multiply it by the index of the given power, for a divisor.

4. Divide the dividend by the divisor, and the quotient will be the next term of the root.

5. Involve the whole root, and subtract and divide as before ; and so on till the whole be finished.

1. Required the square root of $a^4 - 2a^3r + 3a^2r^2 - 2ar^3 + r^4$.

$$\begin{array}{r}
 a^4 - 2a^3r + 3a^2r^2 - 2ar^3 + r^4 \\
 a^4 \\
 \hline
 2a^2) -2a^3r \\
 \hline
 a^4 - 2a^3r + a^2r^2 \\
 \hline
 2a^2) 2a^2r^2 \\
 \hline
 a^4 - 2a^3r + 3a^2r^2 - 2ar^3 + r^4 \\
 \hline
 *
 \end{array}$$

2. Extract the cube root of $r^6 + 6r^5 - 40r^3 + 96r - 64$

$$\begin{array}{r}
 r^6 + 6r^5 - 40r^3 + 96r - 64 \\
 r^6 \\
 \hline
 3r^4) 6r^5 \\
 \hline
 r^6 + 6r^5 + 12r^4 + 8r^3 \\
 \hline
 3r^4) -12r^4 \\
 \hline
 r^6 + 6r^5 - 40r^3 + 96r - 64 \\
 \hline
 *
 \end{array}$$

I N F I N I T E S E R I E S.

An *Infinite Series* is formed from a vulgar fraction, having a compound denominator, or by extracting the root of a surd quantity ; and is such, as, being continued, would run on *ad infinitum*, in the manner of a decimal fraction. And, by obtaining a few of the first terms, the law of the progression will be manifest, so that the series may be continued, without actually performing the whole operation.

PROBLEM

PROBLEM I. To reduce fractional quantities into infinite series.

RULE.—Divide the numerator by the denominator, and the operation continued, as far as shall be thought necessary, will give the operation required.

1. Let $\frac{1}{1+r}$ be thrown into an infinite series.

$$1+r)1 \dots (1-r+r^2-r^3+r^4, \&c.$$

$$\underline{1+r}$$

$$-r$$

$$\underline{-r-r^2}$$

$$+r^2$$

$$\underline{+r^2+r^3}$$

$$-r^3$$

$$\underline{-r^3-r^4}$$

$$+r^4, \&c.$$

2. Let $\frac{1}{1-r}$ be thrown into an infinite series.

$$1-r)1 \dots (1+r+r^2+r^3+r^4, \&c.$$

$$\underline{1-r}$$

$$+r$$

$$\underline{+r-r^2}$$

$$+r^2$$

$$\underline{+r^2-r^3}$$

$$+r^3$$

$$\underline{+r^3-r^4}$$

$$+r^4, \&c.$$

3. Let $\frac{ar}{a-r}$ be proposed.

$$a-r)ar \dots (r+\frac{r^2}{a}+\frac{r^3}{a^2}+\frac{r^4}{a^3}, \&c.$$

$$\underline{ar-r^2}$$

$$r^2$$

$$\underline{r^2-\frac{r^3}{a}}$$

$$\underline{\frac{r^3}{a}}$$

$$\frac{r^3}{a}$$

$$\underline{\frac{r^3}{a}}$$

$$\frac{r^3}{a} - \frac{r^4}{a^2}$$

$$\underline{\frac{r^4}{a^2}}$$

$$\frac{r^4}{a^2}$$

$$\underline{\frac{r^4}{a^2}}$$

$$\frac{r^4}{a^2} - \frac{r^5}{a^3}$$

$$\underline{\frac{r^5}{a^3}}$$

$$\frac{r^5}{a^3}, \&c.$$

4. Let $\frac{a^2}{a^2+2ar+r^2}$ be proposed.

$$(1-\frac{2r}{a}+\frac{3r^2}{a^2}-\frac{4r^3}{a^3})$$

$$\underline{a^2+2ar+r^2}$$

$$-2ar-r^2$$

$$-2ar-4r^2-\frac{2r^3}{a}$$

$$3r^2+\frac{2r^3}{a}$$

$$3r^2+\frac{6r^3}{a}+\frac{3r^4}{a^2}$$

$$-\frac{4r^3}{a}-\frac{3r^4}{a^2}$$

$$-\frac{4r^3}{a}-\frac{8r^4}{a^2}-\frac{4r^5}{a^3}$$

$$\frac{5r^4}{a^2}+\frac{4r^5}{a^3}\&c$$

PROBLEM

PROBLEM II. To reduce a compound surd into an infinite series.

RULE.—Extract the root to such degree of exactness as shall be thought necessary.

Extract the square root of $a^2 + r^2$ in an infinite series.

$$\begin{aligned}
 & a^2 + r^2 \left(a + \frac{r^2}{2a} - \frac{r^4}{8a^3} + \frac{r^6}{16a^5} - \frac{5r^8}{128a^7}, \&c. \right. \\
 & \quad \frac{a^2}{2a + \frac{r^2}{2a}} \Bigg) r^2 \\
 & \quad \frac{r^2 + \frac{r^4}{4a^2}}{2a + \frac{r^2}{a} - \frac{r^4}{8a^3}} \Bigg) - \frac{r^4}{4a^2} \\
 & \quad - \frac{r^4}{4a^2} - \frac{r^6}{8a^4} + \frac{r^8}{64a^6} \\
 & \quad 2a + \frac{r^2}{a} - \frac{r^4}{4a^3}, \&c. \Bigg) \frac{r^6}{8a^4} - \frac{r^8}{64a^6} \\
 & \quad \frac{r^6}{8a^4} + \frac{r^8}{16a^6}, \&c. \\
 & \quad \frac{5r^8}{64a^6}, \&c.
 \end{aligned}$$

ARITHMETICAL PROPORTION.

A Series in Arithmetical Proportion is thus expressed, $a, a+b, a+2b, a+3b, a+4b$, &c. Here the common difference is b . See page 218, &c.

Note. The most useful part of Arithmetical Proportion is contained in the 1st, 3d, and 4th, Theorems.

GEOMETRICAL PROPORTION.

A Series in Geometrical Proportion is thus expressed, a, ar, ar^2, ar^3, ar^4 , &c. Here r is the ratio. See page 235, &c.

Note. The most useful part of Geometrical Proportion is contained in the 1st, 3d, 5th and 8th, Theorems.

SIMPLE EQUATIONS.

An Equation is the comparing of two equal quantities which are differently expressed, together, by means of the sign $=$ placed between them.

Thus $12-7=5$ is an equation, expressing the equality of the quantities $12-7$ and 5 .

A Simple Equation is that which contains only one unknown quantity, without including its power. Thus $r - a + b = c$ is a simple equation, containing only the unknown quantity r .

Reduction of Equations is the method of finding the value of the unknown quantity ; which is shewn in the following rules.

RULE 1.—Any quantity may be transposed from one side of the equation to the other, by changing its sign.

Thus, if $r + 3 = 7$, then will $r = 7 - 3 = 4$. And, if $r - 4 + 6 = 8$, then will $r = 8 + 4 - 6 = 6$. Also, if $r - a + b = c - d$, then will $r = c - d + a - b$. And, if $4r - 8 = 3r + 2c$, then will $4r - 3r = 20 + 8$, or $r = 28$.

RULE 2.—If the unknown term be multiplied by any quantity, it may be taken away by dividing all the other terms of the equation by it.

Thus, if $ar = ab - a$, then will $r = b - 1$. If $2r + 4 = 16$, then will $r + 2 = 8$, and $r = 8 - 2 = 6$. Also, if $ar + 2ba = 3c^2$, then will $r + 2b = \frac{3c^2}{a}$, and $r = \frac{3c^2}{a} - 2b$.

RULE 3 —If the unknown term be divided by any quantity, it may be taken away by multiplying all the other terms of the equation by it.

Thus, if $\frac{r}{2} = 5 + 3$, then will $r = 10 + 6 = 16$. If $\frac{r}{a} = b + c - d$, then will $r = ab + ac - ad$. Also, if $\frac{2r}{3} - 2 = 6 + 4$, then will $2r - 6 = 18 + 12$, and $2r = 18 + 12 + 6 = 36$, or $r = \frac{36}{2} = 18$.

RULE 4.—The unknown quantity in any equation may be made free from surds, by transposing the rest of the terms according to the rule, and then involving each side to such a power as is denoted by the index of the said surd.

Thus, if $\sqrt{r - 2} = 6$, then will $\sqrt{r} = 6 + 2 = 8$, and $r = 8^2 = 64$. If $\sqrt{4r + 16} = 12$, then will $4r + 16 = 144$, and $4r = 144 - 16 = 128$, or $r = \frac{128}{4} = 32$. Also, if $\sqrt[3]{2r + 3} + 4 = 8$; then will $\sqrt[3]{2r + 3} = 8 - 4 = 4$, and $2r + 3 = 4^3 = 64$, and $2r = 64 - 3 = 61$, or $r = \frac{61}{2} = 30\frac{1}{2}$.

RULE 5.—If that side of the equation, which contains the unknown quantity, be a complete power, it may be reduced by extracting the root of said power from both sides of the equation.

Thus, if $r^2 + 6r + 9 = 25$, then will $r + 3 = \sqrt{25} = 5$, or $r = 5 - 3 = 2$. If $3r^2 - 9 = 21 + 3$, then will $3r^2 = 21 + 3 + 9 = 33$,
Q q q
and

and $r^2 = \frac{33}{3} = 11$, or $r = \sqrt{11}$. Also, if $\frac{2r^2}{3} + 10 = 20$, then will $2r^2 + 30 = 60$, and $r^2 + 15 = 30$, or $r^2 = 30 - 15$, or $r = \sqrt{15}$.

RULE 6.—Any analogy, or proportion, may be converted into an equation, by making the product of the two mean terms equal to that of the two extremes.

Thus, if $3r : 16 :: 5 : 10$, then will $3r \times 10 = 16 \times 5$, and $30r = 80$, or $r = \frac{80}{30} = 2\frac{2}{3}$. If $\frac{2r}{3} : a :: b : c$, then will $\frac{2cr}{3} = ab$, and $2cr = 3ab$, or $r = \frac{3ab}{2c}$. Also, if $12 - r : \frac{r}{2} :: 4 : 1$, then will $12 - r = \frac{4r}{2} = 2r$, and $2r + r = 12$, or $r = \frac{12}{3} = 4$.

RULE 7.—If any quantity be found on both sides of the equation with the same sign, it may be taken away from them both; and if every term in an equation be multiplied or divided by the same quantity, it may be struck out of them all.

Thus, if $4r + a = b + a$, then will $4r = b$, and $r = \frac{b}{4}$. If $3ar + 5ab = 8ac$, then will $3r + 5b = 8c$, and $r = \frac{8c - 5b}{3}$. Also, if $\frac{2r}{3} - \frac{8}{3} = \frac{16}{3} - \frac{8}{3}$, then will $2r = 16$, and $r = 8$.

MISCELLANEOUS EXAMPLES.

1. Given $5r - 15 = 2r + 6$, to find the value of r .

First, $5r - 2r = 6 + 15$, then $3r = 21$, and $r = \frac{21}{3} = 7$.

2. Given $40 - 6r - 16 = 120 - 14r$ to find r .

First, $14r - 6r = 120 - 40 + 16$, then $8r = 96$, therefore $r = \frac{96}{8} = 12$.

3. Given $5ar - 3b = 2dr + c$, to find r .

First, $5ar - 2dr = c + 3b$, or $\frac{5a - 2d}{1} \times r = c + 3b$, therefore $r = \frac{c + 3b}{5a - 2d}$.

4. Given $3r^2 - 10r = 8a + r^2$, to find r .

First, $3r - 10 = 8 + r$, then $3r - r = 8 + 10$, therefore $2r = 18$, and $r = \frac{18}{2} = 9$.

5. Given $6ar^3 - 12abr^2 = 3ar^3 + 6ar^2$, to find r .

First, dividing the whole by $3ar^2$, we shall have $2r - 4b = r + 2$, then $2r - r = 2 + 4b$, whence $r = 2 + 4b$.

6. Given

6. Given $\frac{r}{2} - \frac{r}{3} + \frac{r}{4} = 10$, to find r .

First, $r - \frac{2r}{3} + \frac{2r}{4} = 20$, then $3r - 2r + \frac{6r}{4} = 60$, and $12r - 8r + 6r = 240$, therefore $10r = 240$, and $r = \frac{240}{10} = 24$.

7. Given $\frac{r-3}{2} + \frac{r}{3} = 20 - \frac{r+19}{2}$, to find r .

First, $r - 3 + \frac{2r}{3} = 40 - r - 19$, then $3r - 9 + 2r = 120 - 3r - 57$, therefore $3r + 2r + 3r = 120 - 57 + 9$, that is, $8r = 72$, or $r = \frac{72}{8} = 9$.

8. Given $\sqrt{\frac{2}{3}}r + 5 = 7$, to find r .

First, $\sqrt{\frac{2}{3}}r = 7 - 5 = 2$, then $\frac{2}{3}r = 2^2 = 4$, and $2r = 12$, or $r = \frac{12}{2} = 6$.

9. Given $r + \sqrt{a^2 + r^2} = \frac{2a^2}{\sqrt{a^2 + r^2}}$, to find r .

First, $r\sqrt{a^2 + r^2} + a^2 + r^2 = 2a^2$, then $r\sqrt{a^2 + r^2} = a^2 - r^2$, and $r^2 \times \overline{a^2 + r^2} = \overline{a^2 - r^2}^2 = a^4 - 2a^2r^2 + r^4$, or $a^2r^2 + r^4 = a^4 - 2a^2r^2 + r^4$, whence $a^2r^2 + 2a^2r^2 = a^4$, or $3a^2r^2 = a^4$, consequently $r^2 = \frac{a^4}{3a^2}$, and $r = \sqrt{\frac{a^4}{3a^2}} = a\sqrt{\frac{1}{3}}$.

PROBLEM I. To exterminate two unknown quantities, or to reduce the two simple equations, containing them, to a single one.

RULE 1st.—1. Observe which of the unknown quantities is the least involved, and find its value in each of the equations, by the methods already explained.

2. Let the two values thus found be made equal to each other, and there will arise a new equation with only one unknown quantity in it, whose value may be found as before.

1. Given $\begin{cases} 2r + 3u = 23 \\ 5r - 2u = 10 \end{cases}$ to find r and u .

From the first equation, $r = \frac{23 - 3u}{2}$, and from the second, $r = \frac{10 + 2u}{5}$, and consequently $\frac{23 - 3u}{2} = \frac{10 + 2u}{5}$, or $115 - 15u = 20 + 4u$, or $19u = 115 - 20 = 95$, and $u = \frac{95}{19} = 5$, whence $r = \frac{23 - 15}{2} = 4$.

2. Given

2. Given $\begin{cases} r+u = a \\ r-u = b \end{cases}$ to find r and u ,

From the first equation, $r = a - u$, and from the second, $r = b + u$, therefore $a - u = b + u$, or $2u = a - b$, consequently $u = \frac{a-b}{2}$, and $r = a - u = a - \frac{a-b}{2} = \frac{a+b}{2}$.

3. Given $\begin{cases} \frac{r}{2} + \frac{u}{3} = 7 \\ \frac{r}{3} + \frac{u}{2} = 8 \end{cases}$ to find r and u .

From the first equation, $r = 14 - \frac{2u}{3}$, and from the second, $r = 24 - \frac{3u}{2}$, therefore $14 - \frac{2u}{3} = 24 - \frac{3u}{2}$, and $42 - 2u = 72 - \frac{9u}{2}$, or $84 - 4u = 144 - 9u$; whence $5u = 144 - 84 = 60$, and $u = \frac{60}{5} = 12$, and $r = 14 - \frac{2u}{3} = 14 - \frac{24}{3} = 6$.

RULE 2d.—1. Consider which of the unknown quantities you would first exterminate, and let its value be found in that equation where it is the least involved.

2. Substitute the value, thus found, for its equal in the other equation, and there will arise a new equation, with only one unknown quantity, whose value may be found as before.

1. Given $\begin{cases} r+2u = 17 \\ 3r-u = 2 \end{cases}$ to find r and u .

From the first equation, $r = 17 - 2u$, and this value, substituted for r in the second, gives $17 - 2u \times 3 - u = 2$, or $51 - 6u - u = 2$, or, $51 - 7u = 2$; that is, $7u = 51 - 2 = 49$; whence $u = \frac{49}{7} = 7$, and $r = 17 - 2u = 17 - 14 = 3$.

2. Given $\begin{cases} a : b :: r : u \\ r^2 + u^2 = c \end{cases}$ to find r and u .

The first analogy, turned into an equation, is $br = au$, or $r = \frac{au}{b}$, and this value of r , substituted in the second, gives

$$\left(\frac{au}{b}\right)^2 + u^2 = c, \text{ or } \frac{a^2u^2}{b^2} + u^2 = c, \text{ or } a^2u^2 + b^2u^2 = cb^2, \text{ or } u^2 = \frac{cb^2}{a^2 + b^2}, \text{ therefore } u = \sqrt{\frac{cb^2}{a^2 + b^2}}, \text{ and } r = \sqrt{\frac{ca^2}{a^2 + b^2}}.$$

RULE 3.—Let the given equations be multiplied or divided by such numbers or quantities as will make the term, which contains one of the unknown quantities, to be the same in both equations, and then by adding or subtracting the equations, accordingly as is required, there will arise a new equation with only one unknown quantity, as before.

1. Given $\begin{cases} 3r+5u=40 \\ r+2u=14 \end{cases}$ to find r and u .

First, multiply the 2d equation by 3, and we shall have $3r+6u=42$, then from this last equation subtract the first, and it will give $6u-5u=42-40$, or $u=2$, therefore $r=14-2u=14-4=10$.

2. Given $\begin{cases} 5r-3u=9 \\ 2r+5u=16 \end{cases}$ to find r and u .

Let the first equation be multiplied by 2, and the 2d by 5, and we shall have $\begin{cases} 10r-6u=18 \\ 10r+25u=80 \end{cases}$ and if the former of these be subtracted from the latter, it will give $31u=62$, or $u=\frac{62}{31}=2$, consequently, $r=\frac{9+6}{5}=\frac{15}{5}=3$.

Another Method.

Multiply the 1st equation by 5, and the 2d by 3, and we shall have $\begin{cases} 25r-15u=45 \\ 6r+15u=48 \end{cases}$ Now, add these two equations, and it will give $31r=93$, or $r=\frac{93}{31}=3$, consequently $u=\frac{16-6}{5}=\frac{10}{5}=2$, as before.

PROB. II. To exterminate three unknown quantities, or to reduce the three simple equations, containing them, to a single one.

RULE 1. Let r , u , and z , be the three unknown quantities to be exterminated.

2. Find the value of r , from each of the three given equations.

3. Compare the first value of r with the second, and an equation will arise, involving only u and z .

4. Compare the first value of r with the third, and another equation will arise, involving only u and z .

5. Find the value of u and z from these two equations, according to the former rules, and r , u , and z , will be exterminated as required.

Note. Any number of unknown quantities may be exterminated in nearly the same manner.

1. Given $\begin{cases} r+u+z=29 \\ r+2u+3z=62 \\ \frac{r}{2}+\frac{u}{3}+\frac{z}{4}=10 \end{cases}$ to find r , u , and z .

From

From the first equation, $r = 29 - u - z$. From the 2d $r = 62 - 2u - 3z$. From the 3d $r = 20 - \frac{2u}{3} - \frac{z}{2}$, whence $29 - u - z = 62 - 2u - 3z$, and $29 - u - z = 20 - \frac{2u}{3} - \frac{z}{2}$; but from the first of these equations, $u = 62 - 29 - 2z = 33 - 2z$; and from the 2d $u = 27 - \frac{3z}{2}$, therefore $33 - 2z = 27 - \frac{3z}{2}$, or $z = 12$, and $u = 62 - 29 - 2z = 62 - 29 - 24 = 9$, and $r = 29 - u - z = 29 - 12 - 9 = 8$.

$$2. \text{ Given } \left\{ \begin{array}{l} \frac{r}{2} + \frac{u}{3} + \frac{z}{4} = 62 \\ \frac{r}{3} + \frac{u}{4} + \frac{z}{5} = 47 \\ \frac{r}{4} + \frac{u}{5} + \frac{z}{6} = 38 \end{array} \right\} \text{ to find } r, u, \text{ and } z.$$

First, the given equations, cleared from fractions, become

$$12r + 8u + 6z = 1488$$

$$20u + 15u + 12z = 2820$$

$$30r + 24u + 20z = 4560$$

Then, if the 2d equation be subtracted from double the 1st, and three times the 3d from five times the 2d, we shall have

$$4r + u = 156$$

$$10r + 3u = 420.$$

And again, if the second of these be subtracted from three times the first, it will give $12r - 10r = 468 - 420$, or $r = \frac{48}{2} = 24$, therefore $u = 156 - 4r = 60$, and $z = \frac{1488 - 8u - 12r}{6} = 120$.

Questions producing simple Equations.

1. To find two such numbers, as that their sum shall be 40, and their difference 16.

Let r denote the least of the two numbers required, then will $r + 16 =$ the greater, $r + r + 16 = 40$ by the question, that is, $2r = 40 - 16 = 24$, or $r = \frac{24}{2} = 12 =$ least number, and $r + 16 = 12 + 16 = 28 =$ greater number required.

2. What

2. What number is that, whose $\frac{1}{3}$ part exceeds its $\frac{1}{4}$ part by 16?

Let r = number required, then will its $\frac{1}{3}$ part be $\frac{r}{3}$, and its $\frac{1}{4}$ part $\frac{r}{4}$; therefore $\frac{r}{3} - \frac{r}{4} = 16$, by the question, that is $r - \frac{3r}{4} = 48$, or $4r - 3r = 192$; whence $r = 192$ the number required.

3. Divide £1000 between A, B and C, so that A shall have £72 more than B, and C, £100 more than A.

Let r = B's share of the given sum, then will $r + 72$ = A's share, and $r + 172$ = C's share; and the sum of all these shares $r + r + 72 + r + 172$, or $3r + 244 = 1000$, by the question, that is, $3r = 1000 - 244 = 756$, or $r = \frac{756}{3} = £252$ = B's share, and $r + 72 = 252 + 72 = £324$ = A's share, and $r + 172 = 252 + 172 = £424$ = C's share.

Proof, $252 + 324 + 424 = £1000$.

4. A prize of £1000 is to be divided between two persons, whose shares therein are in the proportion of 7 to 9; Required the share of each?

Let r = the first person's share, then will $£1000 - r$ = 2d person's share, and $r : 1000 - r :: 7 : 9$, by the question, that is, $9r = 1000 - r \times 7 = 7000 - 7r$, or $16r = 7000$, whence $r = \frac{7000}{16} = £437 \text{ } 10s.$ = 1st share, and $1000 - r = 1000 - £437 \text{ } 10s. = £562 \text{ } 10s.$ = 2d share.

5. The paving of a square at 2s. per yard, cost as much as the inclosing of it, at 5s. per yard; Required the side of the square?

Let r = side of the square sought, then $4r$ = yards of inclosure, and r^2 = yards of pavement; whence $4r \times 5 = 20r$ = price of inclosing, and $r^2 \times 2 = 2r^2$ = price of paving. But $2r^2 = 20r$, by the question, therefore $r^2 = 10r$, and $r = 10$ = length of the side required.

6. A labourer engaged to serve 40 days upon these conditions, that for every day he worked he should receive 20d. but for every day he was absent, he was to forfeit 8d. Now, at the end of the time, there was due to him £1 11s. 8d.; How many days did he work, and how many was he absent?

Let r be the number of days he worked, then will $40 - r$ be the number of days he was absent; also, $r \times 20 = 20r$ = sum earned, and $40 - r \times 8 = 320 - 8r$ = sum forfeited; whence $20r - 320 - 8r = 380d.$ ($= £1 \text{ } 11s. \text{ } 8d.$) by the question, that is, $20r - 320 + 8r = 380$, or $28r = 380 + 320 = 700$, and $r = \frac{700}{28} = 25$ = number of days he worked; and $40 - r = 40 - 25 = 15$ = number of days he was absent.

7. Out

7. Out of a cask of wine, which had leaked away $\frac{1}{3}$, 21 gallons were drawn; and then, being gauged, it appeared to be half full; How much did it hold?

Let it be supposed to have holden r gallons, then it would have leaked $\frac{r}{3}$ gallons, and consequently there had been taken away $21 + \frac{r}{3}$ gallons. But $21 + \frac{r}{3} = \frac{r}{2}$ by the question, that is, $63 + r = \frac{3r}{2}$, or $126 + 2r = 3r$, hence $3r - 2r = 126$, or $r = 126$, *Answer*.

8. What fraction is that, to the numerator of which if 1 be added, the value will be $\frac{1}{3}$; but if one be added to the denominator, its value will be $\frac{1}{4}$?

Let the fraction be represented by $\frac{r}{u}$ then will $\frac{r+1}{u} = \frac{1}{3}$ and $\frac{r}{u+1} = \frac{1}{4}$, or $3r+3 = u$, and $4r = u+1$; hence $4r - 3r - 3 = u+1-u$, that is, $r-3 = 1$, or $r = 4$, and $u = 3r+3 = 12+3 = 15$. So that $\frac{4}{15}$ = fraction required.

9. A market woman bought a certain number of eggs, at 2 for a cent, and as many, at 3 for a cent, and sold them all out again, at the rate of 5 for 2 cents, and, by so doing, lost 4 cents; What number of eggs had she?

Let r = number of eggs of each sort, then will $\frac{r}{2}$ = price of the 1st sort, and $\frac{r}{3}$ = price of the 2d sort. But $3:2::2r$ (the whole number of eggs): $\frac{4r}{5}$; therefore $\frac{4r}{5}$ = price of both sorts together, at 5 for 2 cents, and $\frac{r}{2} + \frac{r}{3} - \frac{4r}{5} = 4$, by the question; that is, $r + \frac{2r}{3} - \frac{8r}{5} = 8$; or $3r + 2r - \frac{24r}{5} = 24$; or $15r + 10r - 24r = 120$; whence $r = 120$ = number of eggs of each sort required.

10. A person in the afternoon being asked what o'clock it was, answered, that $\frac{3}{5}$ of the time from noon was equal to $\frac{5}{8}$ of the time to midnight; Required the time?

Let r = the time sought from noon, then will $12-r$ = the time to midnight, $\frac{3}{5}$ of the time from noon = $\frac{3r}{5}$, and $\frac{5}{8}$ of the time to midnight = $\frac{60-5r}{8}$, therefore $\frac{3r}{5} = \frac{60-5r}{8}$ by the question; whence, $3r = \frac{300-25r}{8}$ and $24r = 300-25r$, or $24r+25r = 300$ or $r = \frac{300}{49} = 6h. 7' 20'' \frac{40}{49}$, *Answer*.

11. A merchant ships goods for Southcarolina to the amount of £700; What sum, at 5 per cent. should he get insured, to cover his adventure?

Let r = sum to be insured, then will $r - \frac{5r}{100} = 700$, whence $100r - 5r = 70000$, and $r = \frac{70000}{95} = \text{£}736\ 16s. 10\frac{2}{5}d.$ *Ans.*

12. A man lays out 30 cents for apples and pears, buying his apples, at 4, and his pears, at 5 for a cent, and afterwards sold $\frac{1}{2}$ of his apples, and $\frac{1}{3}$ of his pears for 13 cents, which was the prime cost; I demand the number he bought of each?

Let r = the number of apples, and z = the number of pears; then, if 4 apples cost a cent, r will cost $\frac{r}{4}$ cents, and for the same reason z will cost $\frac{z}{5}$ cents, and we shall have $\frac{r}{4} + \frac{z}{5} = 30$, for one fundamental equation. Again, the price of $\frac{r}{2} = \frac{1}{2}$ of his apples will be $\frac{r}{8}$, and the price of $\frac{z}{3} = \frac{1}{3}$ of his pears will be $\frac{z}{15}$; hence $\frac{r}{8} + \frac{z}{15} = 13$, for another fundamental equation: Now, cross multiplying $\frac{r}{4} + \frac{z}{5} = 30$, and then multiplying 30 by 4 and 5 we shall have the first equation $= 5r + 4z = 600$; and doing the same by $\frac{r}{8} + \frac{z}{15} = 13$, we have the 2d equation $= 15r + 8z = 1560$. Subtract the 2d equation from three times the 1st, and we shall have the 3d equation $= 4z = 240$. therefore 4th equation $= z = 60$ = the number of pears: Now, substitute 60 for z , that is, 240 for $4z$, in the 1st equation. $5r + 4z = 600$, we shall have $5r + 240 = 600$, whence, equation 5th $r = 72$ = the number of apples.

QUADRATIC EQUATIONS.

A Simple Quadratic Equation is that which involves the square of the unknown quantity only.

An Adfected Quadratic Equation is that which involves the square of the unknown quantity, together with the product, which arises from multiplying it by some known quantity.

Thus, $ax^2 = b$, is a simple quadratic equation, and $ax^2 + bx = c$ is an adfected quadratic equation.

All adfected quadratic equations fall under the three following forms.

1st. $x^2 + ax = b$. 2d. $x^2 - ax = b$. 3d. $x^2 - ax = -b$. And the rule for finding the value of x , in each of these equations, is as follows:

RULE* 1.—Transpose all the terms, which involve the unknown quantity, to one side of the equation, and the known terms to the other side, and let them be ranged according to their dimensions.

2. Where

* The square root of any quantity may be either $+$ or $-$, and therefore all quadratic equations admit of two solutions. Thus the square root of $+n^2$ is $+n$, or $-n$, for $+n \times +n$, or $-n \times -n$, are each equal to $+n^2$. So, in the first form, where $r + \frac{a}{2}$ is found $= \sqrt{b + \frac{a^2}{4}}$, the root may be either $+\sqrt{b + \frac{a^2}{4}}$, or $-\sqrt{b + \frac{a^2}{4}}$, since either of them being multiplied by itself will produce $b + \frac{a^2}{4}$. And this ambiguity is expressed by writing the uncertain sign \pm before $\sqrt{b + \frac{a^2}{4}}$; thus $r = \pm \sqrt{b + \frac{a^2}{4}} - \frac{a}{2}$. In the first form, where $r = \pm \sqrt{b + \frac{a^2}{4}} - \frac{a}{2}$ the first value of r , viz. $r = +\sqrt{b + \frac{a^2}{4}} - \frac{a}{2}$ is always affirmative.

The second value, viz. $r = -\sqrt{b + \frac{a^2}{4}} - \frac{a}{2}$, will always be negative, because it is composed of two negative terms; therefore, when $r^2 + ar = b$, we shall have $r = +\sqrt{b + \frac{a^2}{4}} - \frac{a}{2}$ for the affirmative value of r , and $r = -\sqrt{b + \frac{a^2}{4}} - \frac{a}{2}$ for the negative value of r .

In the second form, where $r = \pm \sqrt{b + \frac{a^2}{4}} + \frac{a}{2}$, the first value, viz. $r = +\sqrt{b + \frac{a^2}{4}} + \frac{a}{2}$ is always affirmative, since it is composed of two affirmative terms, and the second value, viz. $r = -\sqrt{b + \frac{a^2}{4}} + \frac{a}{2}$ will always be negative; therefore when $r^2 - ar = b$, we shall have $r = +\sqrt{b + \frac{a^2}{4}} + \frac{a}{2}$, for the affirmative value of r , and $r = -\sqrt{b + \frac{a^2}{4}} + \frac{a}{2}$, for the negative value of r .

2. When the square of the unknown quantity has any coefficient prefixed to it, let all the rest of the terms be divided by that coefficient.

3. Add the square of half the coefficient of the second term to both sides of the equation, and that side, which involves the unknown quantity, will be a complete square.

4. Extract the square root from both sides of the equation, and the value of the unknown quantity will be determined.

Note, 1. The square root of one side of the equation is always equal to the unknown quantity, with half the coefficient of the second term subjoined to it.

2. All equations, wherein there are two terms involving the unknown quantity, and the index of one is just double that of the other, are solved like quadratics, by completing the square.

Thus, $r^4 + ar^2 = b$, or $r^n + ar^{\frac{n}{2}} = b$, are the same as quadratics, and the value of the unknown quantity may be determined accordingly.

From this rule may be formed a general theorem with which all particular equations may be compared, and by means whereof they may be more readily resolved.

Suppose $ar^2 = br + c$ be the general quadratic equation proposed to be resolved; where a , b , and c denote known integral quantities, whether affirmative, or negative, and $r =$ the unknown quantity; to find the values of r in this equation.

Here, transposing br , we have $ar^2 - br = c$, then dividing by a , in order to free r^2 the highest power of r from its coefficient, we have $r^2 - \frac{br}{a} = \frac{c}{a}$; this being done, the first side, $r^2 - \frac{br}{a}$ may be considered as an imperfect square raised from a binomial root, and accordingly we may complete that square by adding the square of

In the third form, where $r = \sqrt{\frac{a^2}{4} - b} + \frac{a}{2}$, both the values of r will be positive, supposing $\frac{a^2}{4}$ to be greater than b . Therefore when $r^2 - ar = -b$, we shall have $r = +\sqrt{\frac{a^2}{4} - b} + \frac{a}{2}$, and $-\sqrt{\frac{a^2}{4} - b} + \frac{a}{2}$, both, for the affirmative value of r .

But in this third form, if b be greater than $\frac{a^2}{4}$, the solution of the proposed question will be impossible. For since the square of any quantity is always affirmative, the square root of a negative quantity is impossible.

of half the coefficient of the second term : But if $\frac{bb}{4a^2}$ must be added to the first side of the equation, to complete the square, it must be also added to the other side, to preserve the equality, otherwise by an unequal addition, the equation would be destroyed ; this equal addition therefore being made, the equation will stand thus, $r^2 - \frac{br}{a} + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} + \frac{c}{a}$; but the two fractions $\frac{b^2}{4a^2}$ and $\frac{c}{a}$ when added, give $\frac{ab^2 + 4a^2c}{4a^3}$, which divided by a , gives $\frac{b^2 + 4ac}{4a^2}$; therefore $r^2 - \frac{br}{a} + \frac{b^2}{4a^2} = \frac{b^2 + 4ac}{4a^2}$; therefore the square root of one side will be equal to the square root of the other ; but the square root of $\frac{b^2 + 4ac}{4a^2}$, as it here stands in letters, cannot be extracted, because, although the denominator $4a^2$ be a square, yet there is no literal quantity whatever, which, being multiplied into itself, will produce $b^2 + 4ac$, therefore, to put this numerator into the form of a square, let us suppose $b^2 + 4ac = ss$, and then the equation will be $r^2 - \frac{br}{a} + \frac{b^2}{4a^2} = \frac{ss}{4a^2}$; but the square root of $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$ is $r - \frac{b}{2a}$, and that of $\frac{ss}{4a^2}$ is $\pm \frac{s}{2a}$, therefore this equation will now be reduced to a simple one, and will stand thus, $r - \frac{b}{2a} = \pm \frac{s}{2a}$, therefore $r = \frac{b \pm s}{2a}$, that is, $r = \frac{b+s}{2a}$ and $r = \frac{b-s}{2a}$.

Note. When the quantity c (and consequently $4ac$) is negative, the quantity ss , or $b^2 + 4ac$ must be considered as the sum of the affirmative quantity b^2 and the negative one $4ac$, when added together according to the common rules of Addition.

Examples of the resolution of Adfected Equations with and without the general Theorem.

1. Given $r^2 = 140 - 4r$, to find the values of r .

First, transposing $-4r$, it is $r^2 + 4r = 140$, then, $r^2 + 4r + 4 = 144$ by completing the square ; then $\sqrt{r^2 + 4r + 4} = \sqrt{144}$, by extracting the root ; or $r + 2 = \pm 12$, that is, $r = -2 \pm 12 = +10$, or -14 .

By the general Theorem. a , in the general theorem, answers to 1 in the particular one, that is, to the coefficient of r^2 , b answers to 4, and c , to 140, that is, $a = 1$, $b = -4$, $c = 140$, and $4ac = 560$, therefore ss , or $b^2 + 4ac$ will be the sum of 16 and 560 = 576, therefore

therefore, $s = 24$, $\frac{b+s}{2a} = \frac{-4+24}{2} = +10$, and $\frac{b-s}{2a} = \frac{-4-24}{2} = -14$; therefore the two roots of this equation are 10 and -14.

2. Given $r^2 + 8 = 6r + 80$, to find r .

First, $r^2 - 6r = 72$, by transposition; then $r^2 - 6r + 9 = 72 + 9 = 81$, by completing the square, and $r - 3 = \sqrt{81} = \pm 9$, by extracting the root, therefore $r = +3 \pm 9 = +12$, or -6 .

By the Theorem. $a = 1$, $b = 6$, $c = 72$, and $4ac = 288$, therefore $ss = 36 + 288 = 324$, therefore $s = 18$, $\frac{b+s}{2a} = 12$, and $\frac{b-s}{2a} = -6$.

3. Given $2r^2 - 20 = 70 - 8r$ to find r .

First, $2r^2 + 8r = 70 + 20 = 90$, by transposition, then $r^2 + 4r = 45$, by dividing by the coefficient 2, and $r^2 + 4r + 4 = 45 + 4 = 49$, by completing the square; whence $r + 2 = \sqrt{49} = \pm 7$, therefore, $r = -2 \pm 7 = 5$ or -9 .

By the Theorem. $a = 2$, $b = -8$, $c = 90$, $4ac = 720$, $ss = 64 + 720 = 784$, therefore $s = 28$, $\frac{b+s}{2a} = +5$, $\frac{b-s}{2a} = -9$, so that $+5$, and -9 are the values of r .*

As the general Theorem is sufficiently exemplified in the preceding problems, the following equations will be solved by the Rule only.

4. Given $3r^2 - 3r + 6 = 5\frac{1}{3}$, to find r .

Here $r^2 - r + 2 = 1\frac{2}{3}$ by dividing by 3, and $r^2 - r = 1\frac{2}{3} - 2$, by transposition; also $r^2 - r + \frac{1}{4} = 1\frac{2}{3} - 2 + \frac{1}{4} = \frac{1}{36}$ by completing the square; and $r - \frac{1}{2} = \sqrt{\frac{1}{36}} = \pm \frac{1}{6}$, by evolution; therefore $r = +\frac{1}{2} \pm \frac{1}{6} = \frac{2}{3}$, or $\frac{1}{3}$.

5. Given $\frac{r^2}{2} - \frac{r}{3} + 20\frac{1}{2} = 42\frac{2}{3}$, to find r .

Here $\frac{r^2}{2} - \frac{r}{3} = 42\frac{2}{3} - 20\frac{1}{2} = 22\frac{1}{6}$, by transposition, and $r^2 -$

$\frac{2r}{3} = 44\frac{1}{3}$, by dividing by $\frac{1}{2}$, whence $r^2 - \frac{2r}{3} + \frac{1}{9} = 44\frac{1}{3} + \frac{1}{9} = 44\frac{4}{9}$, by completing the square, and $r - \frac{1}{3} = \sqrt{44\frac{4}{9}} = \pm 6\frac{2}{3}$, therefore $r = +\frac{1}{3} \pm 6\frac{2}{3} = 7$, or $-6\frac{1}{3}$.

6. Given

* In a quadratic equation of this form $ar^2 = br + c$, the sum of the roots will always be $\frac{b}{a}$, and the product of their multiplication $-\frac{c}{a}$; therefore, if $a = 1$, that is, if the equation be $r^2 = br + c$, the sum of the roots will be b , and their product $-c$, or the sum will be the coefficient of the unknown quantity on the second side of the equation, and their product, what is called the absolute term, with its sign changed.

6. Given $ar^2 + br = c$, to find r .

First, $r^2 + \frac{b}{a}r = \frac{c}{a}$, by division; then $r^2 + \frac{b}{a}r + \frac{b^2}{4a^2} = \frac{c}{a} + \frac{b^2}{4a^2}$, by completing the square; and $r + \frac{b}{2a} = \sqrt{\frac{c}{a} + \frac{b^2}{4a^2}}$
 $= \sqrt{\frac{4ac + b^2}{4a^2}}$, by evolution, therefore $r = \pm \sqrt{\frac{4ac + b^2}{4a^2}} - \frac{b}{2a}$.

7. Given $ar^2 - br + c = d$, to find r .

Here, $ar^2 - br = d - c$, by transposition, and $r^2 - \frac{b}{a}r = \frac{d-c}{a}$ by division.

Also, $r^2 - \frac{b}{a}r + \frac{b^2}{4a^2} = \frac{d-c}{a} + \frac{b^2}{4a^2}$ by completing the square;
 and $r - \frac{b}{2a} = \pm \sqrt{\frac{d-c}{a} + \frac{b^2}{4a^2}}$, by evolution; therefore $r = \frac{b}{2a} \pm \sqrt{\frac{d-c}{a} + \frac{b^2}{4a^2}}$.

8. Given $r^4 + 2ar^2 = b$, to find r .

Here, $r^4 + 2ar^2 + a^2 = b + a^2$, by completing the square, and $r^2 + a = \sqrt{b + a^2}$, by evolution; whence $r^2 = \sqrt{b + a^2} - a$, and consequently $r = \sqrt{\sqrt{b + a^2} - a}$.

9. Given $ar^n - br^{\frac{n}{2}} - c = -d$, to find r .

First, $ar^n - br^{\frac{n}{2}} = c - d$, by transposition, and $rn - \frac{b}{a}r^{\frac{n}{2}} = \frac{c-d}{a}$,
 by division. Also, $r^n - \frac{b}{a}r^{\frac{n}{2}} + \frac{b^2}{4a^2} = \frac{c-d}{a} + \frac{b^2}{4a^2}$, by completing the square, and $r^{\frac{n}{2}} - \frac{b}{2a} = \sqrt{\frac{c-d}{a} + \frac{b^2}{4a^2}}$, by evolution;
 therefore $r^{\frac{n}{2}} = \frac{b}{2a} \pm \sqrt{\frac{c-d}{a} + \frac{b^2}{4a^2}}$ and consequently $r = \left(\frac{b}{2a} \pm \sqrt{\frac{c-d}{a} + \frac{b^2}{4a^2}} \right)^{\frac{2}{n}}$.

QUESTIONS producing QUADRATIC EQUATIONS.

1. To find two numbers, whose difference is 8, and product 240.

Let r = the least number, then will $r + 8$ = the greater, and
 $r \times r + 8 = r^2 + 8r = 240$, by the question; whence $r^2 + 8r + 16 = 240 + 16 = 256$ by completing the square; also $r + 4 = \sqrt{256} = 16$, by evolution, and therefore $r = 16 - 4 = 12$ = the least number, and $12 + 8 = 20$ = the greater.

2. To divide the number 60 into two such parts, as that their product may be 864.

Let r = greater part, then will $60 - r$ = the less, and $r \times 60 - r = 60r - r^2 = 864$, by the question, that is, $r^2 - 60r = -864$; whence $r^2 - 60r + 900 = -864 + 900 = 36$, by completing the square: Also $r - 30 = \sqrt{36} = 6$, by extracting the root; therefore $r = 6 + 30 = 36$ = greater part, and $60 - r = 60 - 36 = 24$ = the less part.

Sold a piece of cloth for 24*l*. and gained as much per cent. as the cloth cost me; What was the price of it?

Let r = pounds the cloth cost, then $24 - r$ = whole gain, but $100 : r :: r : 24 - r$, by the question, or $r^2 = 100 \times 24 - r = 2400 - 100r$, that is, $r^2 + 100r = 2400$; whence $r^2 + 100r + 2500 = 2400 + 2500 = 4900$, by completing the square, and $r + 50 = \sqrt{4900} = 70$, by extraction of roots, consequently $r = 70 - 50 = 20$ = price of the cloth.

4. A person bought a number of oxen for 80*l*. and if he had bought 4 more for the same money, he would have paid 1*l*. less for each; How many did he buy?

Suppose he bought r oxen, then $\frac{80}{r}$ = price of each, and $\frac{80}{r+4}$ = price of each, if $r+4$ had cost 80*l*. But $\frac{80}{r} = \frac{80}{r+4} + 1$, by the question, or $80 = \frac{80r}{r+4} + r$, or $80r + 320 = 80r + r^2 + 4r$, that is, $r^2 + 4r = 320$; whence $r^2 + 4r + 4 = 320 + 4 = 324$, by completing the square, and $r + 2 = \sqrt{324} = 18$, by evolution, consequently $r = 18 - 2 = 16$ = number of oxen required.

5. What two numbers are those, whose sum, product and difference of their squares, are all equal to each other?

Let r = greater number, and u = less; then $\left\{ \begin{array}{l} r+u = ru \\ r+u = r^2 - u^2 \end{array} \right\}$, by the question, and $1 = \frac{r^2 - u^2}{r+u} = r - u$, or $r = u + 1$, from the 2d equation: Also $u + 1 + u = u + 1 \times u$, from the first equation; or $2u + 1 = u^2 + u$, that is, $u^2 - u = 1$; whence $u^2 - u + \frac{1}{4} = 1\frac{1}{4}$, by completing the square: Also $u - \frac{1}{2} = \sqrt{1\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ by evolution, consequently $u = \frac{\sqrt{5}}{2} + \frac{1}{2} = \frac{\sqrt{5} + 1}{2}$, and $r = u + 1 = \frac{\sqrt{5} + 3}{2}$.

6. There are four numbers in Arithmetical Progression whereof the product of the two extremes is 45, and that of the means 77; What are the numbers?

Let

Let r = less extreme, and u = common difference, then $r, r+u, r+2u, r+3u$ will be the 4 numbers, and

$$\left\{ \begin{array}{l} r \times r + 3u = r^2 + 3ru = 45 \\ r + u \times r + 2u = r^2 + 3ru + 2u^2 = 77 \end{array} \right\} \text{ by the question ;}$$

whence $2u^2 = 77 - 45 = 32$, and $u^2 = \frac{32}{2} = 16$, by subtraction

and division, or $u = \sqrt{16} = 4$ by evolution, therefore $r^2 + 3ru = r^2 + 12r = 45$, by the first equation; also $r^2 + 12r + 36 = 45 + 36 = 81$, by completing the square, and $r + 6 = \sqrt{81} = 9$, by the extraction of roots, consequently $r = 9 - 6 = 3$, and the numbers are 3, 7, 11 and 15.

RECAPITULATION of the PRINCIPLES of ARITHMETIC and ALGEBRA.

AXIOM 1. Since whole numbers increase in a decuple proportion, 10 is the universal ratio of any series of numbers whatever; and the reason for carrying at 10 in Addition and Multiplication is selfevident, since 10 in any place to the right is equal to 1 in the next place to the left. Hence also the reason for carrying according to the subdivisions of any integer when several denominations are to be added.

AXIOM 2. If two whole numbers be equally increased, their difference is always the same. Hence the reason of borrowing 10 in one place to the right, and paying it back by carrying one to the next place. Hence likewise the reason will be evident, for placing the first figure to the right of the product of every particular multiplier directly below its own multiplier.

AXIOM 3. The multiplicand will be increased or diminished in proportion to the multiplier, when the same multiplicand is used. Hence the reason why the multiplicand is increased, when it is multiplied by any thing greater than unity, and decreased, when it is multiplied by a fraction.

AXIOM 4. The dividend will be increased or diminished in proportion to the divisor, when the same dividend is used. Hence, to divide by any thing greater than unity, will quote a number less than the dividend; and, on the contrary, to divide by any thing less than unity, will quote a number greater than the dividend.

AXIOM 5. The whole is equal to all its parts taken together. Hence one sum may be made equal to several by Addition, and subtraction may be proved by adding the difference to the least given sum.

AXIOM 6. If equal quantities be added to, taken from, multiplied or divided by, equal quantities, the sums, remainders, products, and quotients, will respectively be equal. Hence the reason of reducing equations by Addition, Subtraction, Multiplication, and Division, and of abridging commensurable terms, and canceling equal quantities and numbers.

AXIOM 7. To multiply, or divide, any quantity or number by other quantities or numbers continually, is the same as to multiply by the product of those other numbers. Hence the reason of multiplying, or dividing by component parts.

AXIOM 8. If four numbers or quantities be proportional, the rectangle or product of the extremes will be equal to the product of the means ; and *vice versa*, if the product of the extremes be equal to that of the means, the numbers or quantities are proportional.

AXIOM 9. The quotient of any two succeeding powers, when the next higher is divided by the next lower, exhibits the root of these powers. On the contrary, if any power be multiplied by the root of that power, the product will be the next higher power of the root : And if a higher power be divided by the root, the quotient will exhibit the next lower power. Again, if a proportional part of a higher power be divided by a proportional part of the next lower power, the quotient will exhibit a proportional part of the root. Hence the first figure or figures in the root of any power being raised to the power next lower than that whose root is wanted, and that power multiplied by a number expressing the proportion, which the given power bears to its root, produces a proportional divisor, whose ratio, compared with the dividend, is a proportional part of the root, which being annexed to the former part of the root, and raised to the full power of the given number, will be either the whole or a proportional part of the given power, discoverable by Subtraction, &c. Hence we have a general rule for extracting the root of any power whatever.

AN INTRODUCTION TO CONIC SECTIONS.

SECTION I.

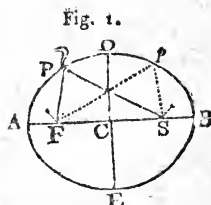
Of the ELLIPSIS.

Definition 1.

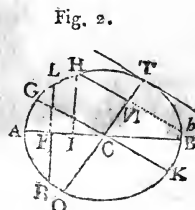
IF two pins be fixed at the points F, S ; and a thread $PSFP$, put about them and knotted at P ; then if the thread be drawn tight, and the point P and the thread be moved about the fixed centres F, S ; the point P will describe the curve $PDpBEAP$, called an *Ellipsis*. See Fig. 1.

Def. 2. The points or centres F, S , are called the *foci*.

Def. 3. The line A, B , drawn through the foci to the curve, is called the *transverse axis*.



Def. 4. The point C in the middle of the axis AB , is the *centre*. See Fig. 2.



Def. 5. The line DE , (drawn through the centre C) perpendicular to the transverse AB , is called the *conjugate axis*. See Fig. 2.

Def. 6. Any line TO , drawn through the centre C to the curve, is called a *diameter*. And the extremity T (or O) its *vertex*.

Def. 7. If TO be a diameter, then the diameter GK , drawn parallel to the tangent at its vertex T , is called its *conjugate*. And the two diameters TO, GK , are said to be *conjugates* to one another.

Def. 8. The line LR (drawn through the focus F , perpendicular to the transverse axis AB ,) is called the *parameter* or *latus rectum*.

Def.

Def. 9. A line drawn from any point of the curve (as HI) perpendicular to the transverse axis, is called an *ordinate* to the transverse. And, in general, any line drawn from the curve to any diameter TO, parallel to its conjugate GK, (as HN,) is an *ordinate* to that principal diameter TO. If it go quite through the figure, as Hh, it is called a *double ordinate*.

Def. 10. A right line meeting the ellipsis in one point M, but does not cut it, is called a *tangent* to it in that point, as TM.

Def. 11. The part of the diameter between the vertex and the ordinate, is called the *abscissa*, TN, AI. And the *vertex* is the extremity of any diameter.

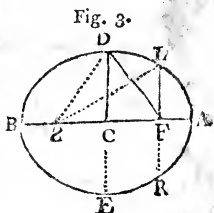
PROPOSITION I. The sum of the lines FP, SP drawn from the foci, to any point of the curve, is equal to the transverse axis AB. See Fig. 1.

For by construction, $PF + PS = AF + AS = AF + AF + FS = 2AF + FS$. And the same $PF + PS = 2BS + FS$; therefore $2AF + FS = 2BS + FS$, and $2AF = 2BS$, or $AF = BS$. Whence $PF + PS = 2AF + FS = AF + BS + FS = AB$.

COR. The two foci are equally distant from the vertices, and also from the centre: $AF = BS$; and $FC = SC$. For it is proved that $AF = BS$; and since $AC = CB$ (Def. 4,) therefore $AC - AF = CB - BS$, or $FC = SC$.

PROP. II. A line drawn from the end of the conjugate axis, to the focus, is equal to half the transverse; $DF = CA$. See Fig. 3.

Draw DS to the other focus. Then the two right angled triangles CDF and CDS are similar and equal. For $SC = CF$, the angles at C are right, and CD common: therefore $SD = DF$; and since the sum $SD + DF =$ the transverse (Prop. 1,) and of them $DF =$ half the transverse CA.



COR. The distance of the foci is a mean proportional between the sum and difference of the transverse and conjugate axis, $SF^2 = \overline{BA + DE} \times \overline{BA - DE}$: For $CA^2 = DF^2 = DC^2 + CF^2$; and $CF^2 = CA^2 - CD^2 = \overline{CA + CD} \times \overline{CA - CD}$; and $4CF^2$ or $SF^2 = \overline{2CA + 2CD} \times \overline{2CA - 2CD}$.

PROP. III. The rectangle of the focal distances, from either vertex, is equal to the square of the semiconjugate: $AF \times FB = DC^2$. See Fig. 3.

For $DC^2 = DF^2 - CF^2 = (\text{Prop. 2.}) \overline{CA^2 - CF^2} = \overline{CA + CF} \times \overline{CA - CF} = \overline{BC + CF} \times \overline{CA - CF} = BF \times FA$.

PROP.

PROP. IV. As the transverse axis to the conjugate, so the conjugate to the latus rectum of the transverse : $AB : DE :: DE : LR$. See Fig. 3.

For $SL + LF = BA = 2CA$ (Prop. 1.) ; and $SL = 2CA - LF$, and by squaring $SL^2 = 4CA^2 - 4CA \times LF + LF^2$. And in the right angled triangle SLE , $SL^2 = SE^2 + LE^2$; whence $4AC^2 - 4CA \times LF + LF^2 = SE^2 + LF^2$, and $4AC^2 - 4CA \times LF = SE^2 + LF^2 - LF^2 = SE^2$, and $4AC^2 = 4CA \times LF + SE^2 = 4CA \times LF + 4DF^2 - 4DC^2$, and $4AC^2 + 4DC^2 = 4CA \times LF + 4DF^2$; but $CA^2 = DF^2$ (Prop. 2.) ; therefore $4DC^2 = 4CA \times LF = 2CA \times 2LF$; that is, $DE^2 = BA \times LR$.

COR. 1. As the semitransverse is to the semiconjugate, so the semiconjugate to half the latus rectum ; $CA : DC :: DC : LF$.

COR. 2. As the semitransverse, to the distance of the focus from the centre ; so is the same distance, to the difference between the semitransverse and half the latus rectum : $FC^2 = CA \times CA - LF$.

For $CF^2 = DF^2 - DC^2 = CA^2 - CD^2 = CA^2 - CA \times LF$.

COR. 3. The rectangle $BFA =$ half the transverse \times half the latus rectum $= CA \times FL$. By Cor. 1. and Prop. 3. See Fig. 3.

SCHOLIUM. Since the transverse axis is to the conjugate, so the conjugate to the latus rectum, of the transverse axis. Therefore, in any other diameters, the third proportional, to the diameter and its conjugate, is called the *latus rectum* of that diameter.

PROP. V. From any point M in the curve, drawing the line MF, MS , to the two foci ; and the ordinate MP perpendicular to the transverse axis BA ; it will be,

As the semitransverse, CA :

To the distance of the focus from the centre, CF ::

So the distance of the ordinate from the centre, CP :

To half the difference of the lines MF, MS , or $\frac{MS - MF}{2}$.

For, make $SD = CA$, then $SM = CA + DM$, and $FM = 2CA - SM = CA - DM$.

In the right angled triangle SMP , SM^2 or $CA^2 + 2CA \times DM + DM^2 = SP^2 + PM^2 =$

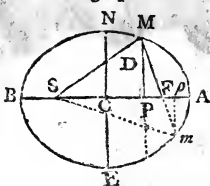
$CF^2 + CP^2 + PM^2 = CF^2 + 2CF \times CP + CP^2 + PM^2$, and in the right angled triangle FMP ,

FM^2 or $CA^2 - 2CA \times DM + DM^2 = FP^2 + PM^2 = CF^2 - 2CF \times CP + CP^2 + PM^2$; then subtracting the latter equation from the former, $SM^2 - FM^2 = 4CA \times DM = 4CF \times CP$, and $CF \times CP = CA \times DM$. But since $SM = CA + DM$, and $FM = CA - DM$;

therefore $SM - FM = 2DM$; therefore $CF \times CP = CA \times \frac{SM - FM}{2}$.

COR. 1. If F, S be the foci, MP an ordinate ; then it is $CA : CF :: CP : CA - MF$ or $SM - CA$. See Fig. 4.

Fig. 4.



For

For $CF \times CP = CA \times DM$, and $DM = SM - CA = CA - FM$.

COR. 2. If F, S , be the foci, MP an ordinate; then the difference of the squares of the lines SM, FM ; that is, $SM^2 - FM^2 = 4CF \times CP$.

COR. 3. If F, S , be the foci, MP an ordinate; then $CA \times SM - FM = 2CF \times CP$.

For $SM^2 - FM^2 = \overline{SM + FM} \times \overline{SM - FM} = 2CA \times \overline{SM - FM} = 4CF \times CP$, and $CA \times \overline{SM - FM} = 2CF \times CP$.

SCHOLIUM. If PM fall on the other side of F , as pm , then $pF = Cp - CF$, and its square the same as before, and the rest of the demonstration the same.

PROP. VI. If an ordinate MP be drawn to the transverse axis; it will be,

As the square of the transverse, BA^2 :

To the square of the conjugate, NE^2 ::

So the rectangle of the segments of the transverse BPA :

To the square of the ordinate, PM^2 . See Fig. 4.

For make $SD = CA$, then DM is half the difference of SM and MF ; therefore by Prop. 5. $CA : CF :: CP : DM$, and $CA : CA + CF$ or $BF :: CP : CP + DM$, and $CA : CP :: BF : CP + DM$, and $CA : CA + CP$ or $BP :: BF : BF + CP + DM$. But $BF = BC + CF = SD + CF$; and $BF + CP + DM = SD + CF + CP + DM = SM + CS + CP = SM + SP$; whence $CA : BP :: BF : SM + SP$. Again, since $CA : CF :: CP : DM$; then $CA : (CA - CF) AF :: CP : CP - DM$; and $CA : CP :: AF : CP - DM$. And $CA : (CA - CP) PA :: AF : AF - CP + DM$. But $AF = CA - CF = SD - SC$; therefore $AF - CP + DM = SD - SC - CP + DM = SM - SP$; therefore $CA : PA :: AF : SM - SP$, and we had before, $CA : BP :: BF : SM + SP$; then multiplying these proportions together, we have $CA^2 : BP \times PA :: BF \times FA : SM^2 - SP^2$.

But (Prop. 3.) $BF \times FA = CN^2$; and $SM^2 - SP^2 = PM^2$; therefore $CA^2 : BPA :: CN^2 : PM^2$, or alternately, $CA^2 : CN^2 :: BPA : PM^2$, or $BA^2 (4CA^2) : NE^2 (4CN^2) :: BPA : PM^2$.

COR. 1. $CA^2 : CN^2 :: BFA : PM^2$.

COR. 2. As the transverse BA : to its latus rectum :: So the rectangle BPA : to square of the ordinate PM^2 .

For (Prop. 4.) latus rectum $= \frac{NE^2}{AB}$, whence, since $BA^2 : EN^2 ::$

$BFA : PM^2$, therefore, $BA : \frac{NE^2}{BA}$ or latus rectum :: $BFA : PM^2$.

COR. 3. The rectangles of the segments of the transverse are as the squares of the ordinates.

For every rectangle is to the square of its ordinate, in the given ratio of CA^2 to CN^2 , or of BA to the latus rectum.

COR,

COR. 4. As the square of the semitransverse CA^2 :
 Rectangle of the focal distances from vertex BFA : :
 So rectangle of the segments BPA :
 To square of the ordinate PM^2 .

SECTION II.

Of the PARABOLA.

Definition 1. If one end of a thread, equal in length to CH , be fixed at the point F , and the other end fixed at H , the end of the square DCH . And if the side CD of the square be moved along the right line BD , and always coincide with it, then, if the string FGH be always kept tight, and close to the side GH of the square, the point or pin G (where it leaves the square) will describe a curve $MRALGK$ called a Parabola. See Fig. 5.

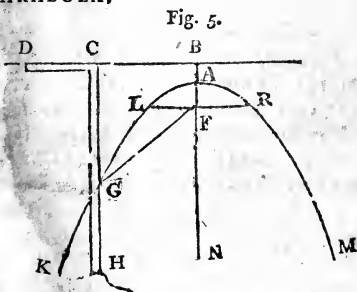


Fig. 5.

Def. 2. The fixed point F is called the *focus*.

Def. 3. The right line BD is called the *directrix*.

Def. 4. If the line BN be drawn through the focus F , perpendicular to BD ; then AN is called the *axis* of the parabola, and A the *vertex*.

Def. 5. A line drawn through the focus F , perpendicular to the axis, as LR , is called the *parameter* or *latus rectum*.

Def. 6. Any line drawn within the curve, parallel to the axis, as GH , is called a *diameter*. And the point G , where it cuts the curve, is the *vertex*.

Def. 7. A right line drawn from any diameter to the curve, and parallel to the tangent at the vertex, as PM , is called an *ordinate*. If it go quite through the curve, it is called a *double ordinate*. See Fig. 6.

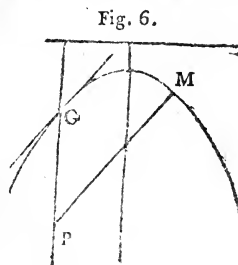
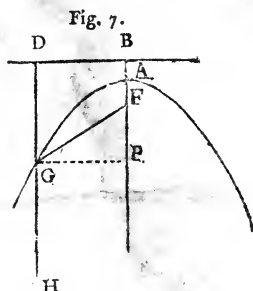


Fig. 6.

Def. 8. The part of the diameter between the vertex and ordinate, as GP, is called the *abscissa*.

Def. 9. A right line meeting the curve in one point G, but does not cut it, is called a *tangent* in that point.

PROPOSITION I. If BD be the directrix, G any point in the curve, the line GD drawn to the directrix, parallel to the axis, is equal to the line GF drawn from the same point G to the focus; $GD = GF$. See Fig. 7.



For $HG + GF = \text{length of the string} = HD$; take away GH from both, and then $GD = GF$.

COR. 1. The distances of the focus, and of the directrix from the vertex are equal. $AB = AF$. For when D is at B, G will be at A; consequently $AB = AF$.

COR. 2. If GP be an ordinate to the axis; then $AP + AF = FG$; For $AP + AF = BP = GD$.

COR. 3. $FG - FP = \text{half the latus rectum}$.

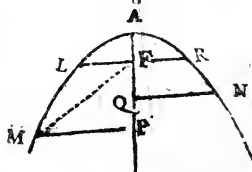
PROP. II. The distance of the focus from the vertex is $\frac{1}{4}$ the latus rectum: $AF = \frac{1}{4}LR = \frac{1}{2}LF$. See Fig. 5.

For when the pin G comes to L, then $LF = FB$ (Prop. 1. Cor. 1.) $= 2FA$, and $AF = \frac{1}{2}FL$. For the same reason $FA = \frac{1}{2}FR$, therefore $FA = \frac{1}{4}LR$.

SCHOLIUM. As the latus rectum to the axis is four times the distance of the vertex A from the focus F: So in any other diameter GH, four times the distance of its vertex from the focus, or AFG, is called its *latus rectum*.

PROP. III. The square of any ordinate to the axis is equal to the rectangle of the latus rectum and abscissa: $PM^2 = LR \times AP$. See Fig. 8.

Fig. 8.



For $MF = AF + AP = (\text{Prop. 2.}) AP + \frac{1}{4}LR$, and $FP = AP - AF = AP - \frac{1}{4}LR$. And in the right angled triangle MFP , $MP^2 = MF^2 - FP^2 = MF + FP \times MF - FP = 2AP \times \frac{1}{2}LR = AP \times LR$.

COR. 1. If F be the focus, $MP^2 = AP \times 4AF$.

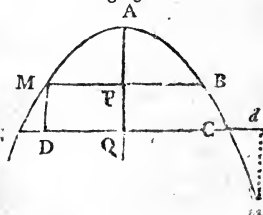
COR. 2. The abscissas are as the squares of their ordinates.

$AP : AQ :: PM^2 : QN^2$. For $AP : AQ :: AP \times LR : AQ \times LR :: PM^2 : QM^2$.

COR. 3. The latus rectum is a third proportional to the abscissa and ordinate. $AP : PM : LR ::$.

PROP. IV. As the latus rectum to the sum of any two ordinates ; so their difference, to the difference of the abscissæ. Lat. rect. : $CD :: ND : PQ$. See Fig. 9.

Fig. 9.



Let $L =$ latus rectum, then (Prop. 3.) $L \times AP = PM^2$; and $L \times AQ = NQ^2$. And by subtraction, $L \times AQ - L \times AP = NQ^2 - PM^2$; therefore $L : NQ + PM :: NQ - PM : AQ - AP$; that is, $L : DC :: ND : PQ$.

COR. 1. If MD be the axis, NC an ordinate to it; then the rectangle $NDC = MD \times$ parameter.

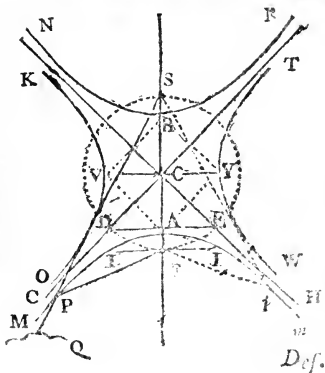
COR. 2. The rectangle NDC is every where as MD .

S E C T I O N III.

Of the HYPERBOLA.

Fig. 10.

Definition 1. If the ends of two threads SPQ , FPO , be fastened at the points S , F , and be made to pass through a small bead, or pin P , and knotted together at Q ; then taking hold of Q , and drawing the threads tight; if the bead be moved along the threads, the point P will describe the curve mp APM , called an hyperbola. See Fig. 10



Def. 2. And if the end of the long thread be fixed at F, and that of the short one at S; and the curve NBR be described after the same manner, that curve is called the *opposite hyperbola*; and both curves together, MAm, NBR, are called *opposite sections*, or *opposite hyperbolas*.

Def. 3. The two fixed points F, S, are called the *foci*.

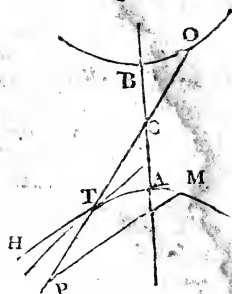
Def. 4. The line AB (passing through the foci, when continued) contained between the two parts of the curve, is called the *transverse axis*.

Def. 5. The middle point of AB, that is, C, is called the *centre* of the hyperbola, or of the opposite sections.

Def. 6. If VY be drawn through the centre C perpendicular to AB; and with radius CF, and centre A, an arch be described, cutting VY in V, and Y; then VY is called the *conjugate axis*.

Fig. 11.

Def. 7. Any line TO drawn through the centre C, and terminated at the opposite sections, is called a *diameter*; and the extremity T (or O) its *vertex*. And the line drawn through the centre, parallel to the tangent at the vertex, is called its *conjugate diameter*. See Fig. 10.



Def. 8. If any diameter OT be continued within the curve, the part within, TP, is called the *abscissa*.

Def. 9. Any line PM, drawn parallel to the tangent at the vertex T, and terminated at the abscissa and curve, is called an *ordinate* to that diameter TO. And if it go quite through the curve, it is called a *double ordinate*.

Def. 10. The line LI, drawn through the focus F, perpendicular to the transverse axis AB, and terminating at the curve, is called the *parameter* or *latus rectum*. See Fig. 10.

Def. 11. If the ends of the two axes be joined by the lines BY, BV; and through the centre C, two lines CH, CG, be drawn parallel to BY, BV; or (which is the same) if VY be placed at A, perpendicular to BA; and the lines CH, CG, be drawn from the centre C, through the ends E, D; these lines CH, CG, are called the *asymptotes* of the hyperbola, or of the opposite hyperbolas.

Def.

COR. 1. As the semitransverse, to the semiconjugate ; so the semiconjugate to half the latus rectum, $CA : CY : LF$.

COR. 2. As the semitransverse to the distance of the focus from the centre ; so is the same distance, to the sum of the semitransverse and half the latus rectum, $CA : CF :: CF : CA + LF$.

For, (Prop. 2.) $CF^2 = CA^2 + CY^2 = CA^2 + CA \times LF = CA \times CA + FL$.

COR. 3. The rectangle $BFA = \frac{1}{2}$ transverse $\times \frac{1}{2}$ latus rectum $= CA \times FL$. By Cor. 1. and Prop. 3.

SCHOLIUM. Since the transverse axis is to the conjugate, as the conjugate to the latus rectum of the transverse axis ; therefore, in any other diameters, the third proportional, to any diameter and its conjugate, is called the *latus rectum of that diameter*. Therefore, in a right angled hyperbola, every diameter is equal to its latus rectum.

F I N I S.

234



99566

QA

101

p55

1797

copy 2

